The Convex Hull of Points on a Sphere is a Spanner

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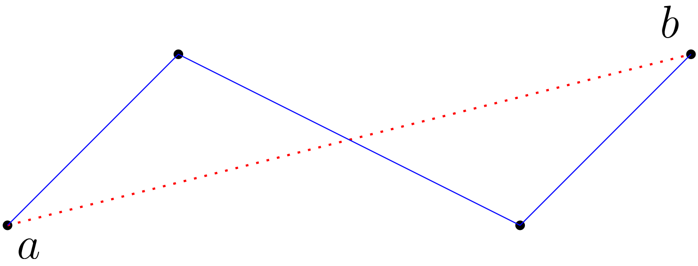
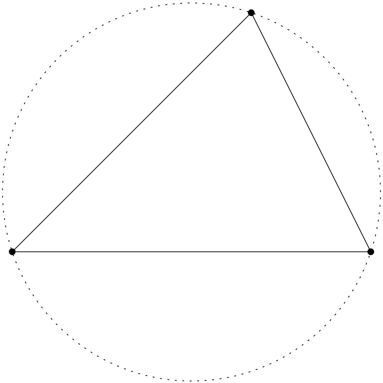
Euclidean spanners

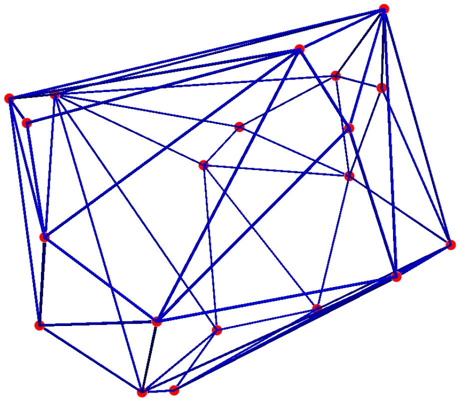


$$d_G(a, b) \leq t \cdot d(a, b)$$

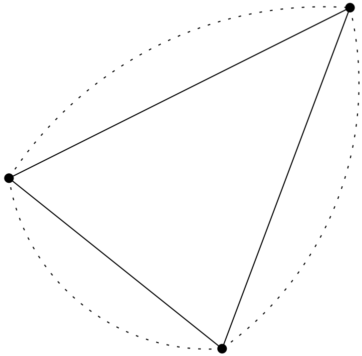
Dobkin et al. 1987



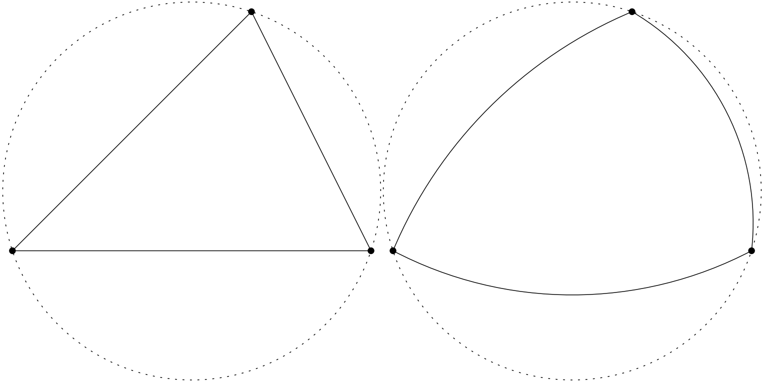
Raghavan's suggestion



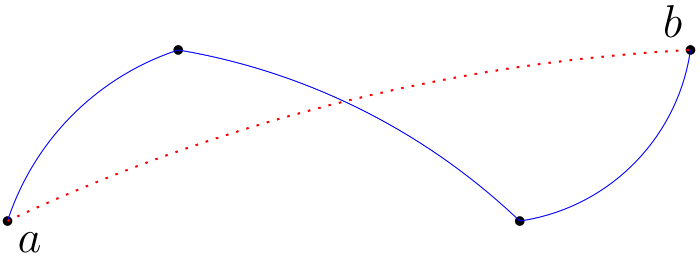
Convex hull and spherical Delaunay



Spherical Delaunay Triangulation

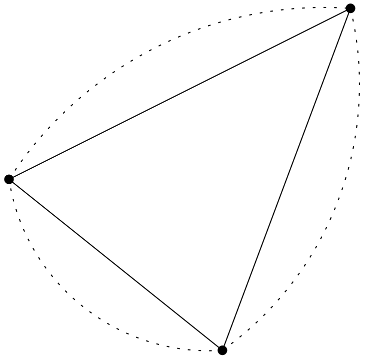


Spherical spanners



