Three Approaches to Building Time-Windowed Geometric Data Structures

Simon Pratt

MMath Thesis

Problem Statement

Preprocess a set of n geometric objects each associated with a time value, s.t.:

given a query time window $[t_1, t_2]$, solve a geometric problem on the subset of objects whose associated time values are within that window.

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e.g. closest pair

Motivation

- *Geographic Information Systems* (GIS) data
  (e.g., latitude, longitude, altitude, time)
- Timestamped data
  (e.g., social network data)

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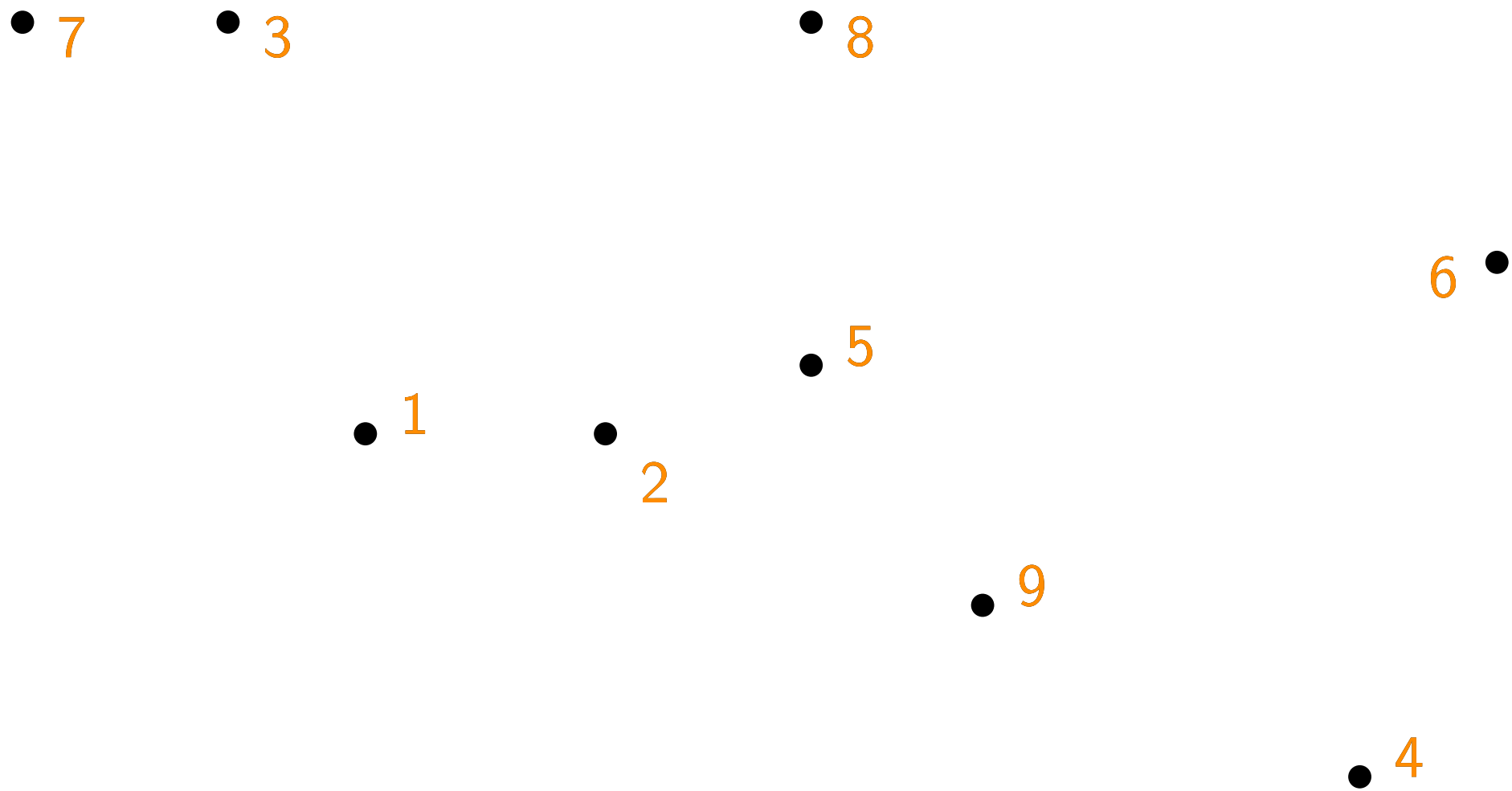

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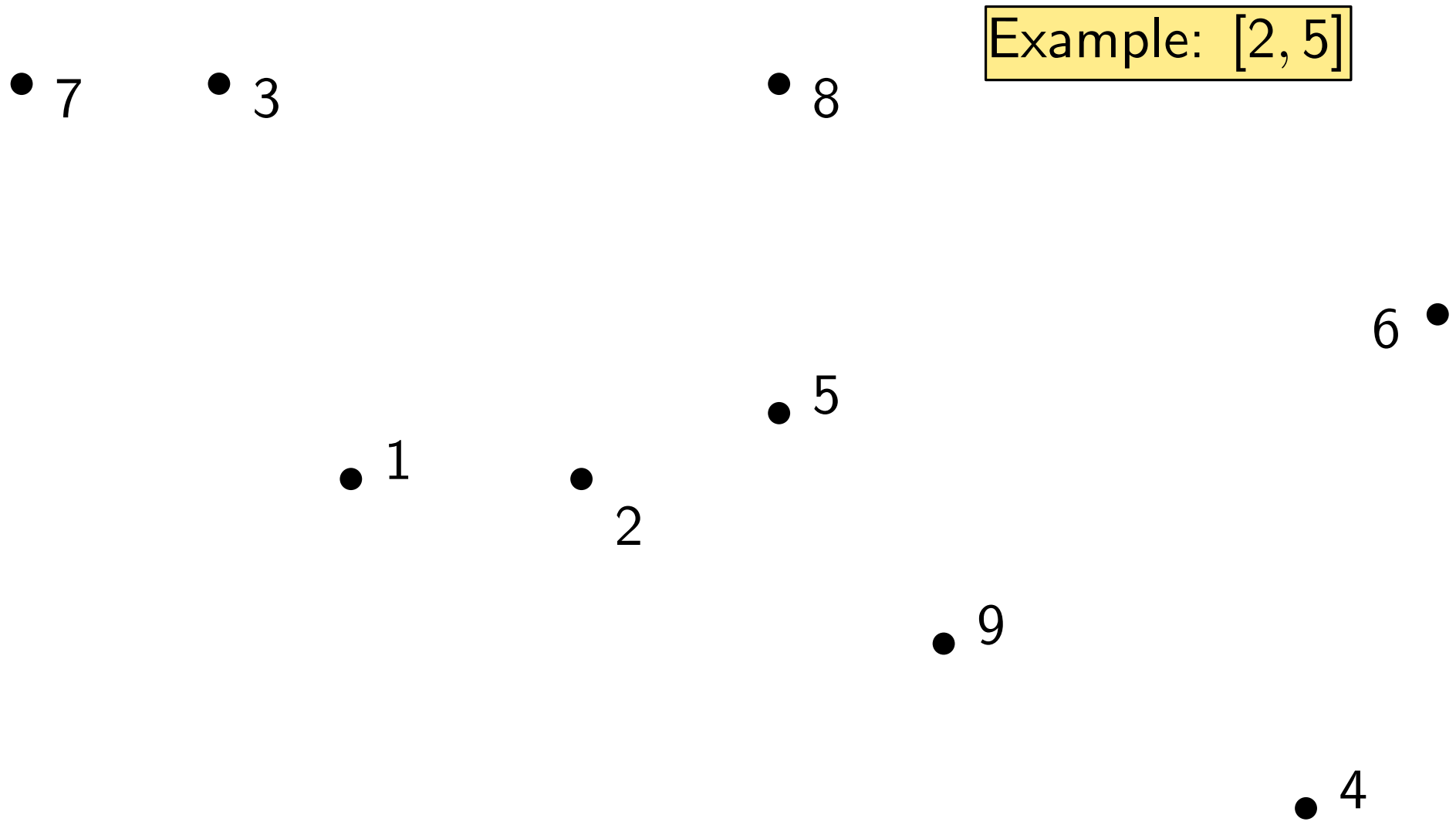

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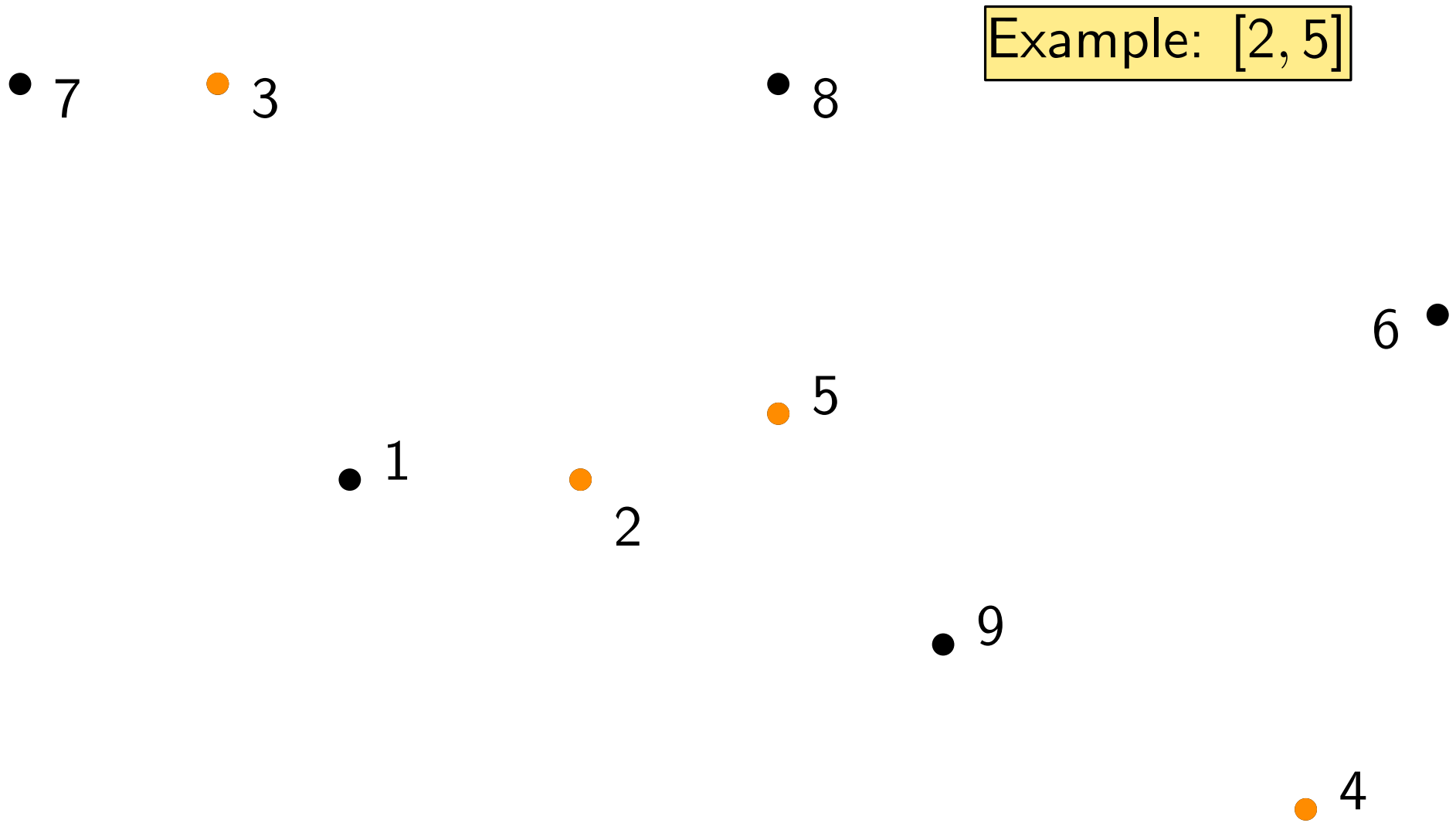

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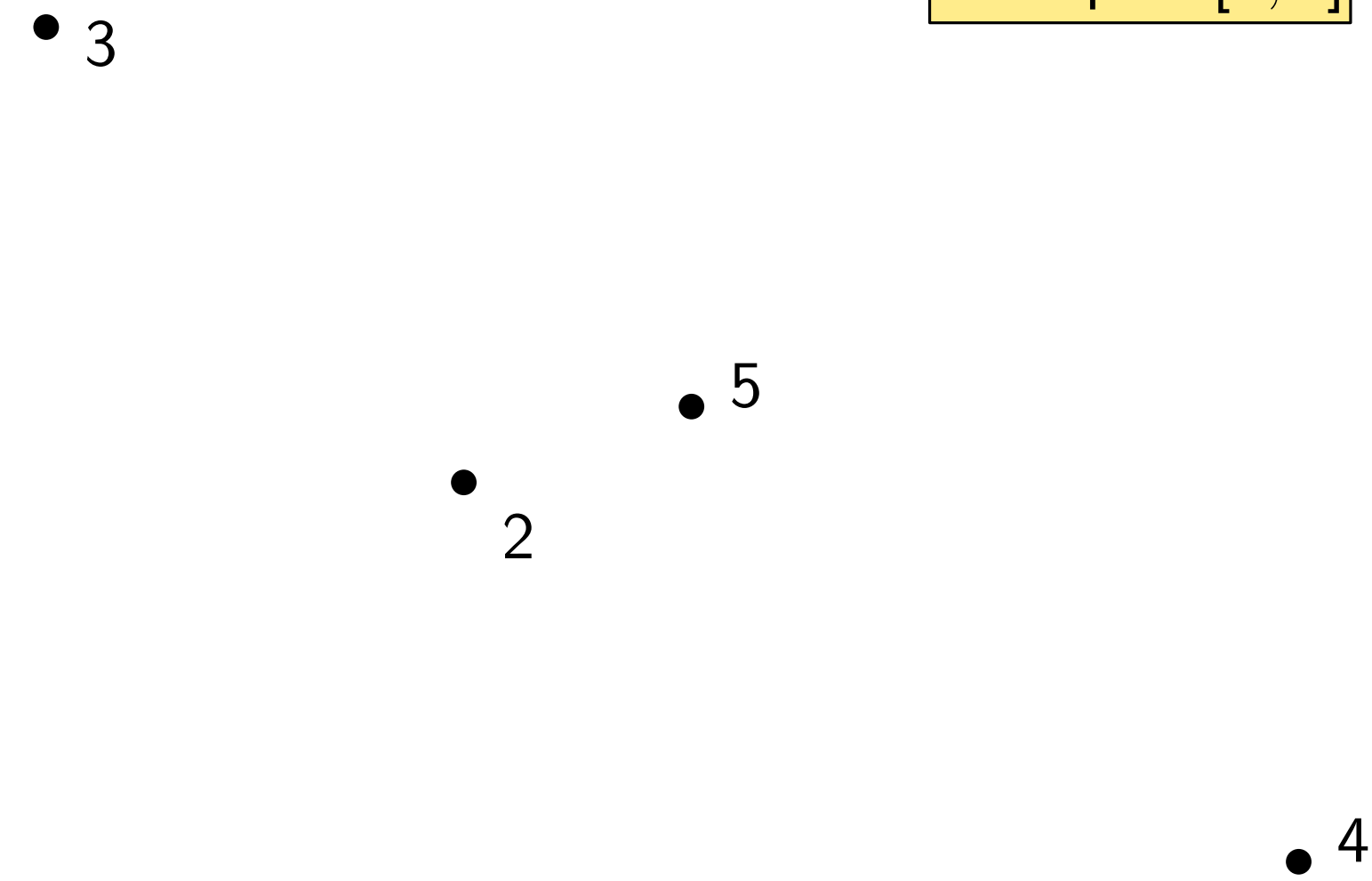