$$\breve{d}_G(a, b) \leq t \cdot \breve{d}(a, b)$$

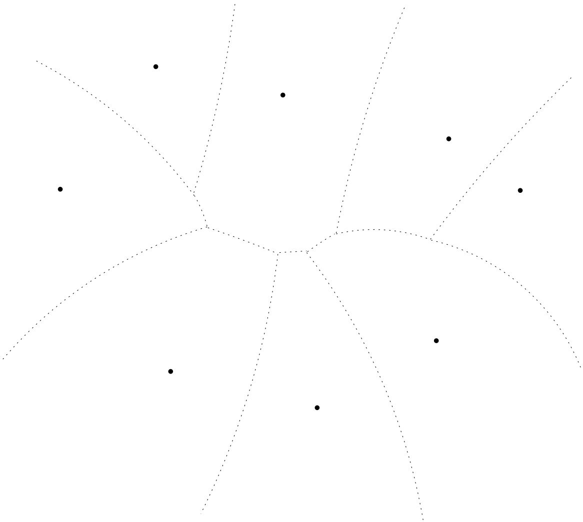
Convex hull and spherical Delaunay



So if we prove the SDT is a t-spanner, then the CH is a $(\pi/2)t$-spanner.

Constructing a path

Constructing a path



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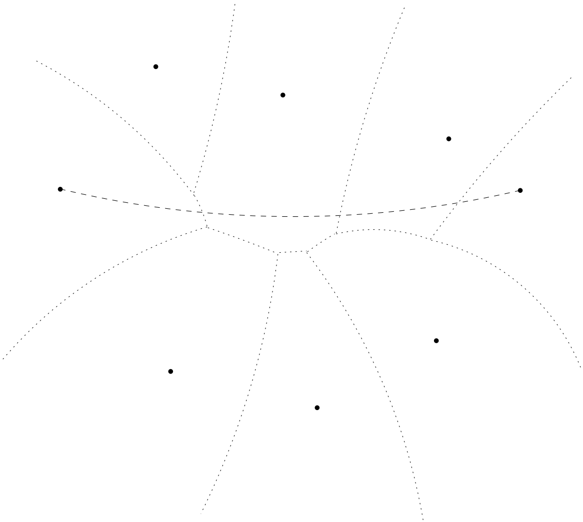

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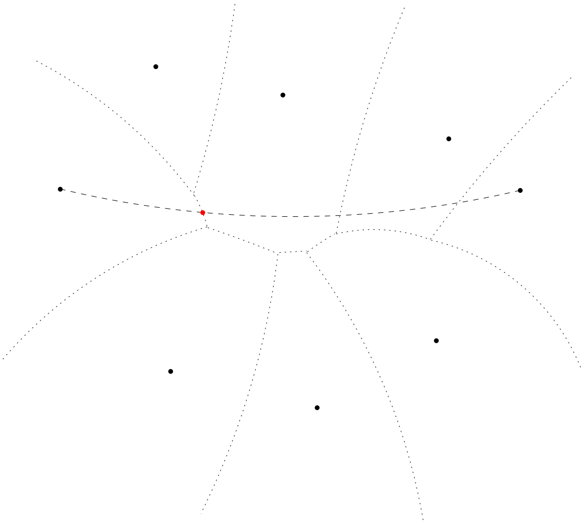

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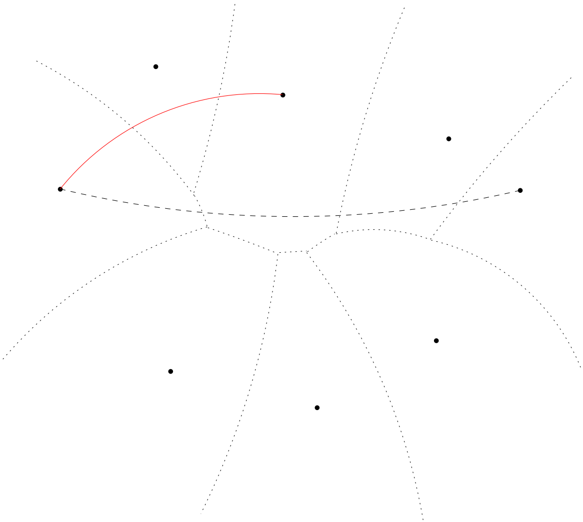
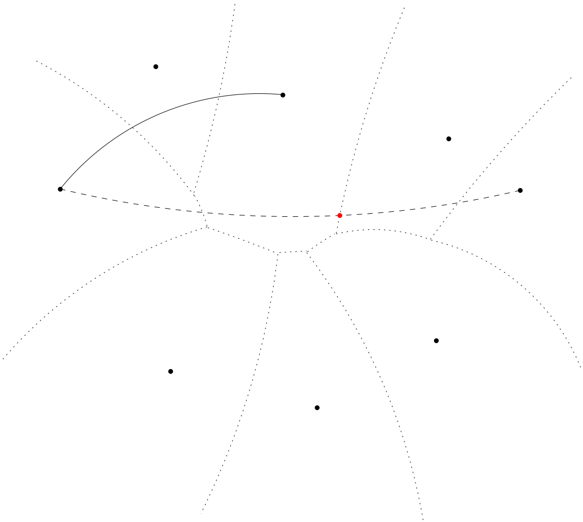