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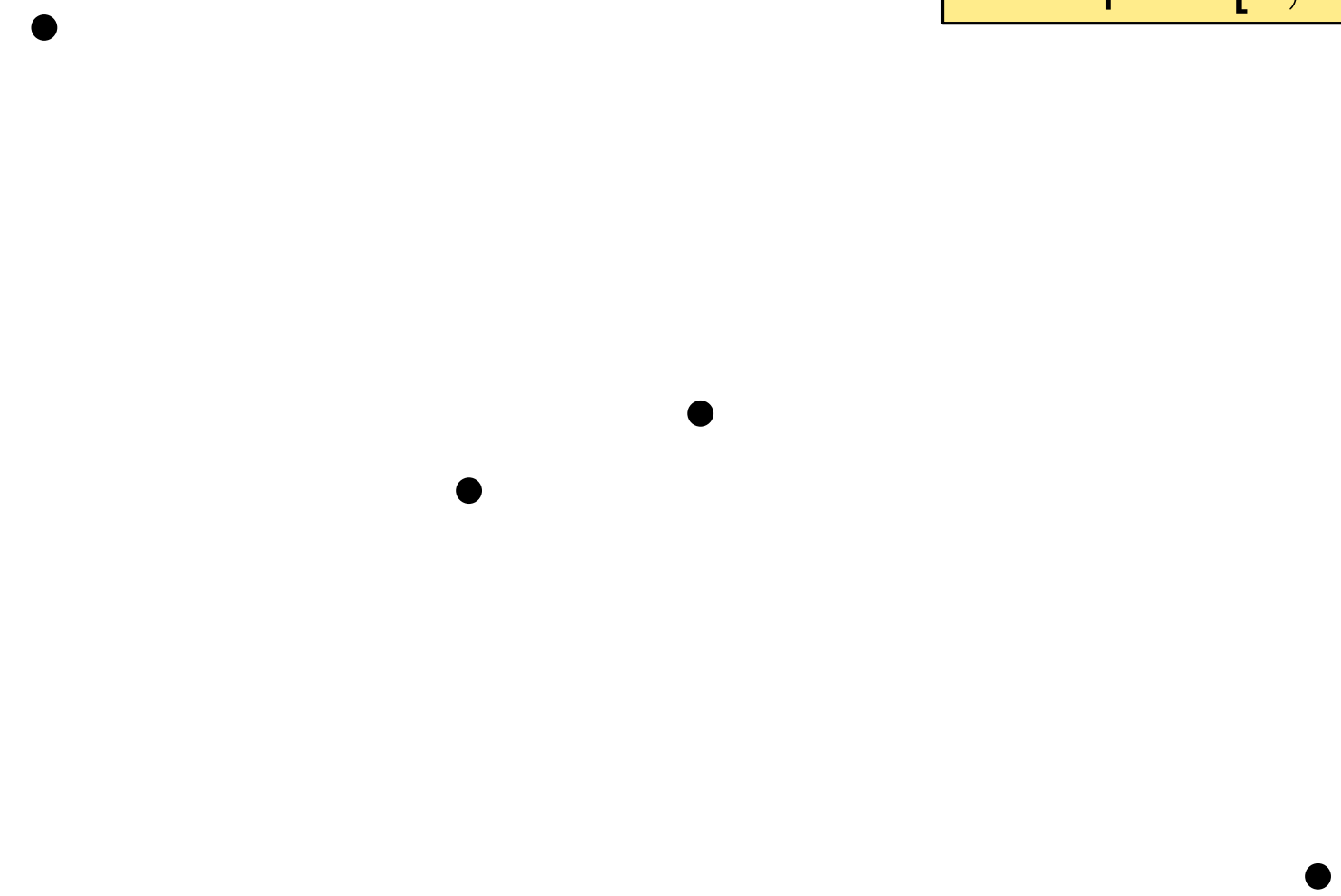

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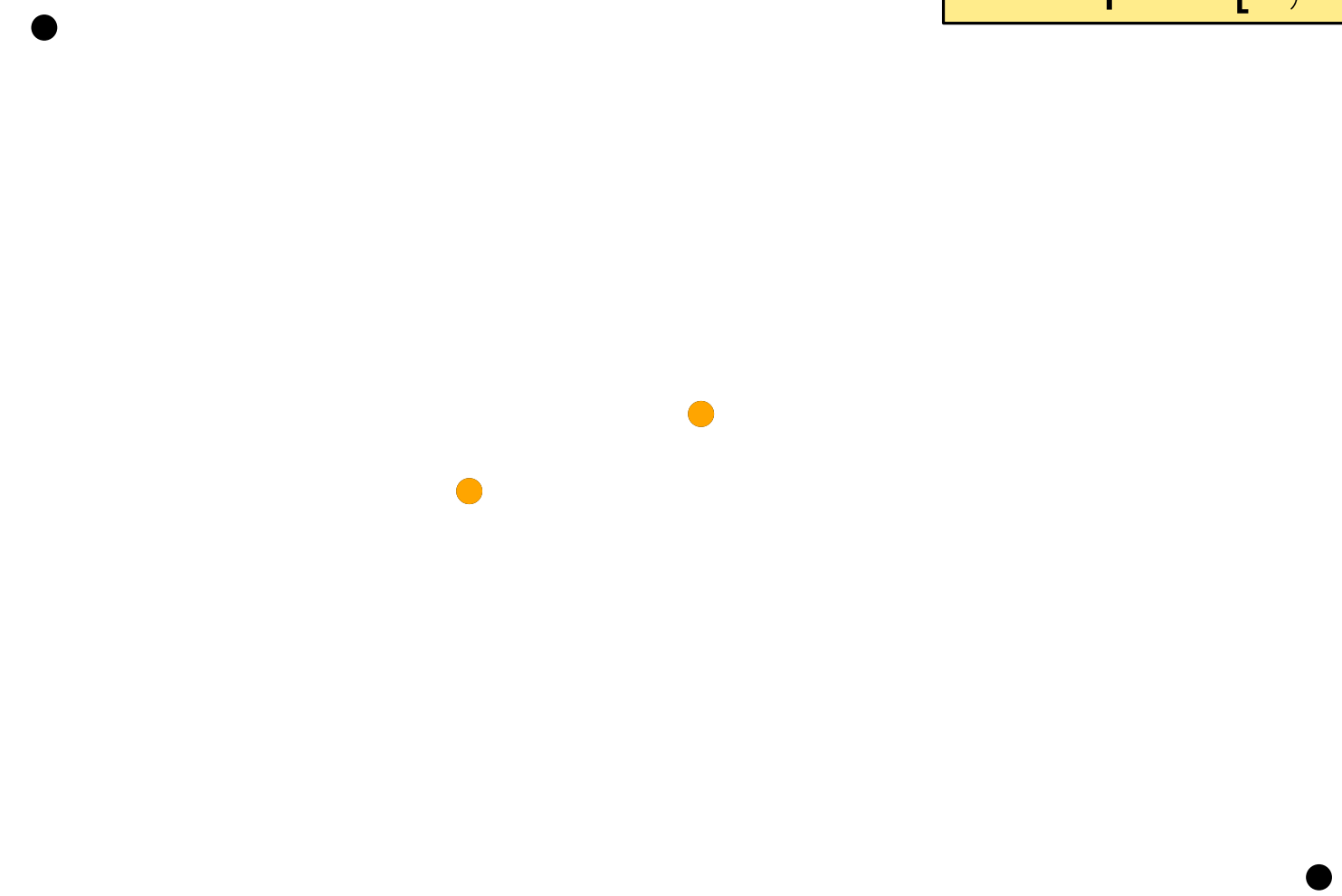

Previous Results

Bannister, Devanny, Goodrich, Simons, Trott; *CCCG 2014*:

- Time-windowed convex hull
  - ◇ Gift wrapping & tangent $O(\log^2 \omega)$
  - ◇ Linear programming $O(\log \omega)$
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$\omega = t_2 - t_1$, the size of the query time window

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Approach #1: Grids & Quadtrees

Problem	Preprocessing	Space	Query
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3D diameter decision	$O(n \lg^2 n)$
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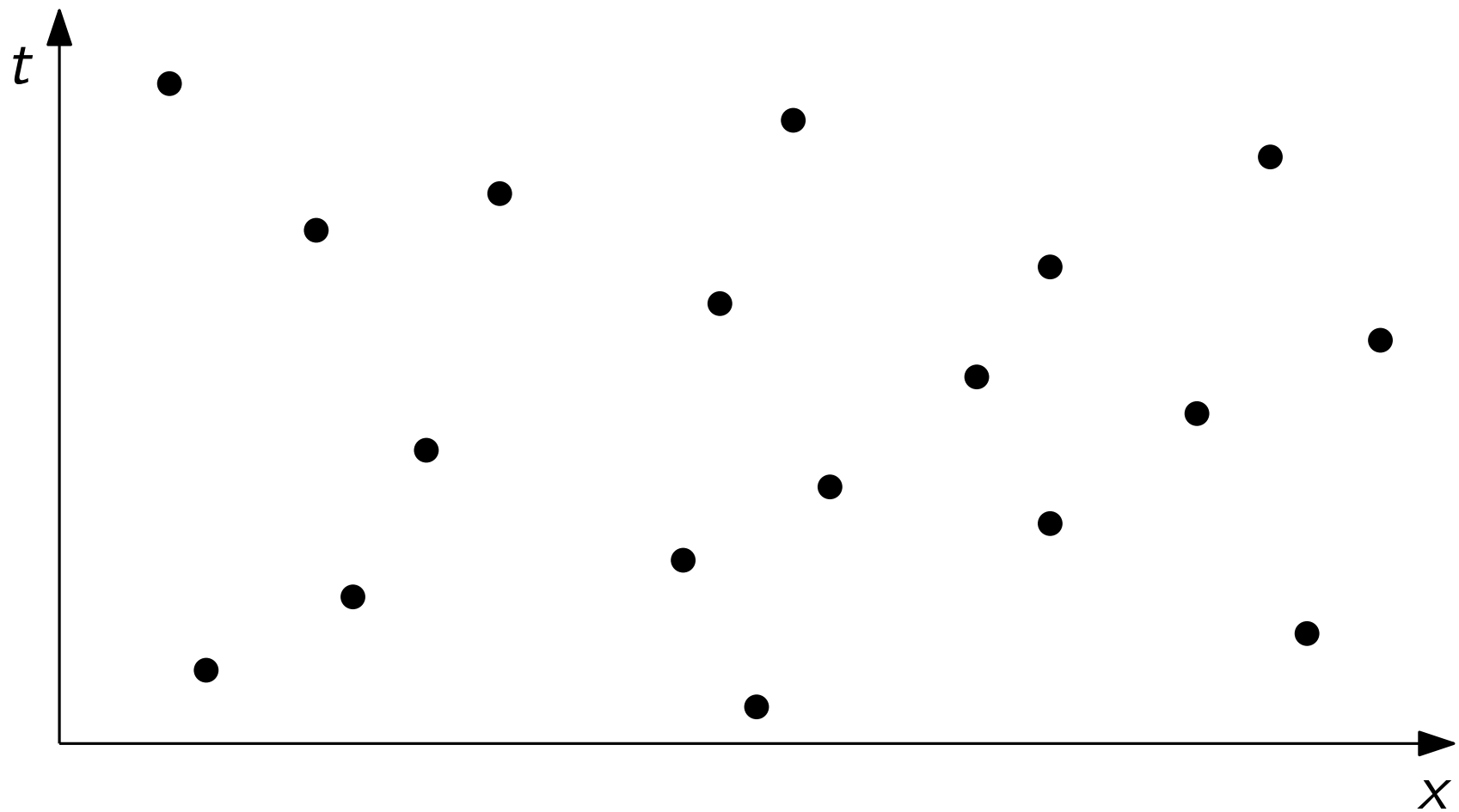

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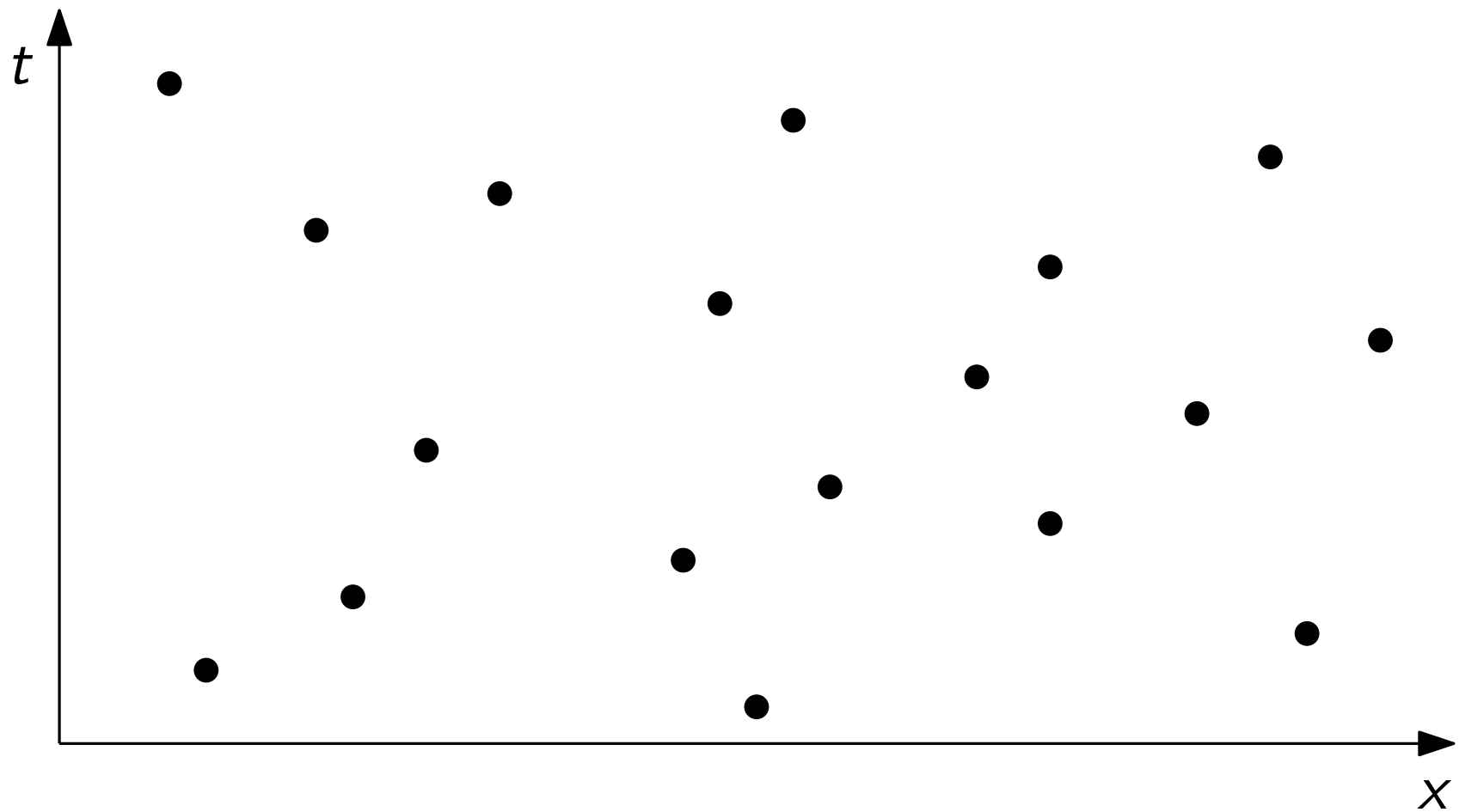