Constructing a path



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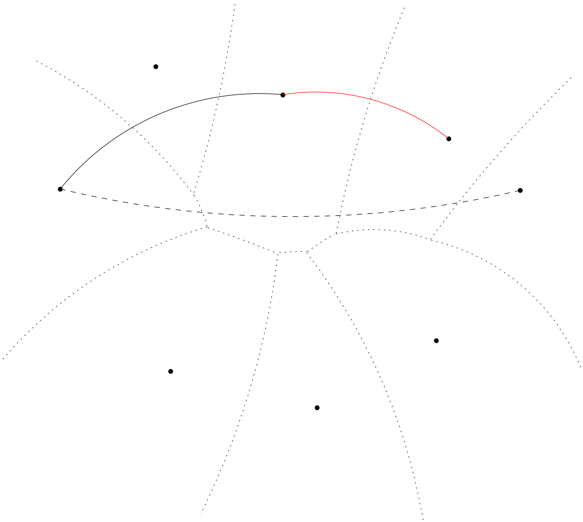

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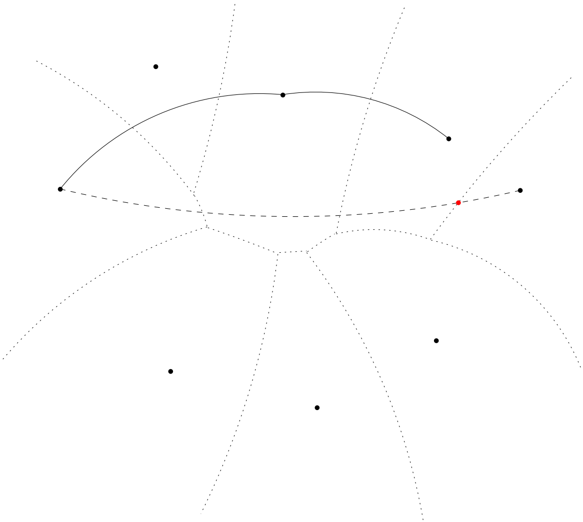

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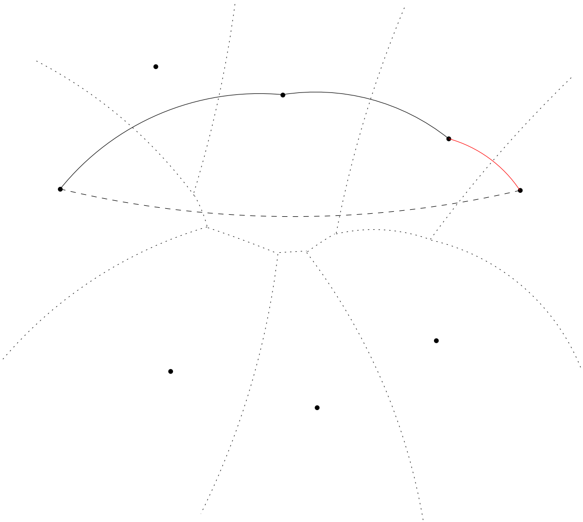