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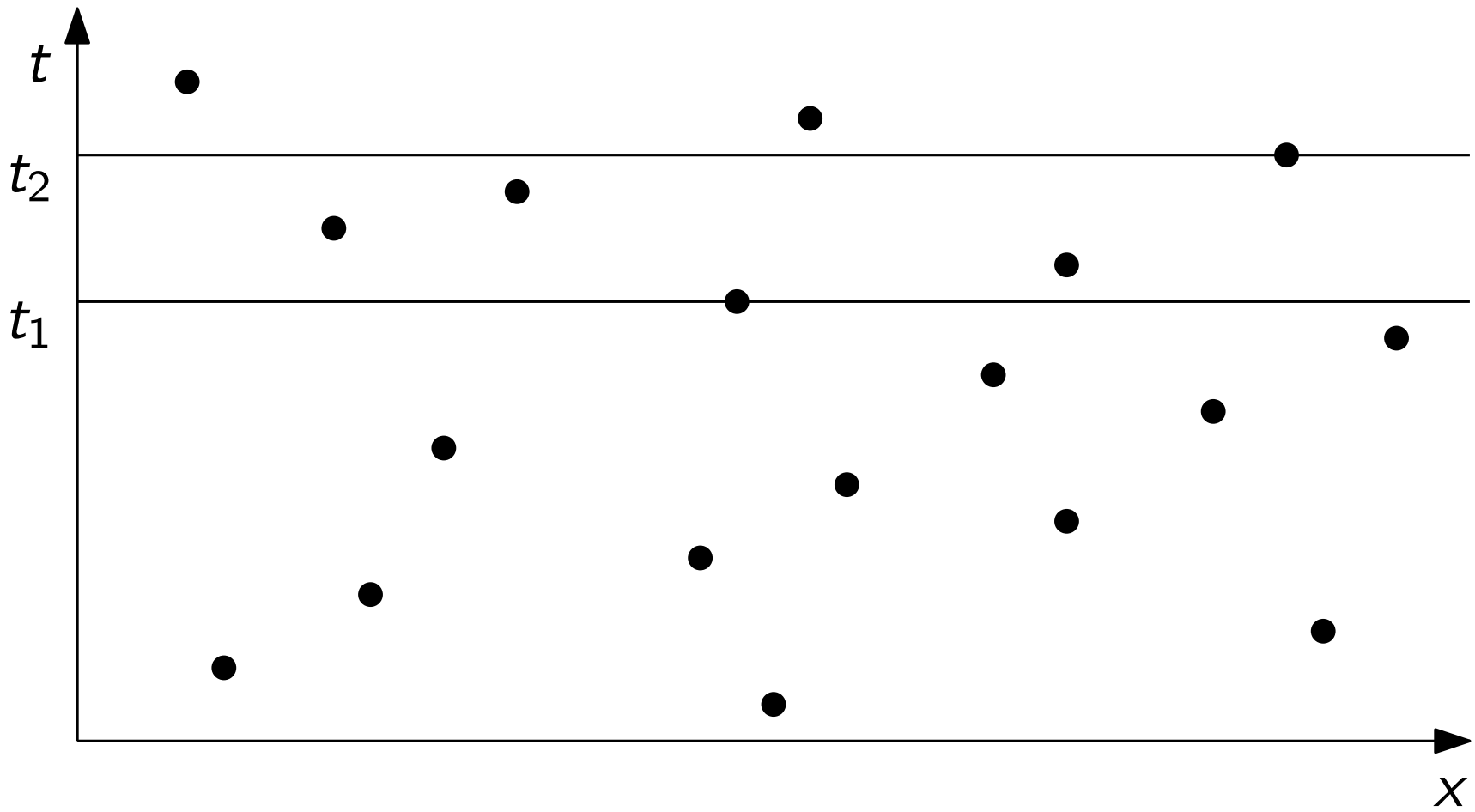

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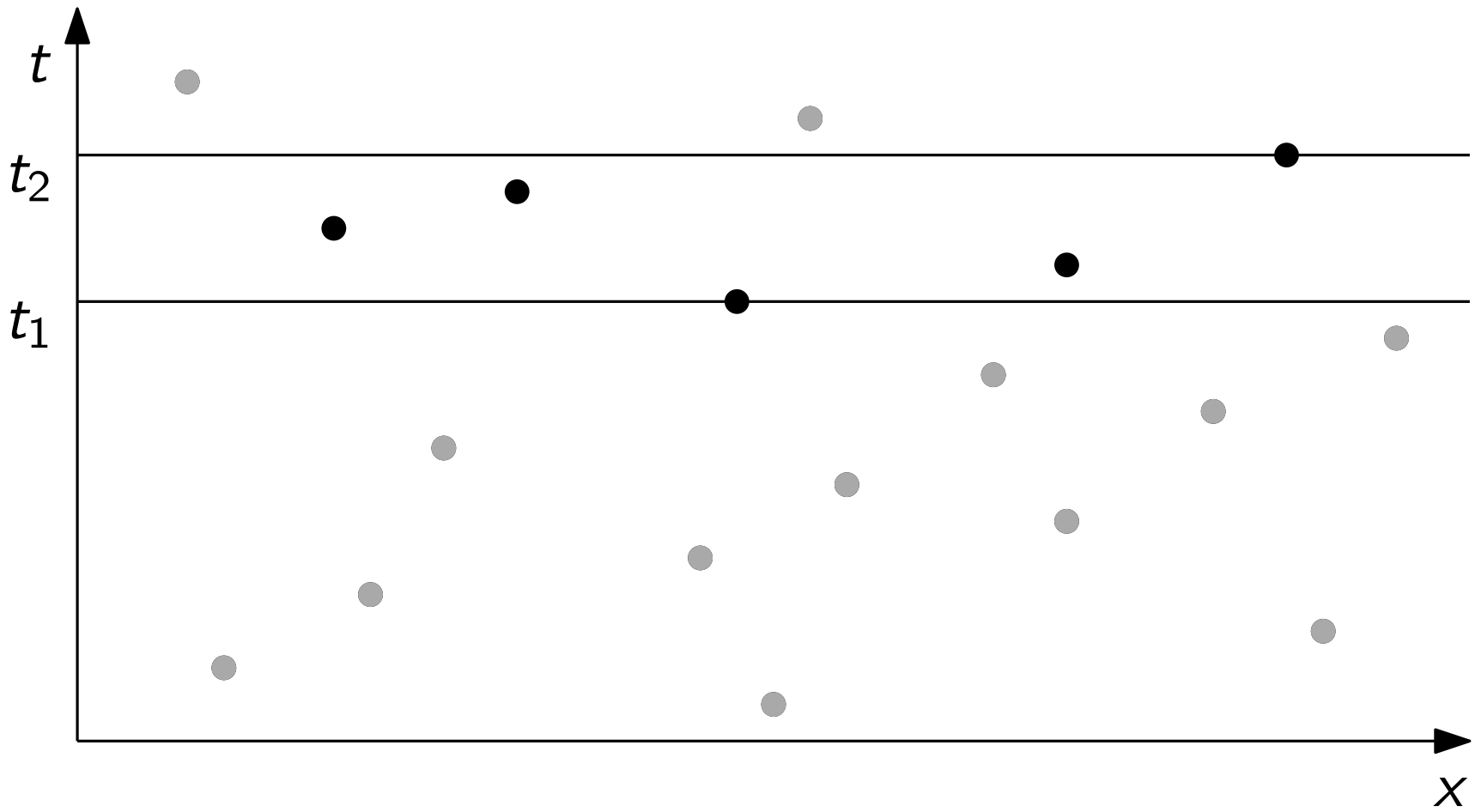

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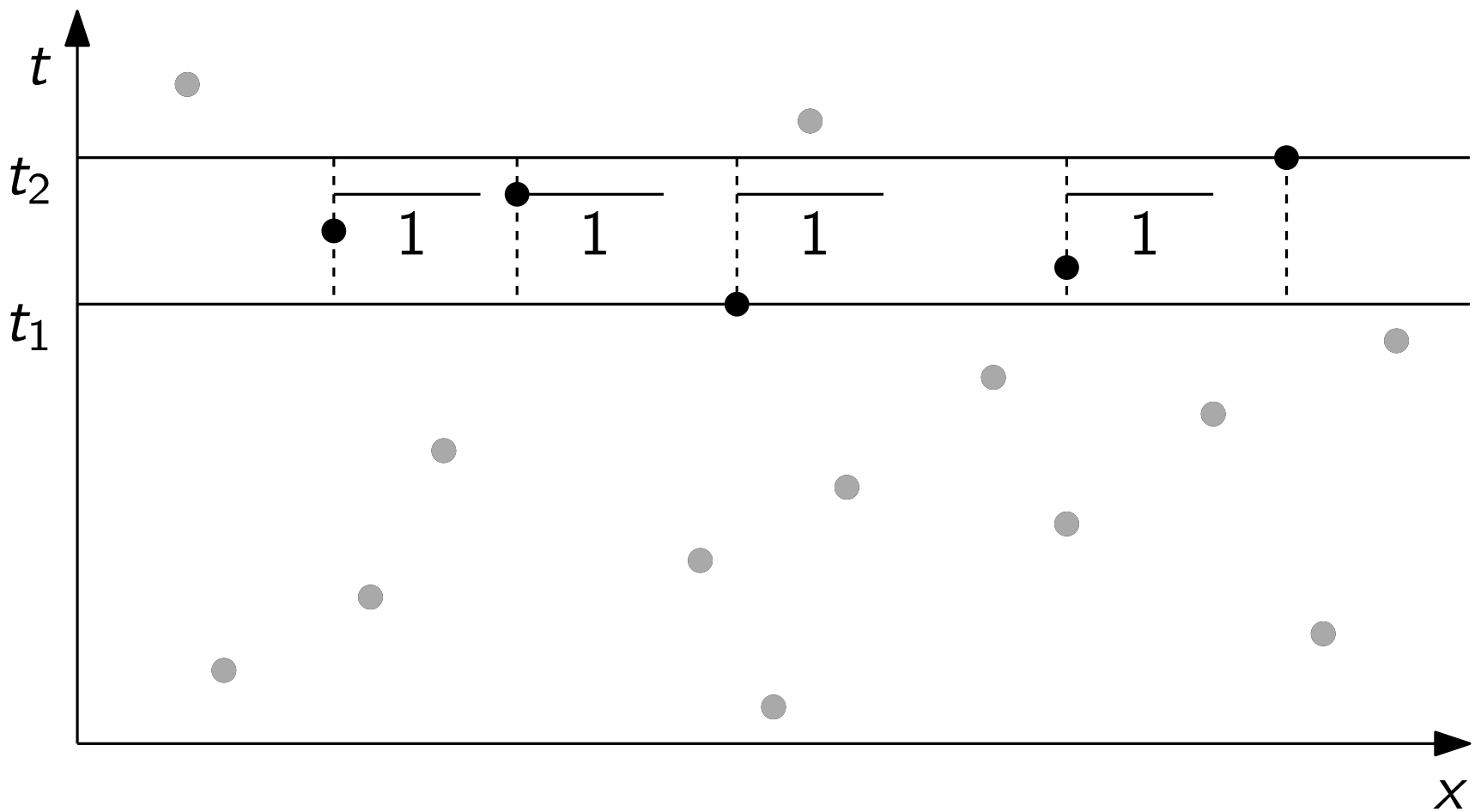

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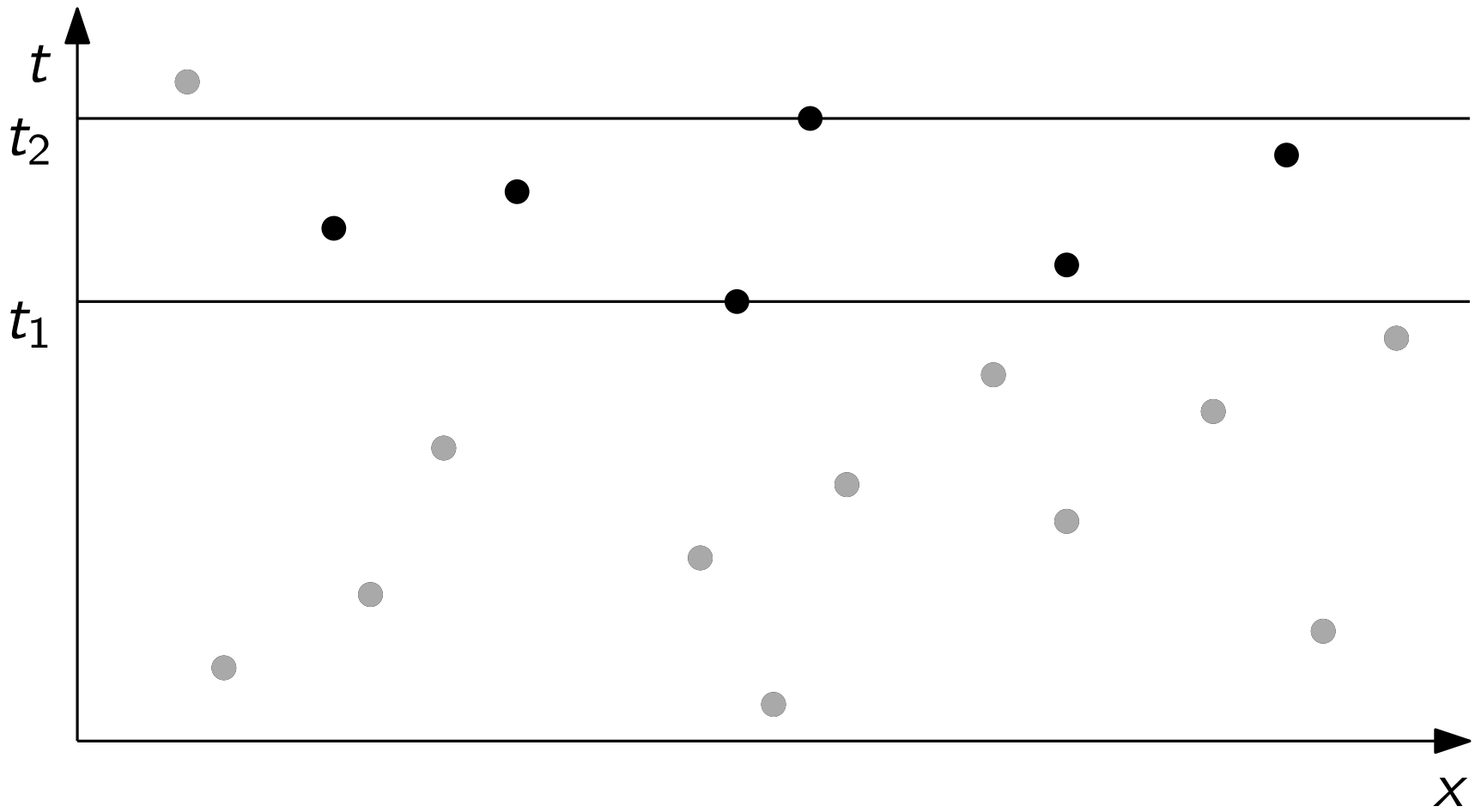

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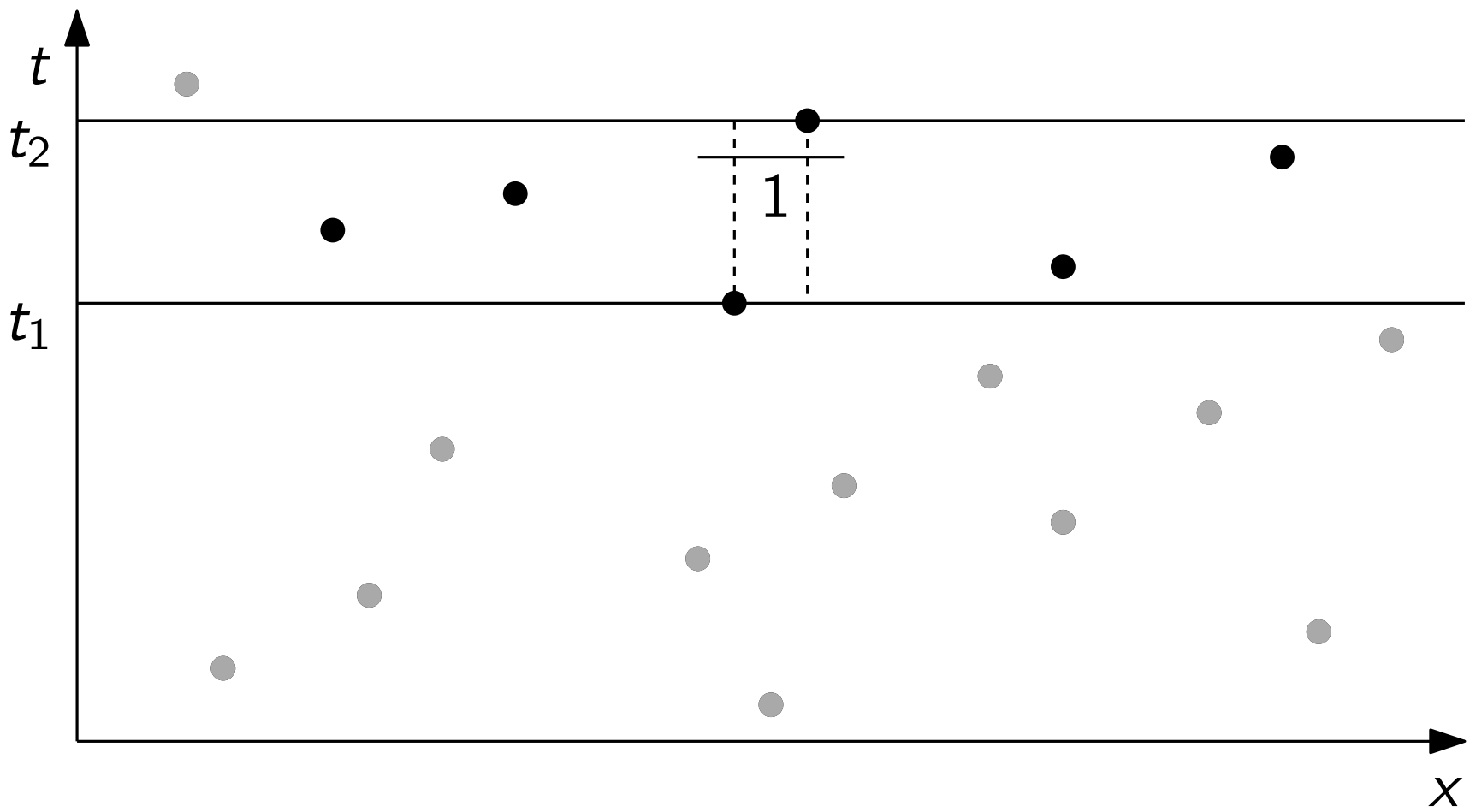

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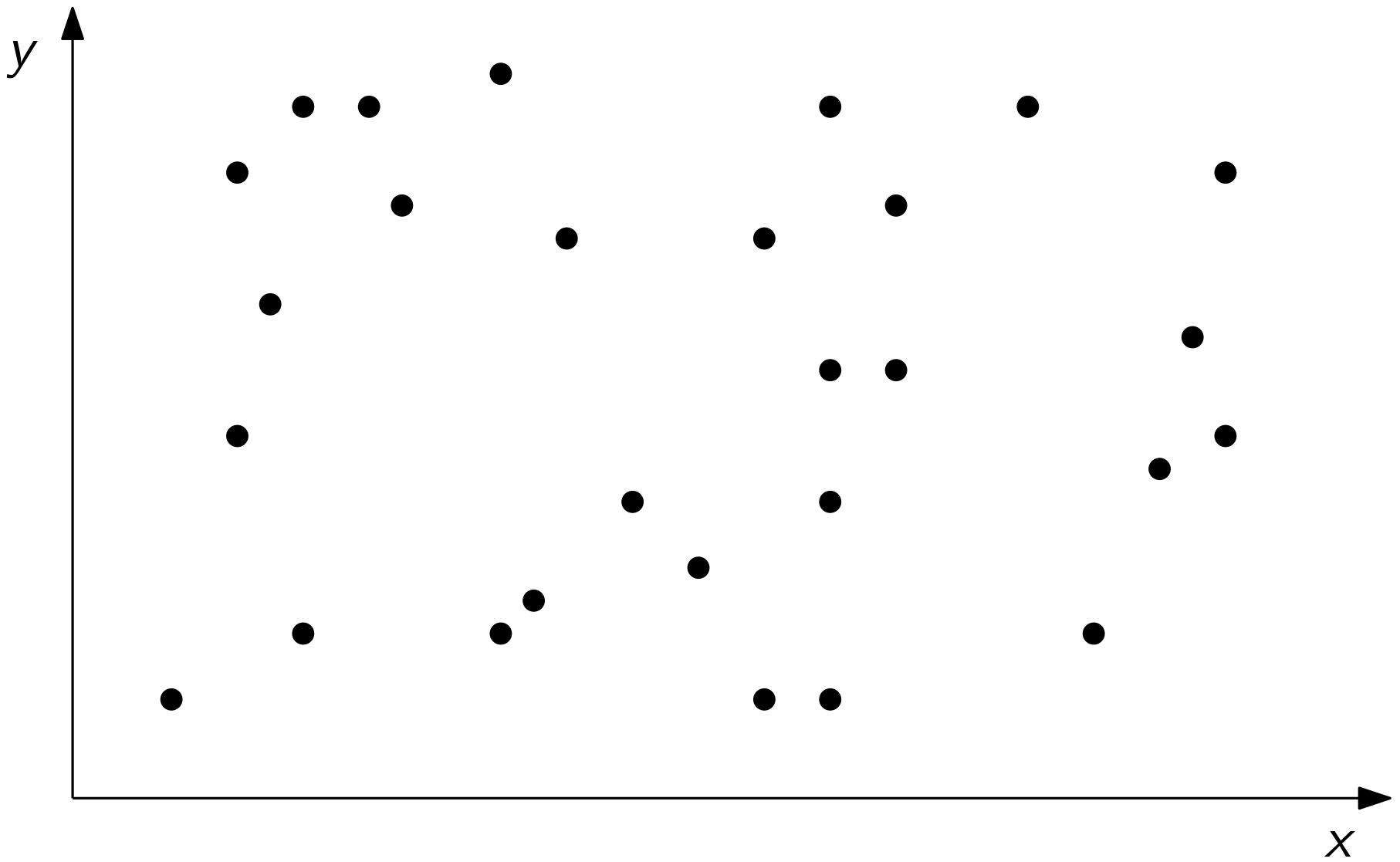

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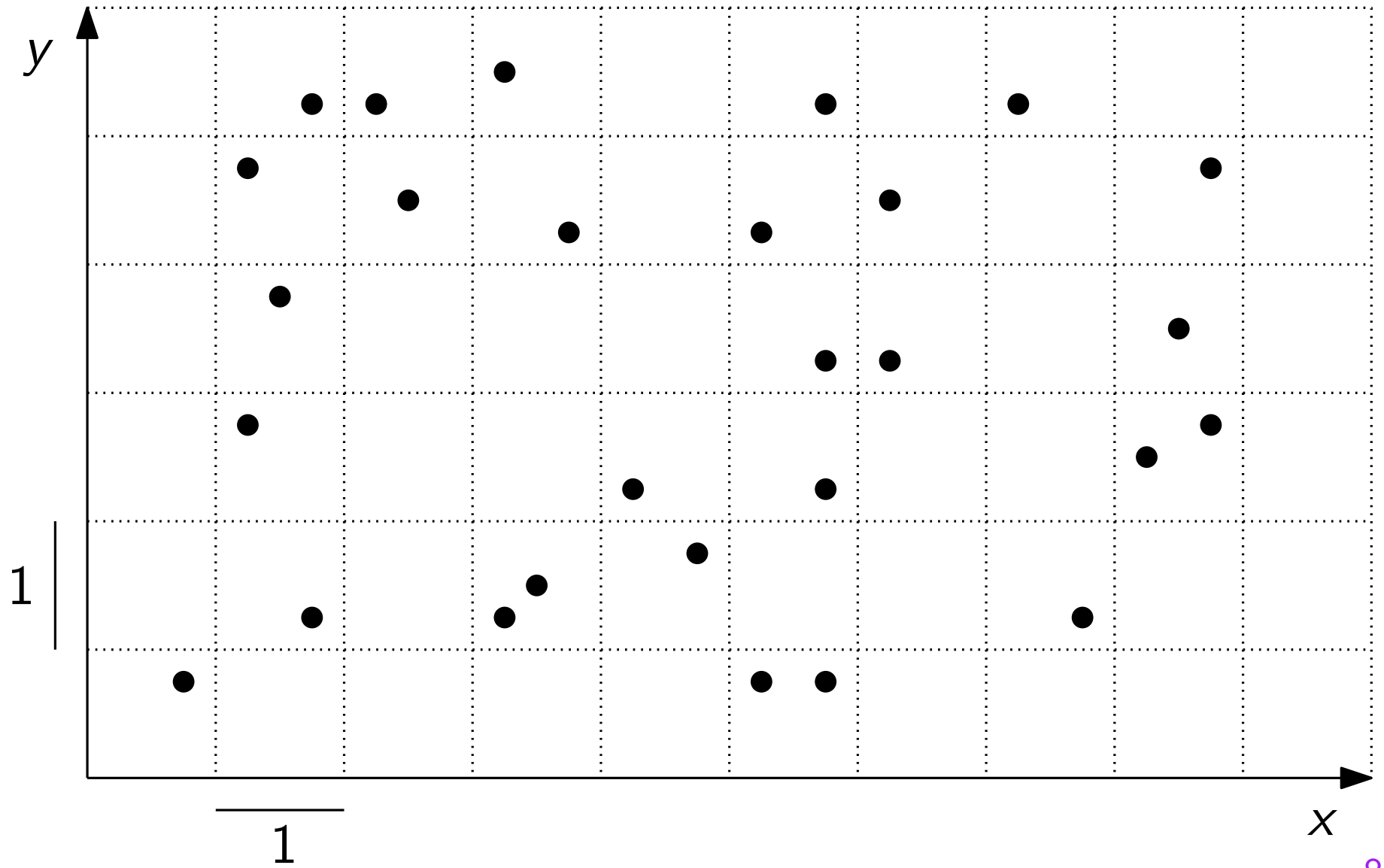

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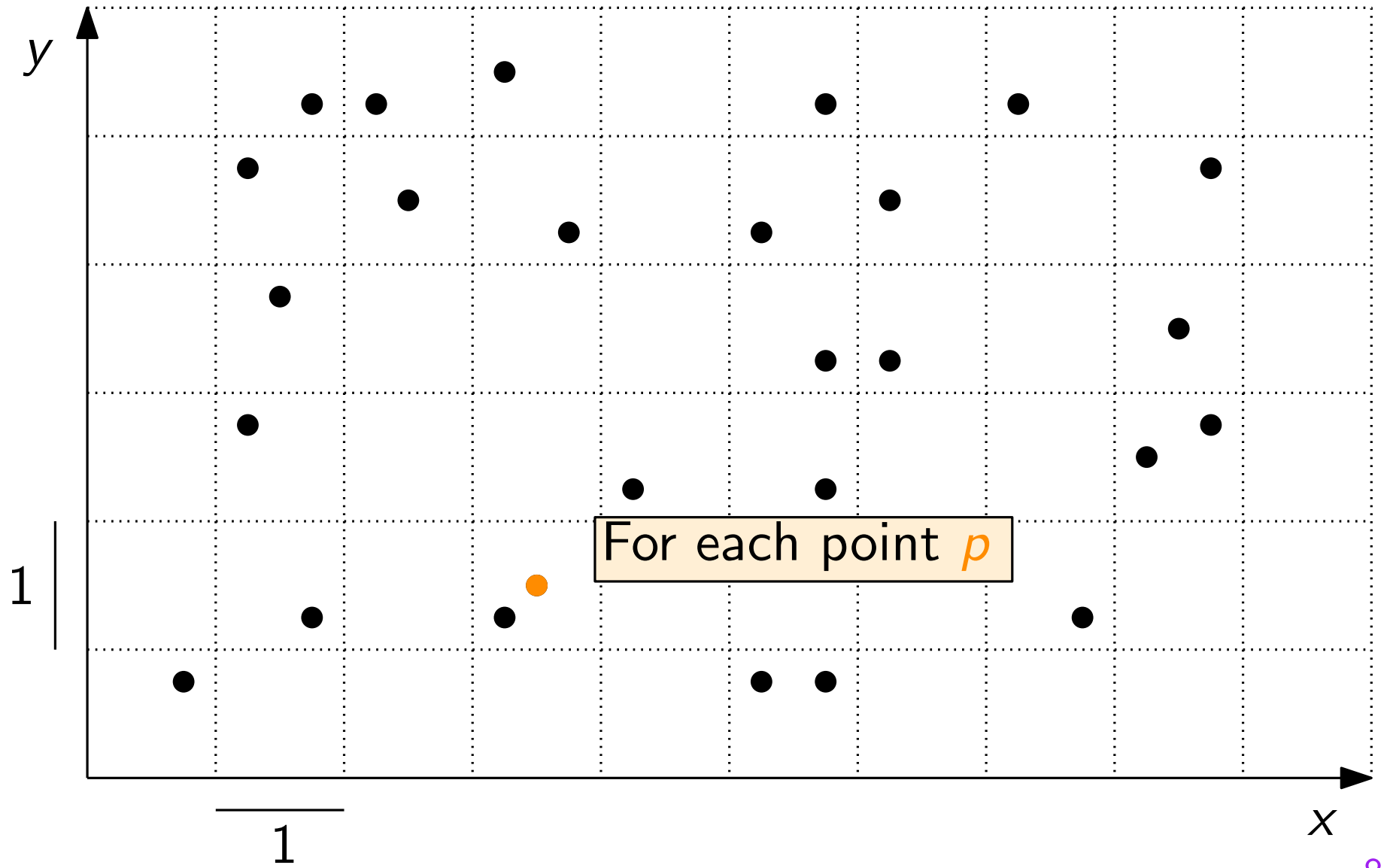

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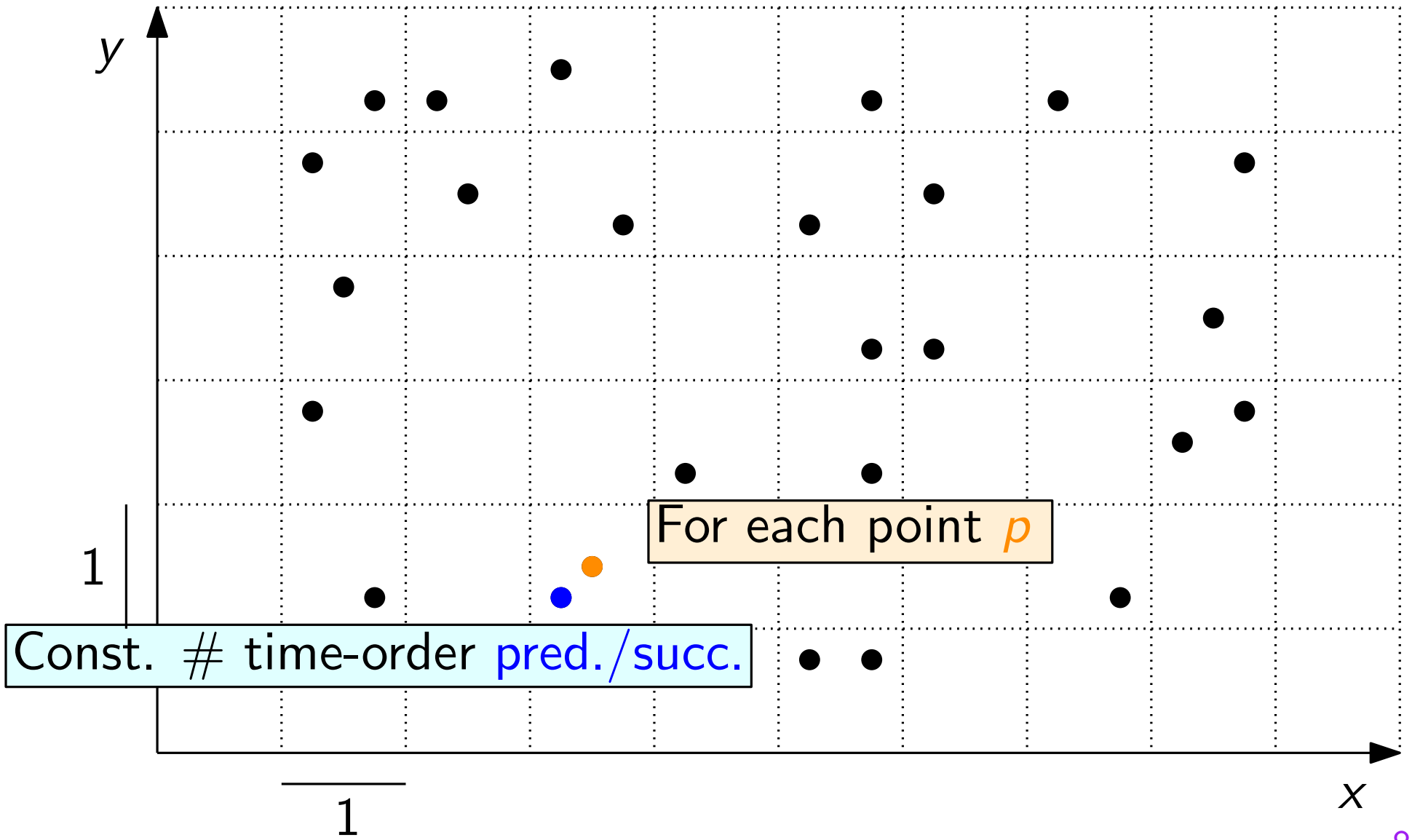