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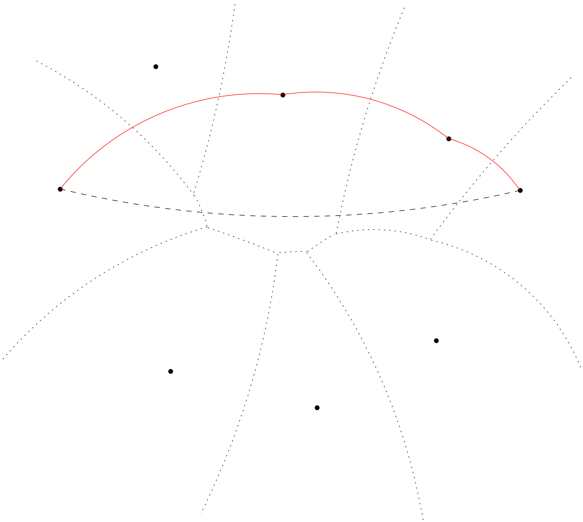
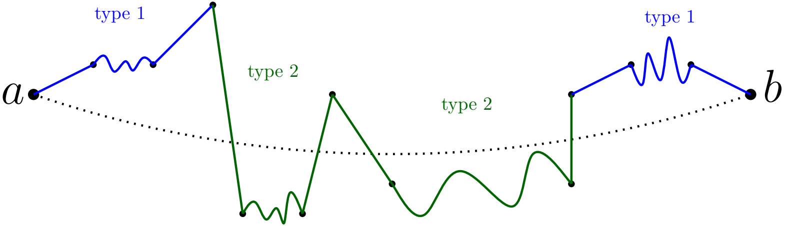
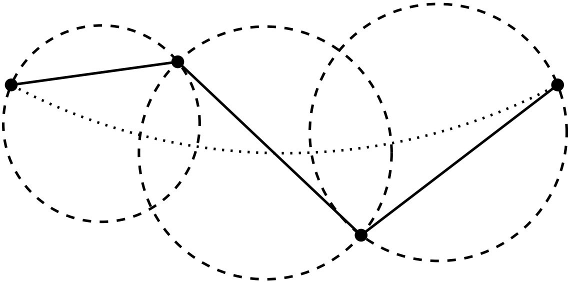


Deconstructing a path into subpaths



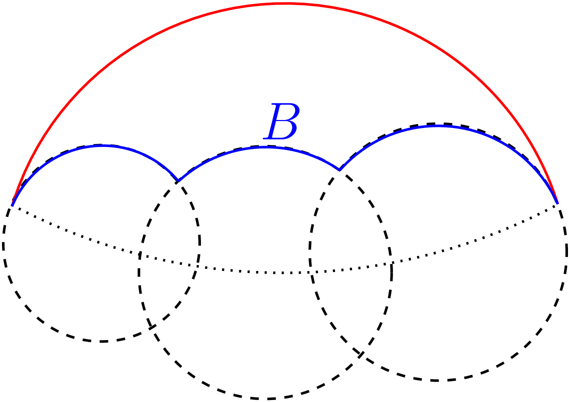
Useful spherical caps



For each pair of points on a path, draw a spherical cap such that:

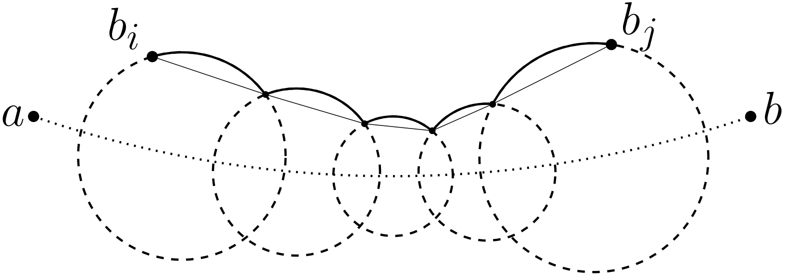
- the center is at the intersection of the direct arc and the Voronoi boundary
- the pair of points are on the boundary