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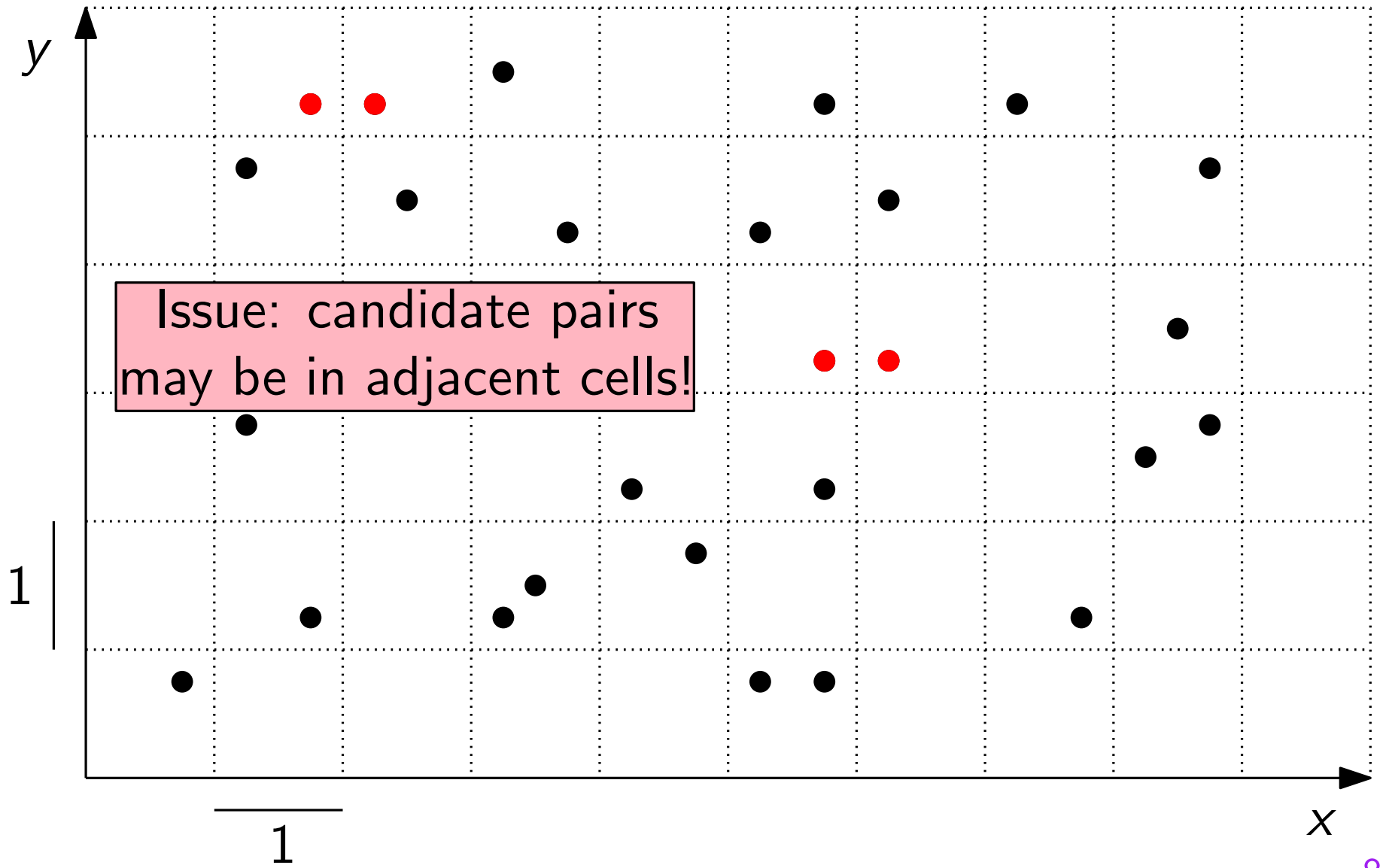

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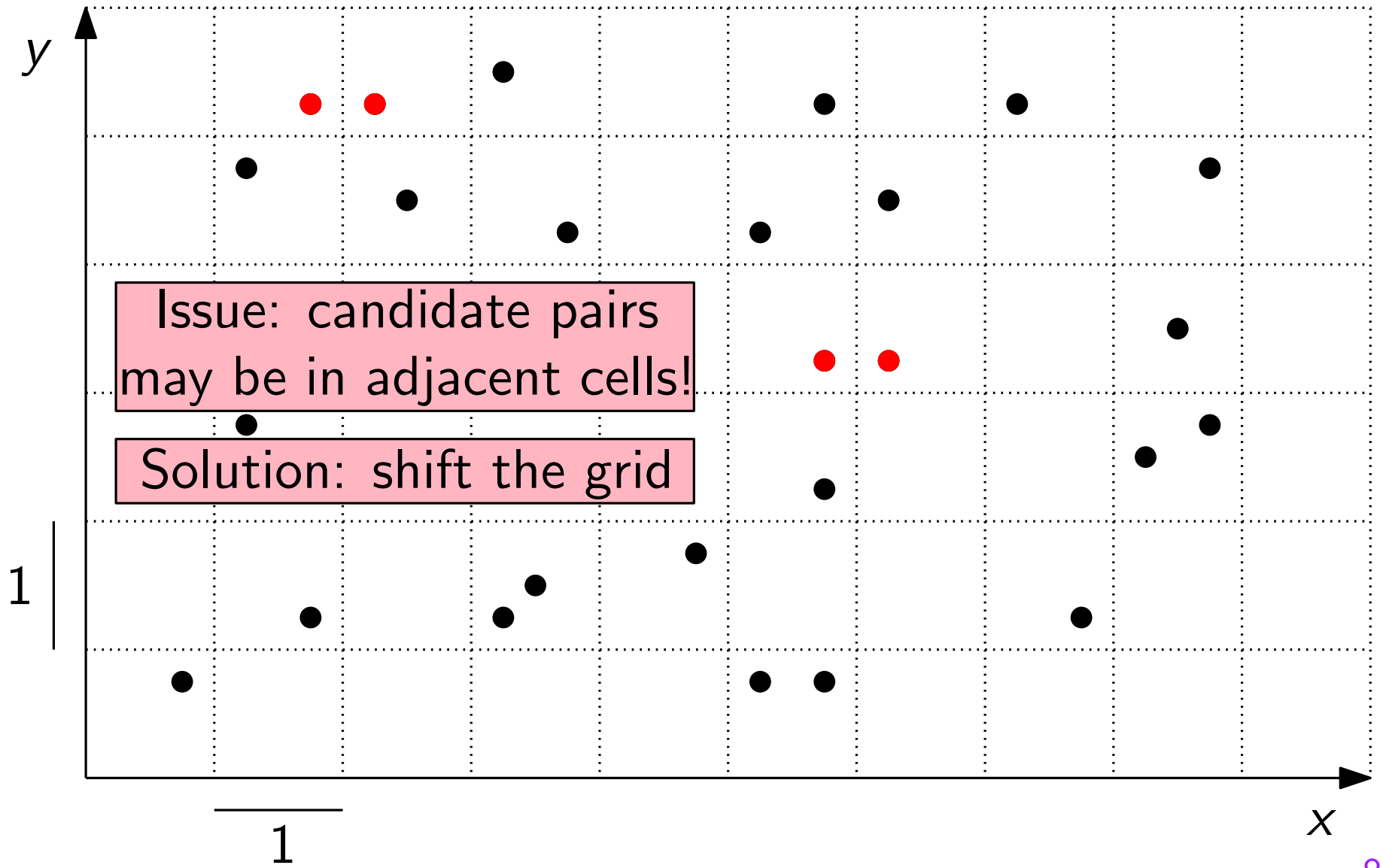

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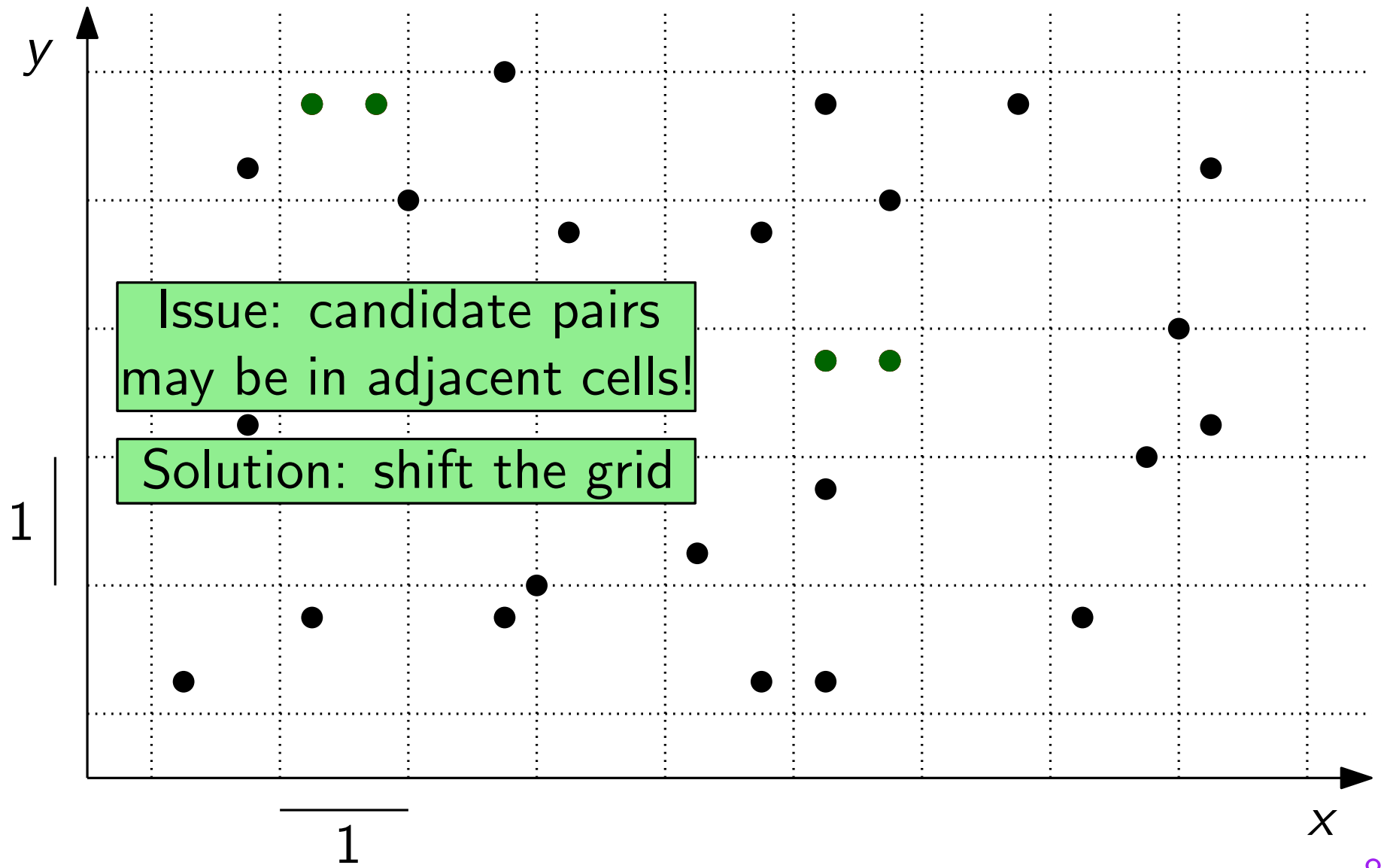

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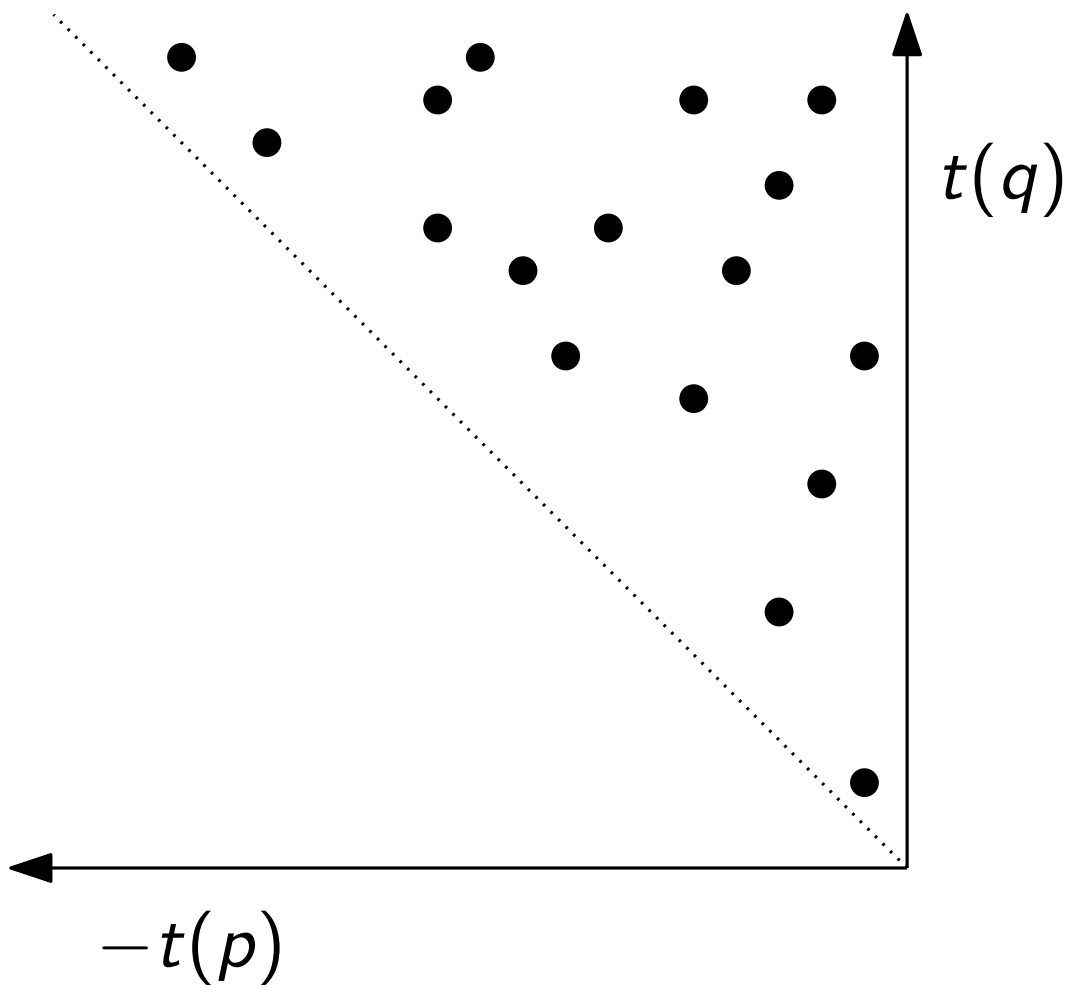