Boundary of unioned spherical caps is bounded

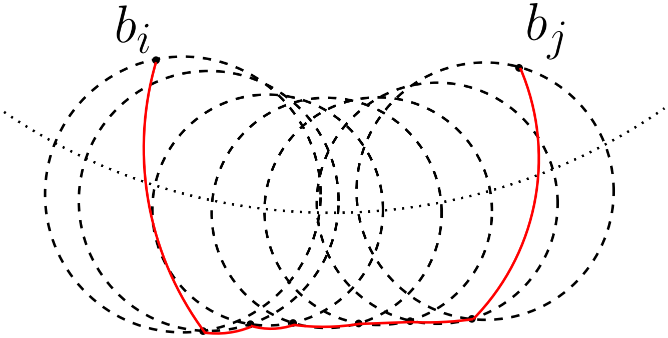


$B \leq \pi/2 \cdot d(a, b)$

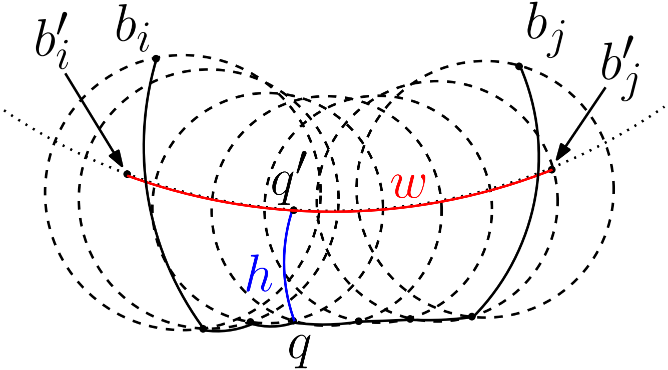
Type 1 subpaths



Type 2 subpaths

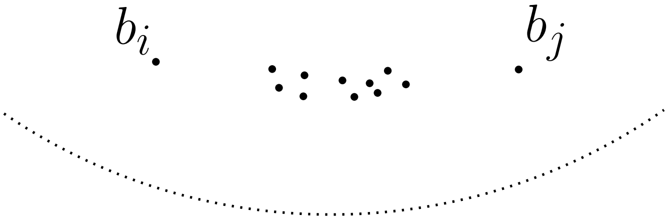


Type 2 subpaths



Some type 2 subpaths are not very wide with respect to their height. Specifically: $h > w/4$.

Constructing a shortcut subpath



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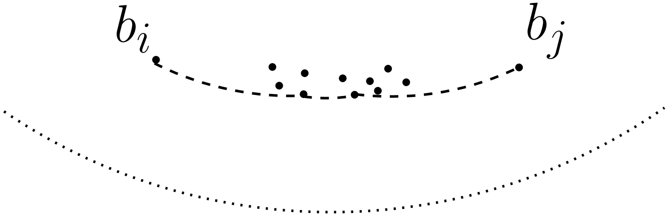
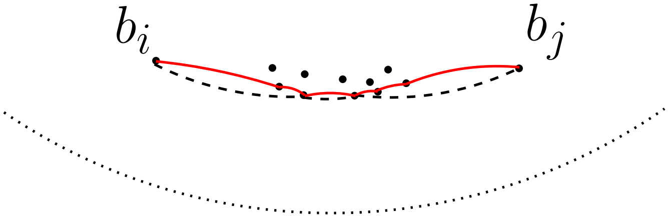


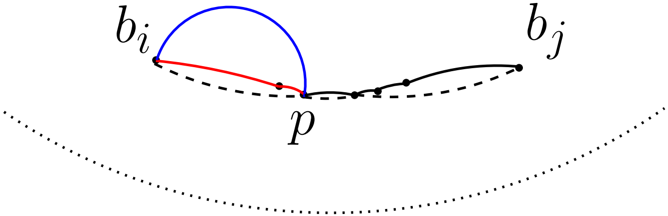
Take the lower spherical convex hull on those points.

Constructing a shortcut subpath



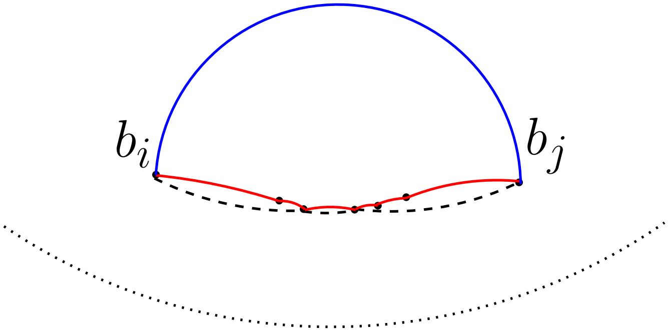
Construct a path between each pair of points on the hull by following spherical Delaunay edges corresponding to Voronoi regions.

Shortcuts have bounded length



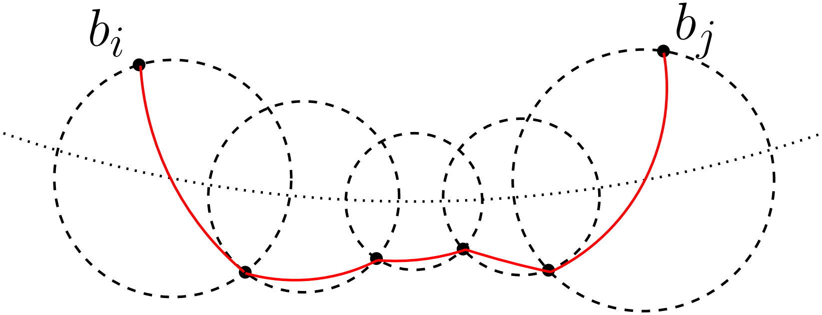
$$\breve{d}_G(b_i, p) \leq \pi/2 \cdot \breve{d}(b_i, p)$$

Shortcuts have bounded length

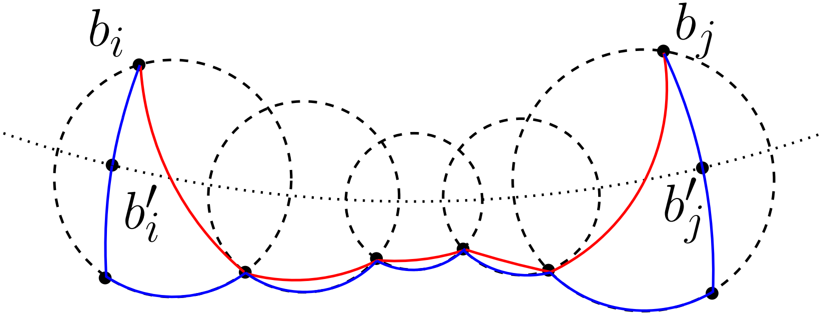


$$\color{red}{\breve{d}_G(b_i, b_j)} \leq \pi/2 \cdot \color{blue}{\breve{d}(b_i, b_j)}$$

Wide type 2 subpaths



Wide type 2 subpaths



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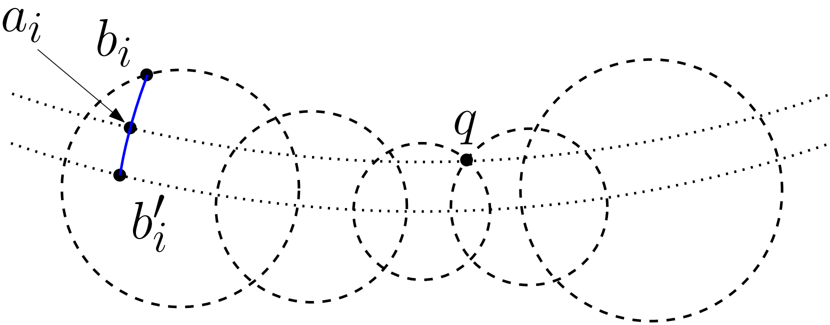

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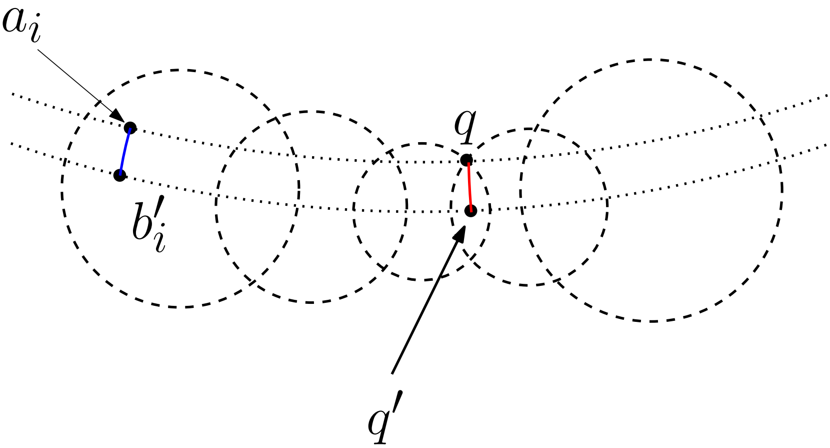

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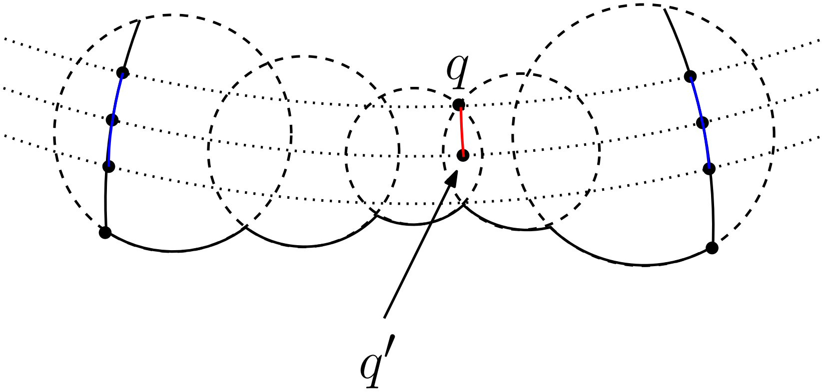