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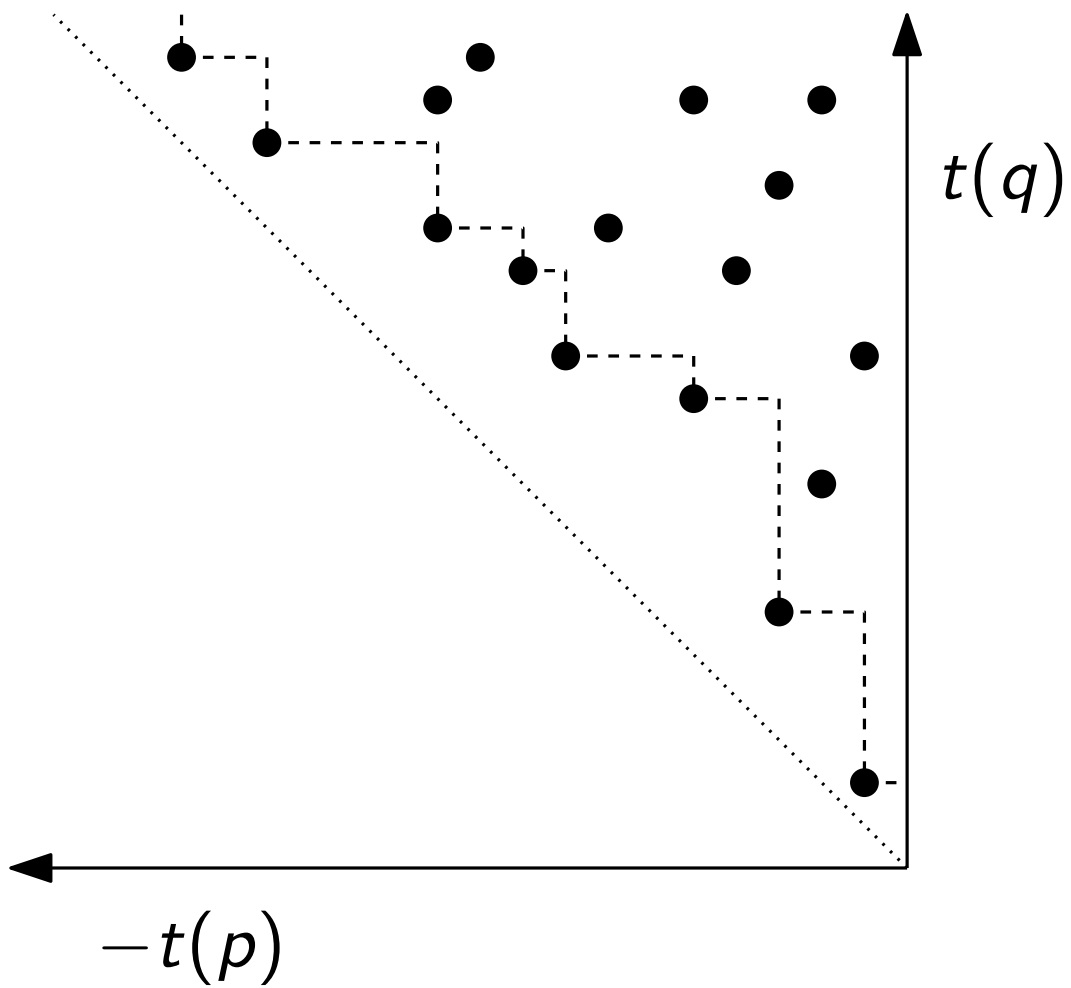

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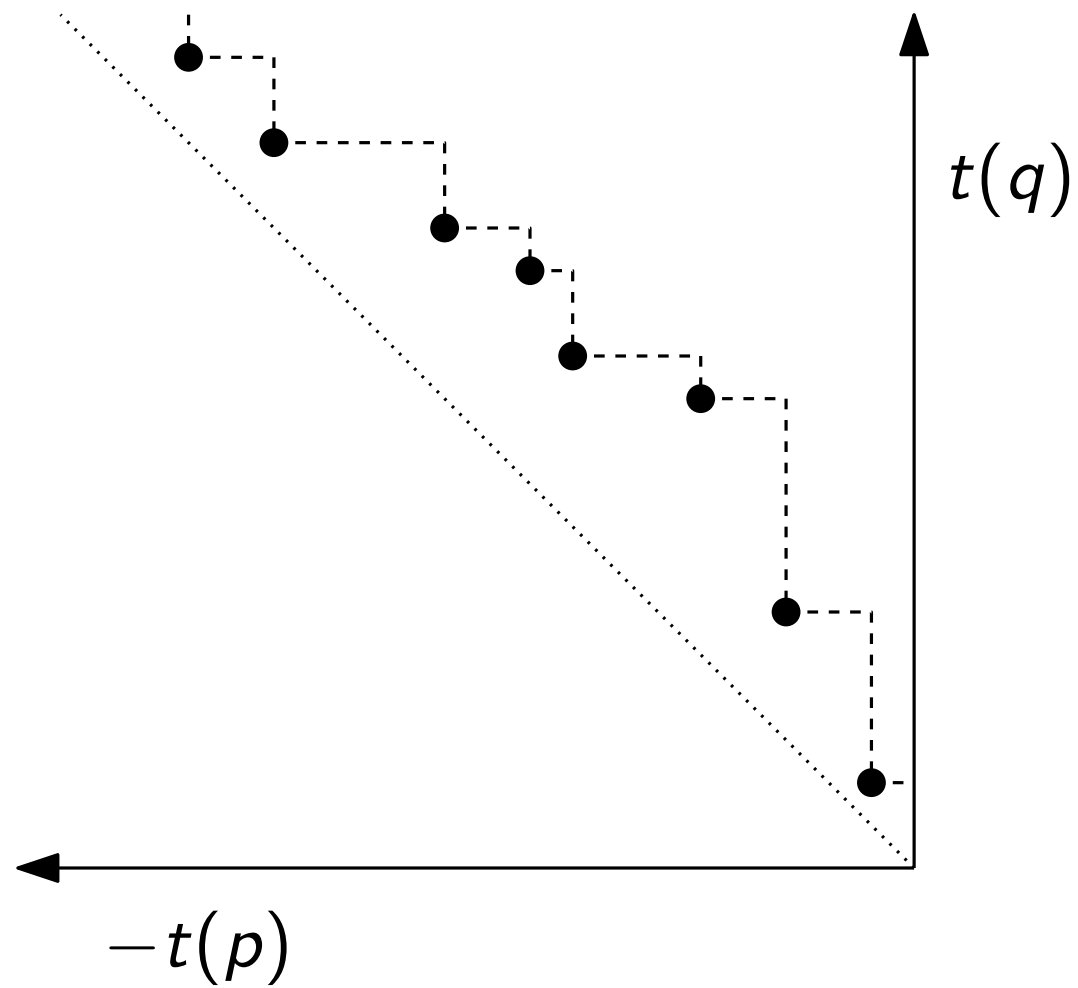

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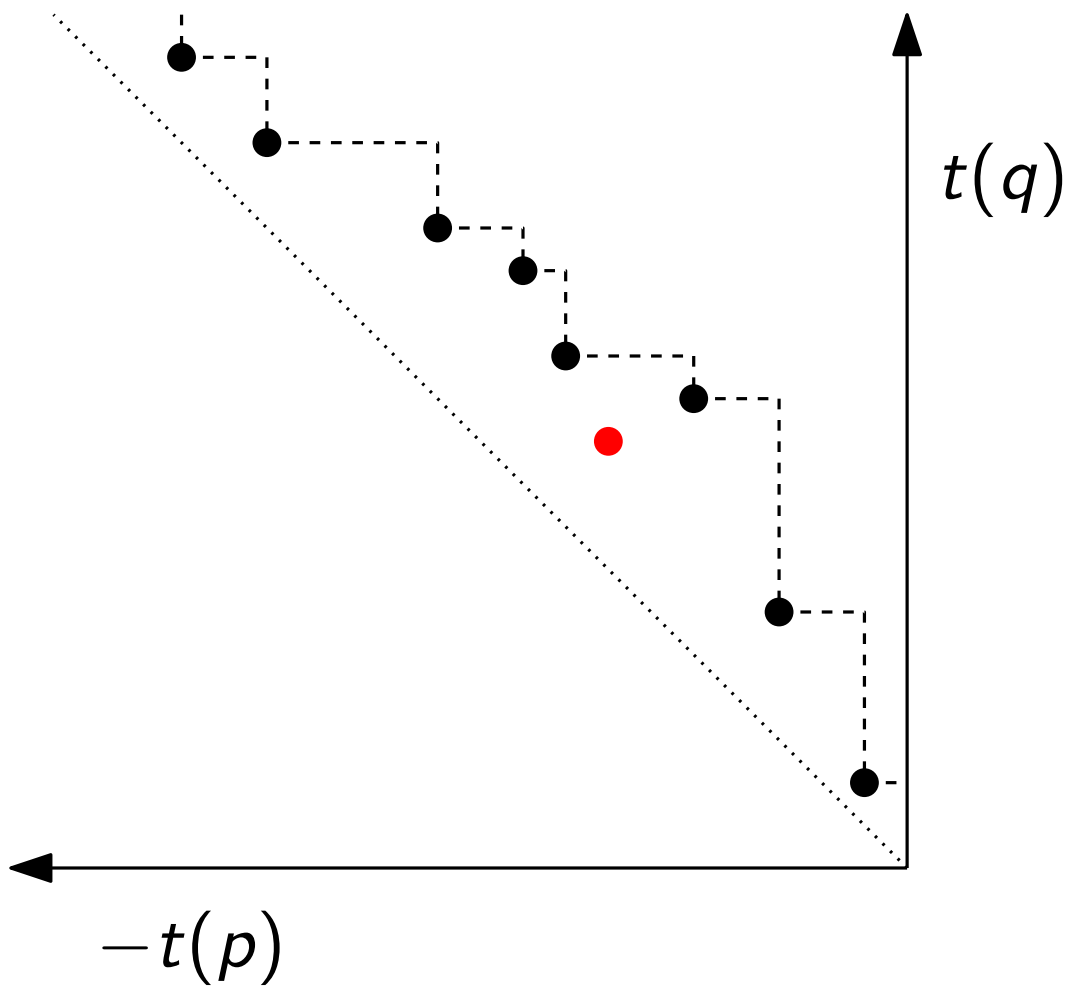

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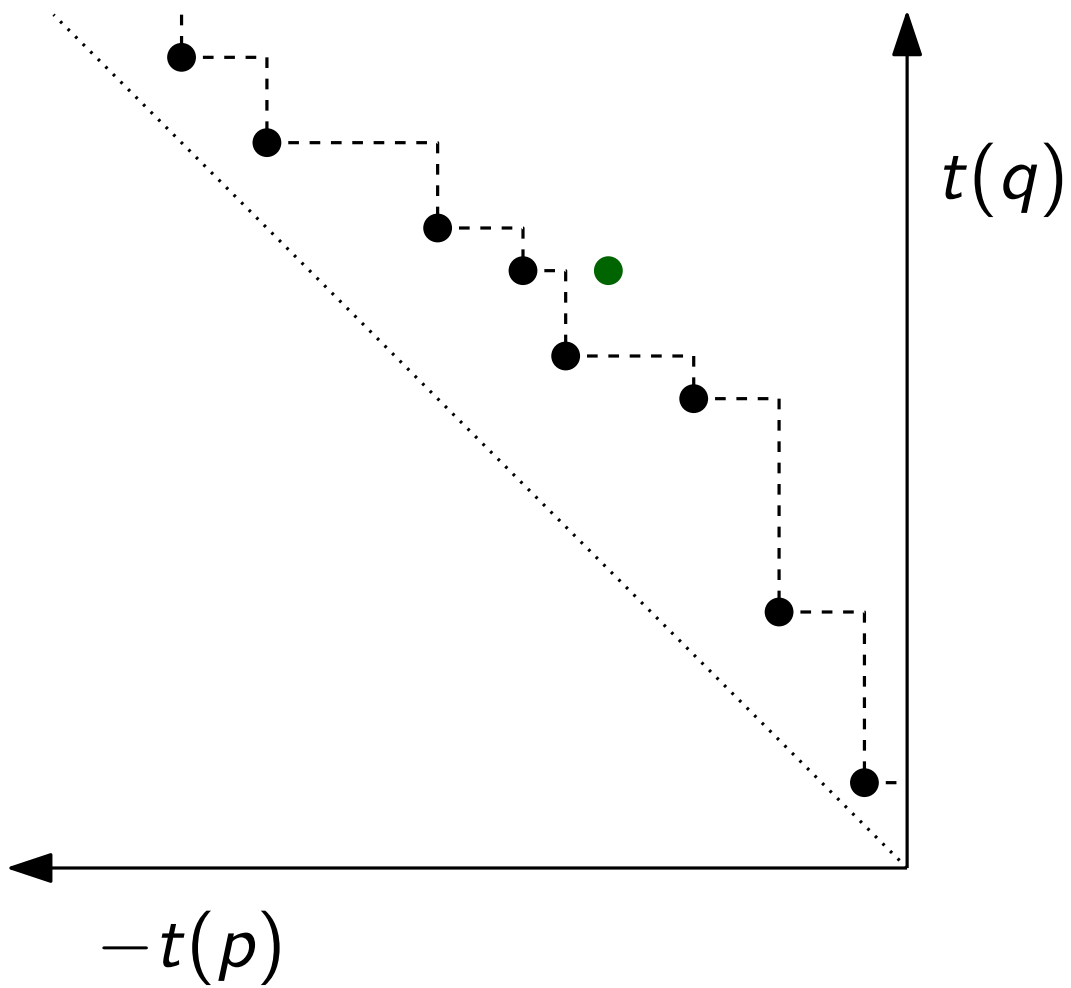

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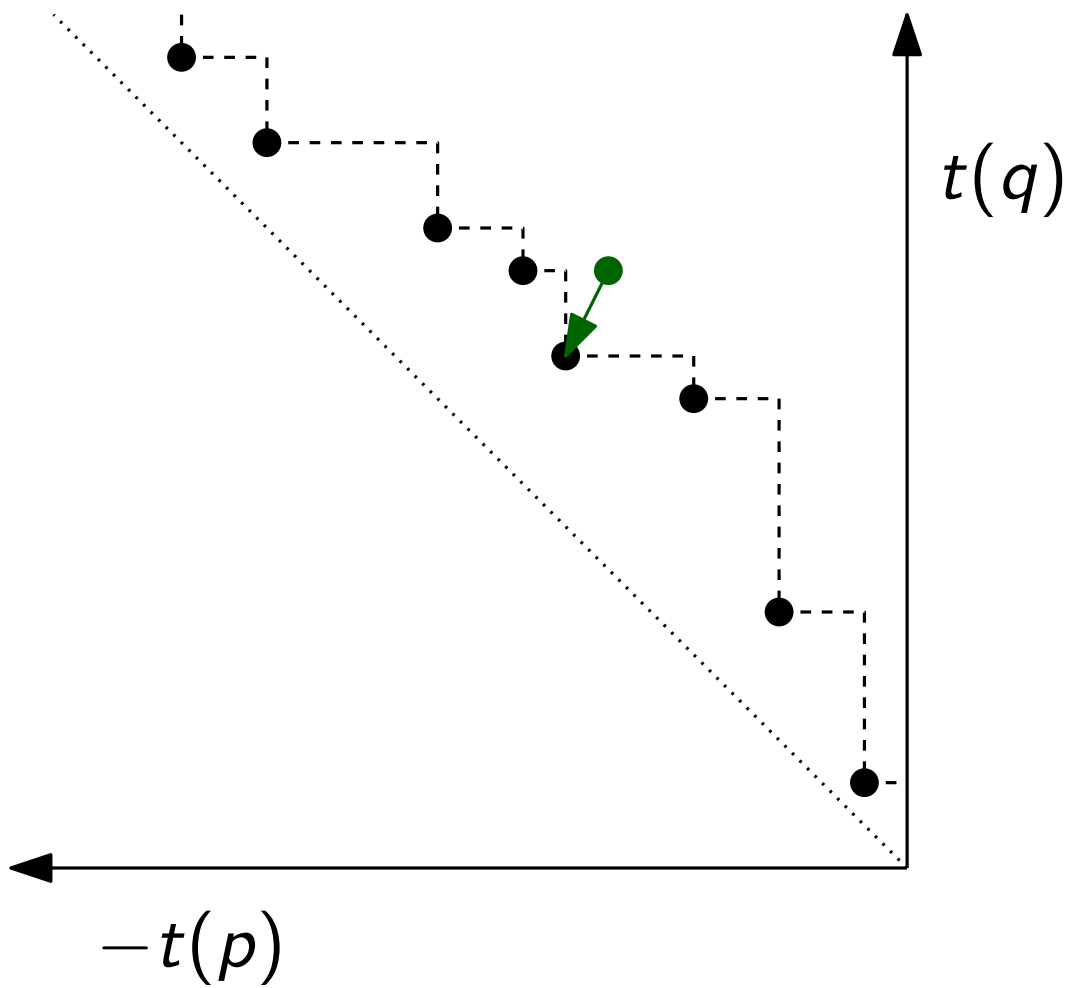
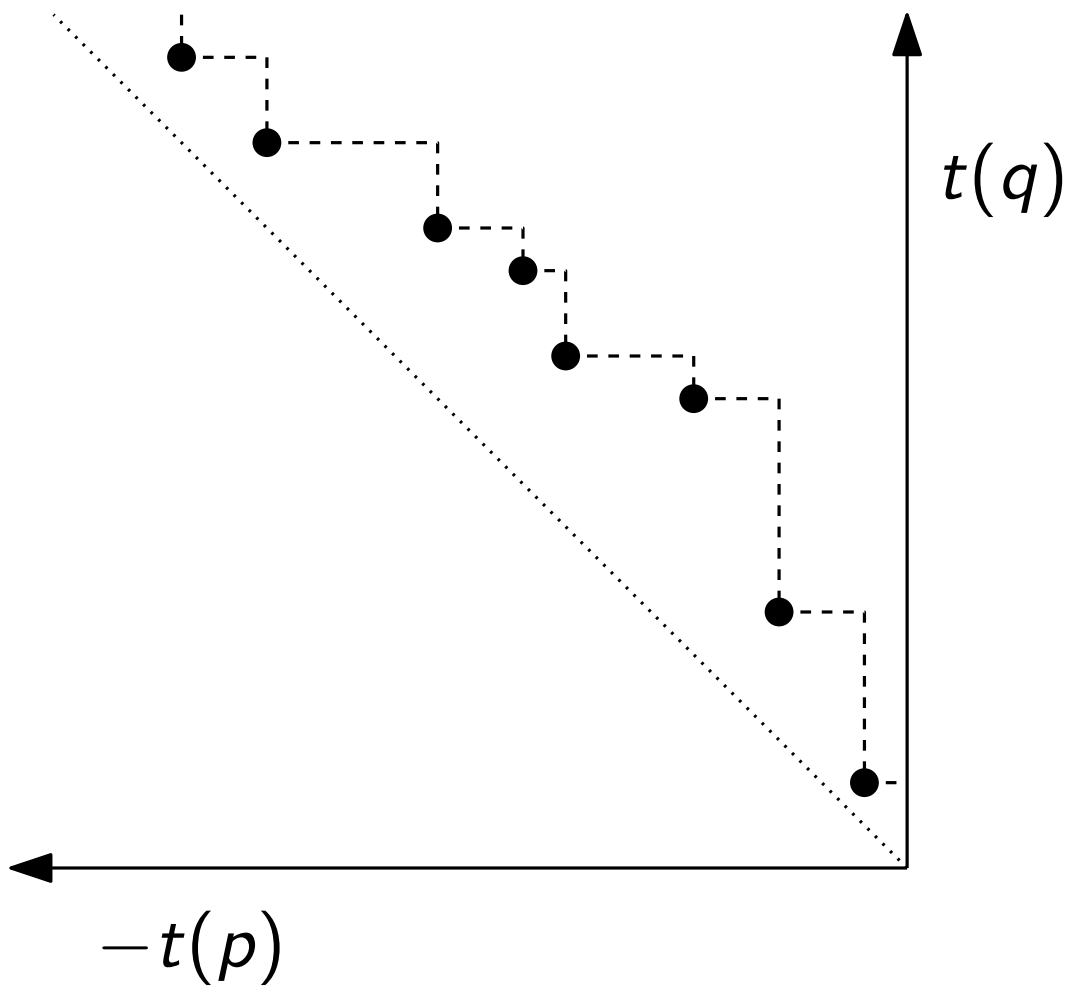
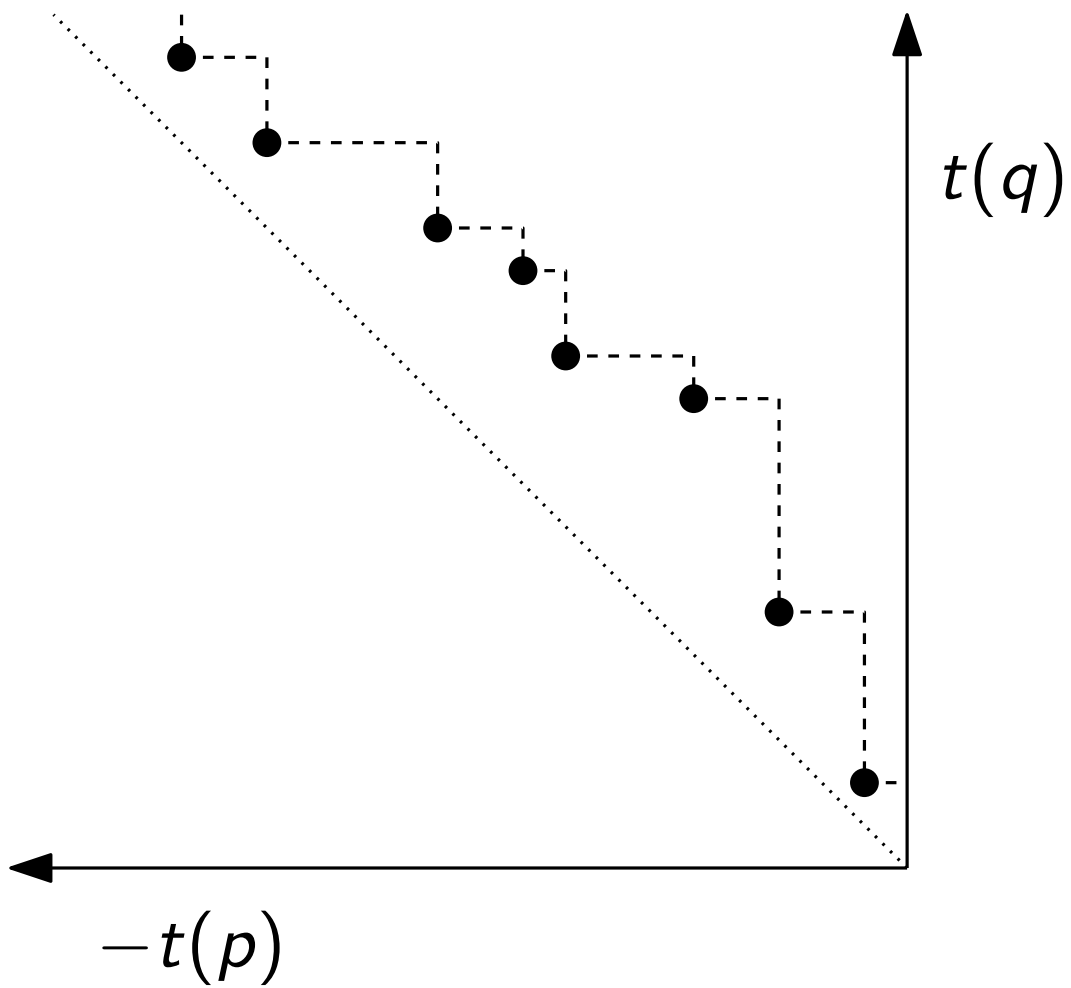


Succinct Representation



Succinct Representation

- Monotone chain



Succinct Representation

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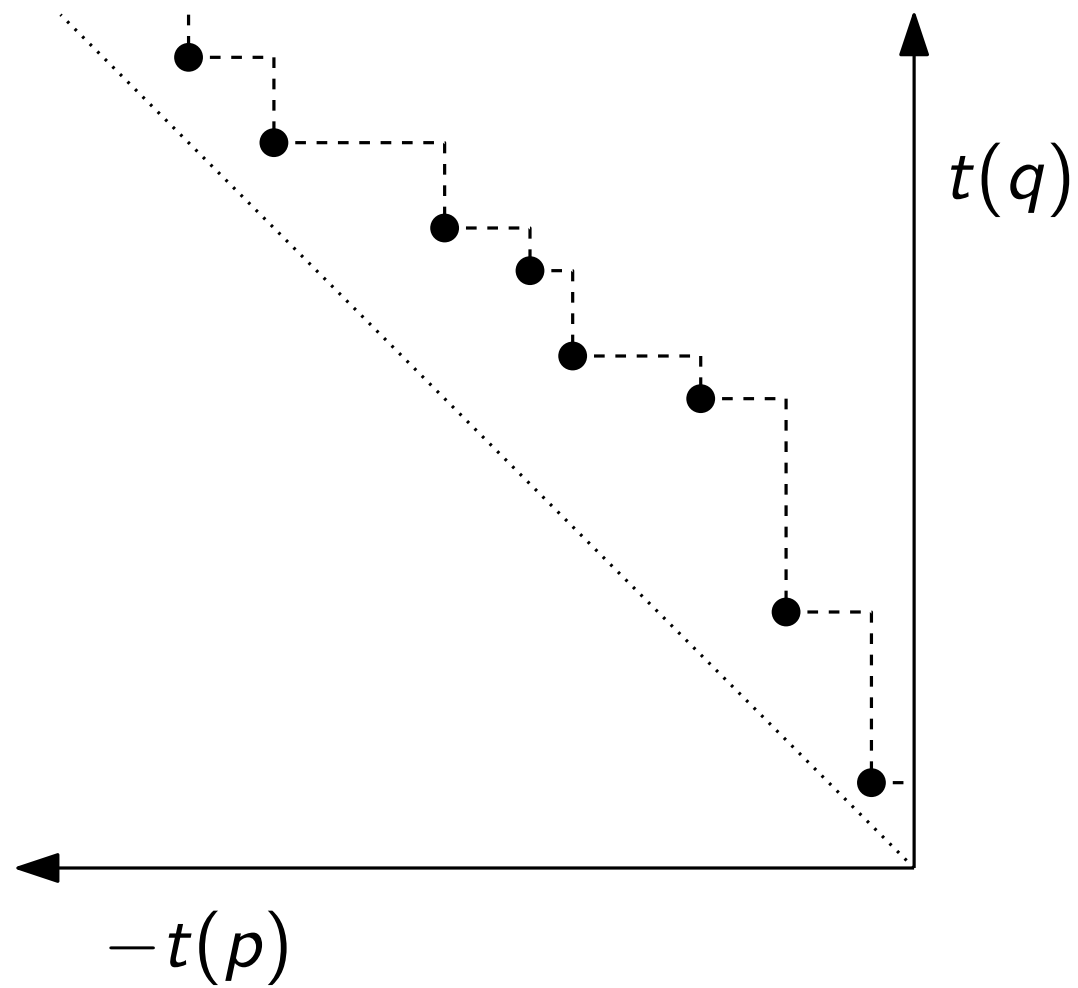
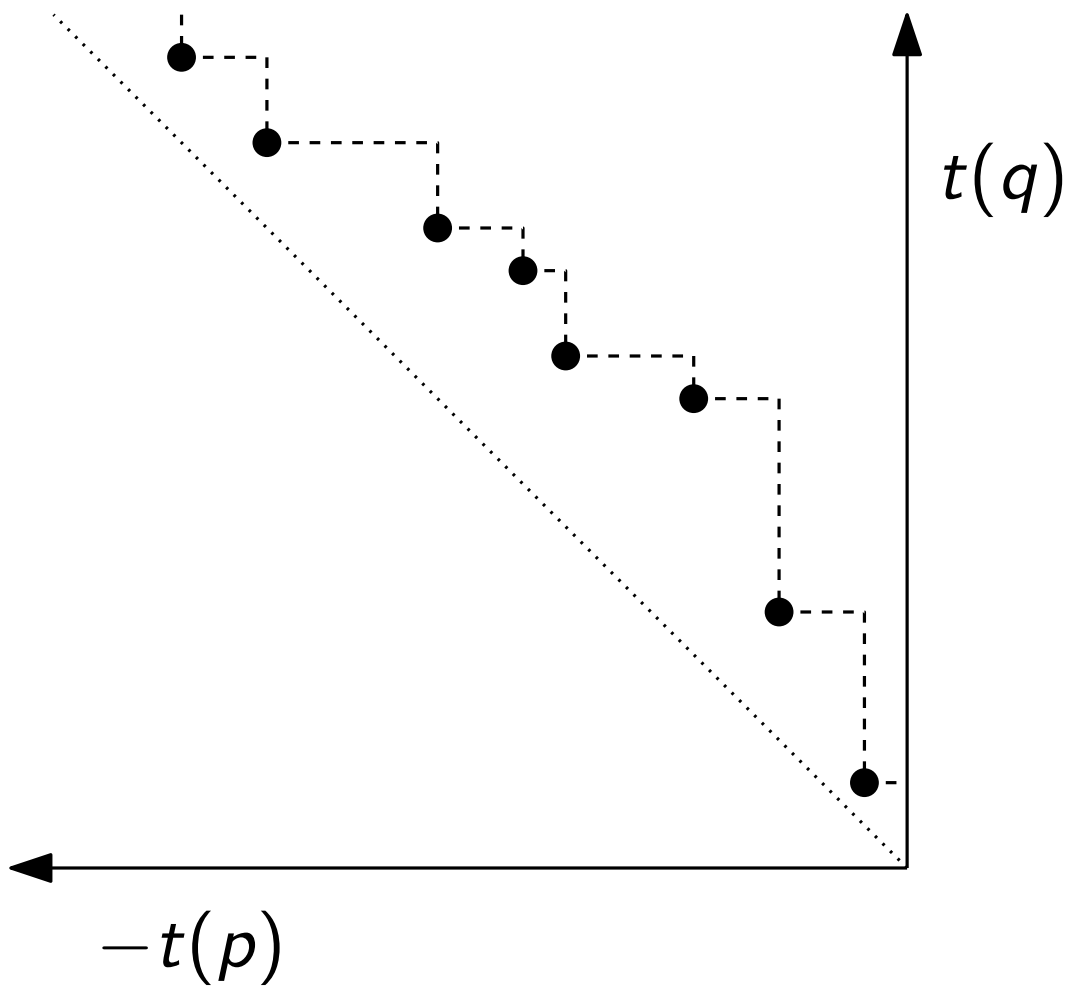