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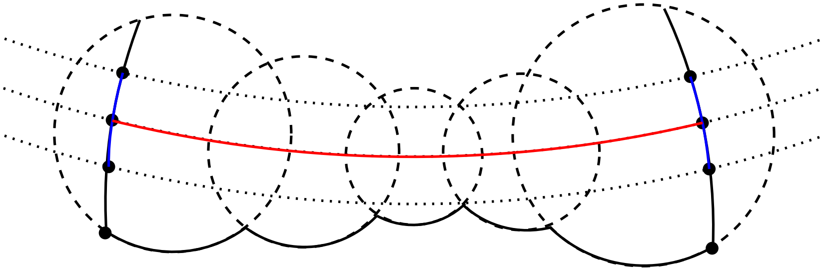

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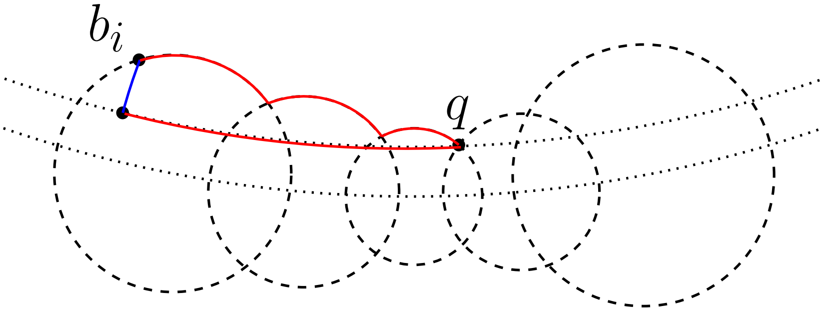

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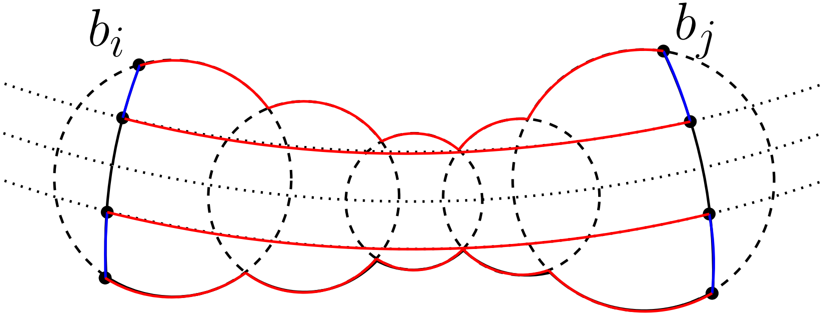

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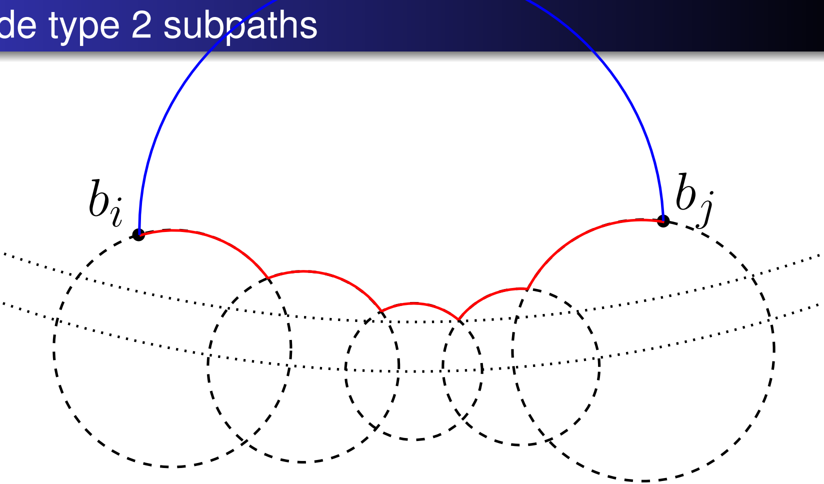

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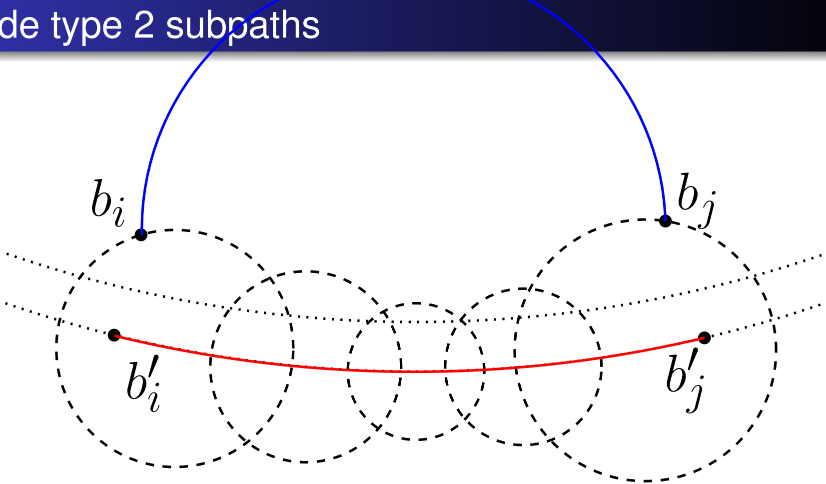

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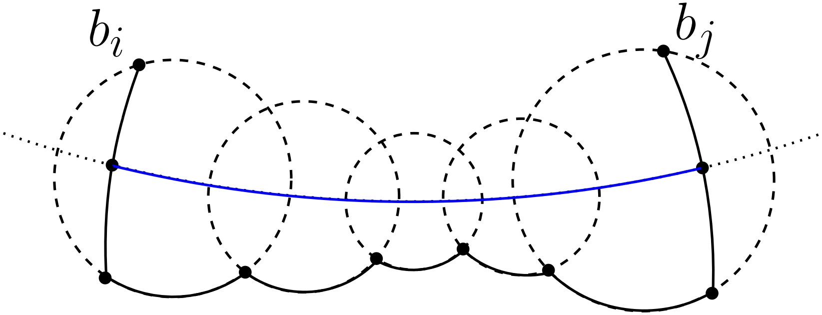
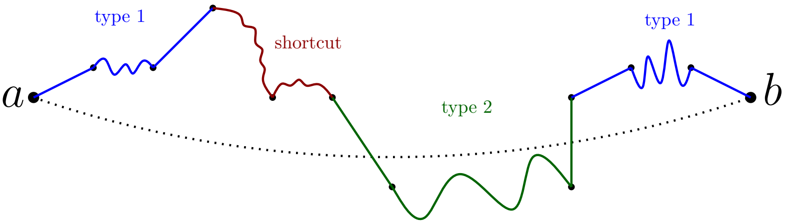


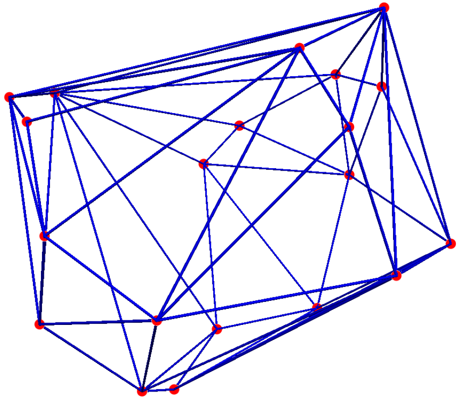
$$\breve{d}_G(b_i, b_j) \leq 3(\pi/2 + 1)\breve{d}(b_i', b_j')$$

The total length of a path is bounded



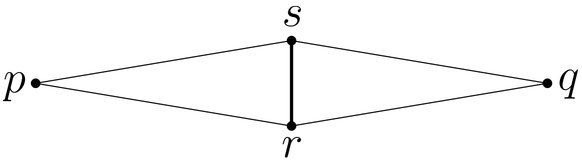
$$\breve{d}_G(a,b) \leq 3(\pi/2+1) \cdot \breve{d}(a,b)$$

The convex hull of points on a sphere is bounded



$$d_G(a, b) \leq 3(\pi/2)(\pi/2 + 1) \cdot d(a, b)$$

The convex hull of points *almost* on a sphere is *not* a spanner



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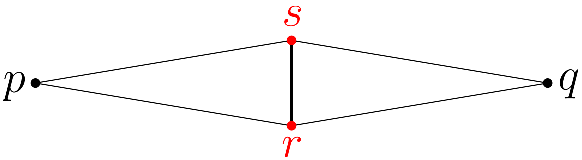

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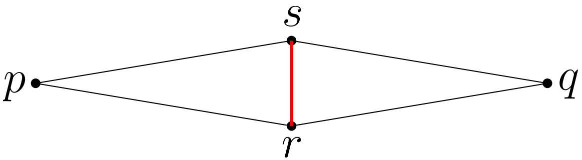

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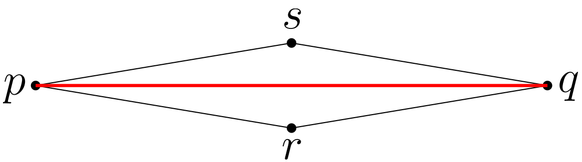

That's All!

Are there any questions?