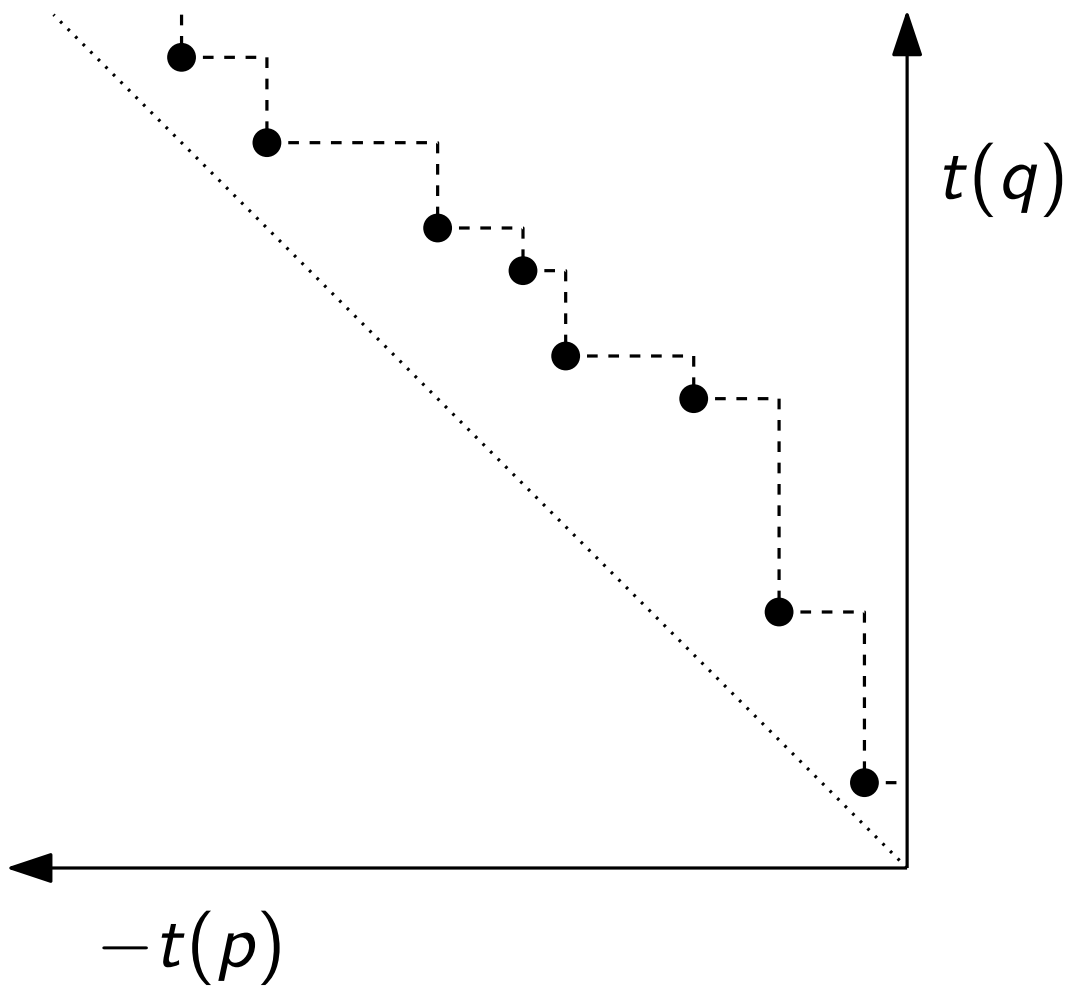
Succinct Representation

- Monotone chain
- Starts at $0, 0$
- Each step moves:



Succinct Representation

- Monotone chain
- Starts at $0, 0$
- Each step moves:
  - ◇ Horizontally



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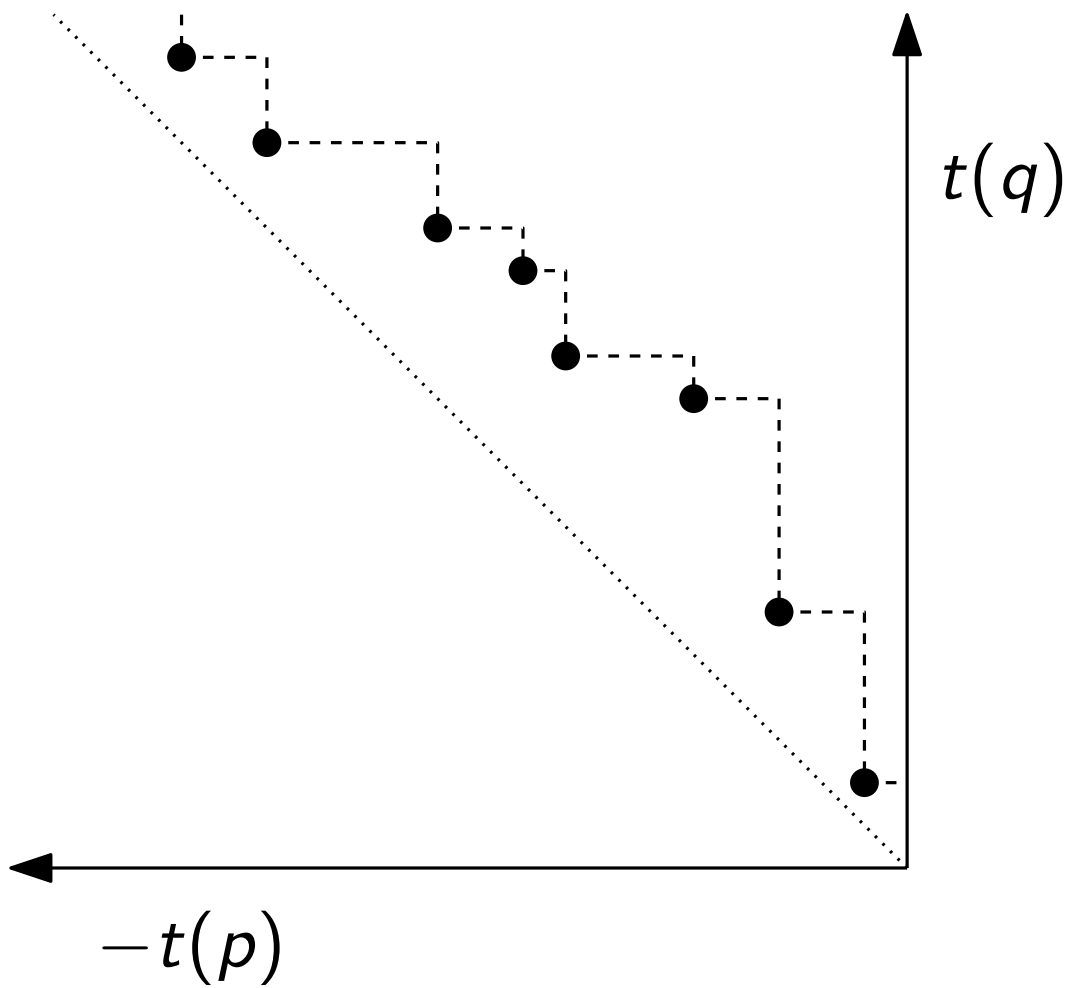

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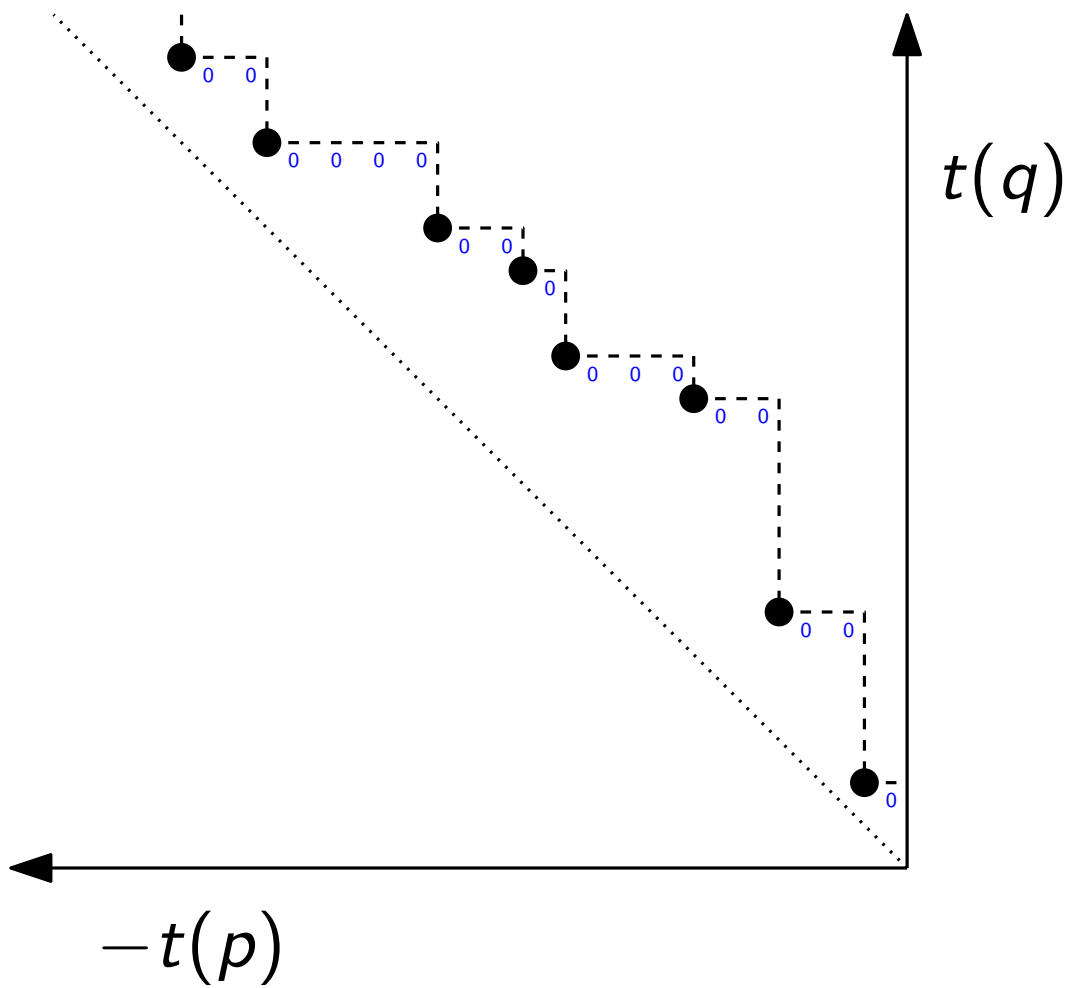

Succinct Representation

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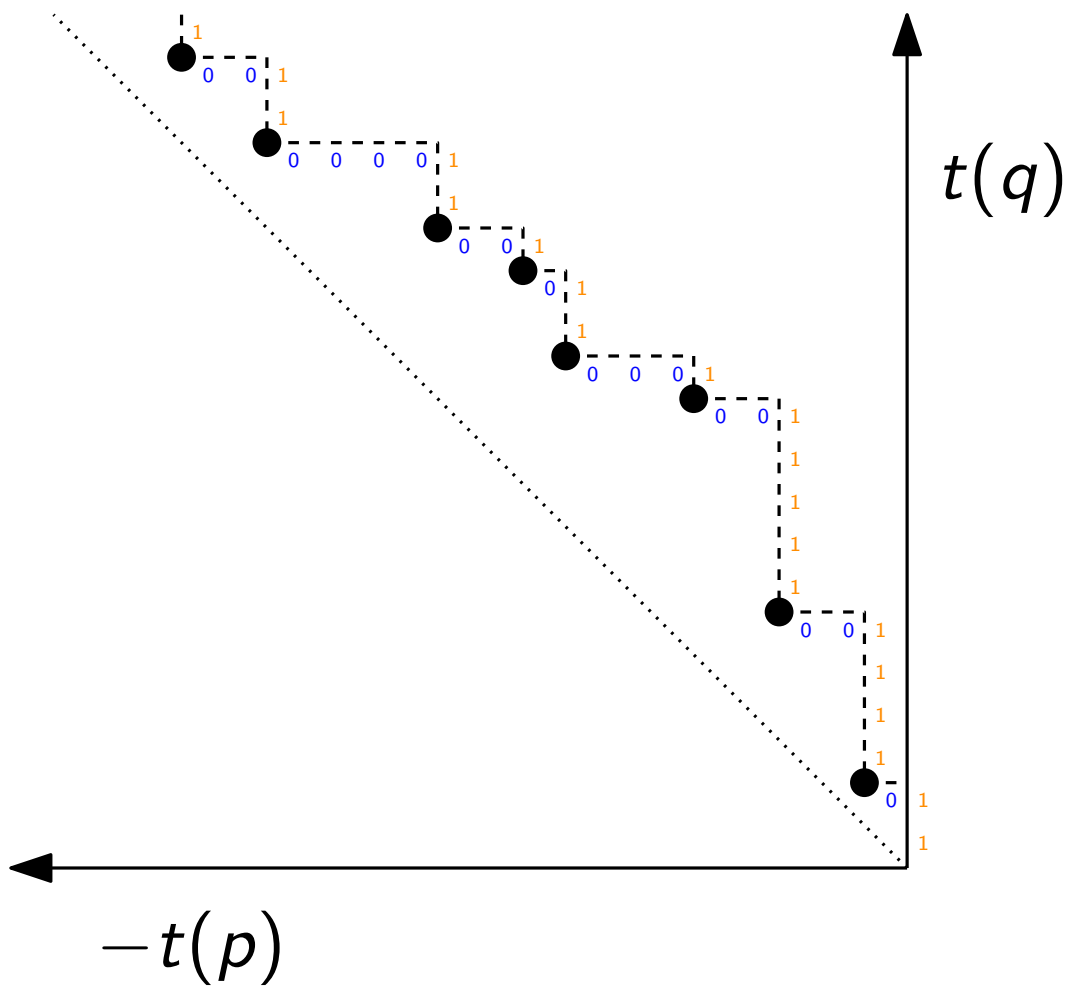

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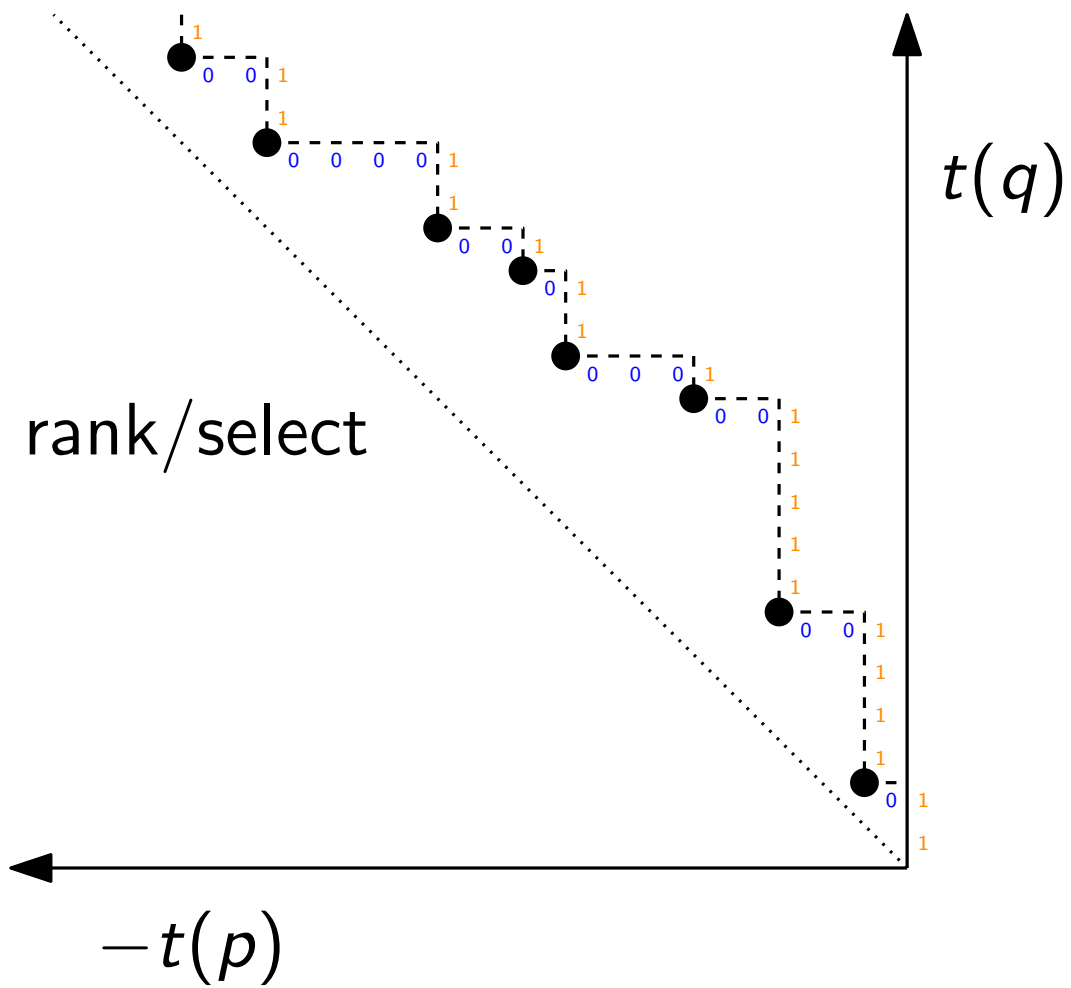
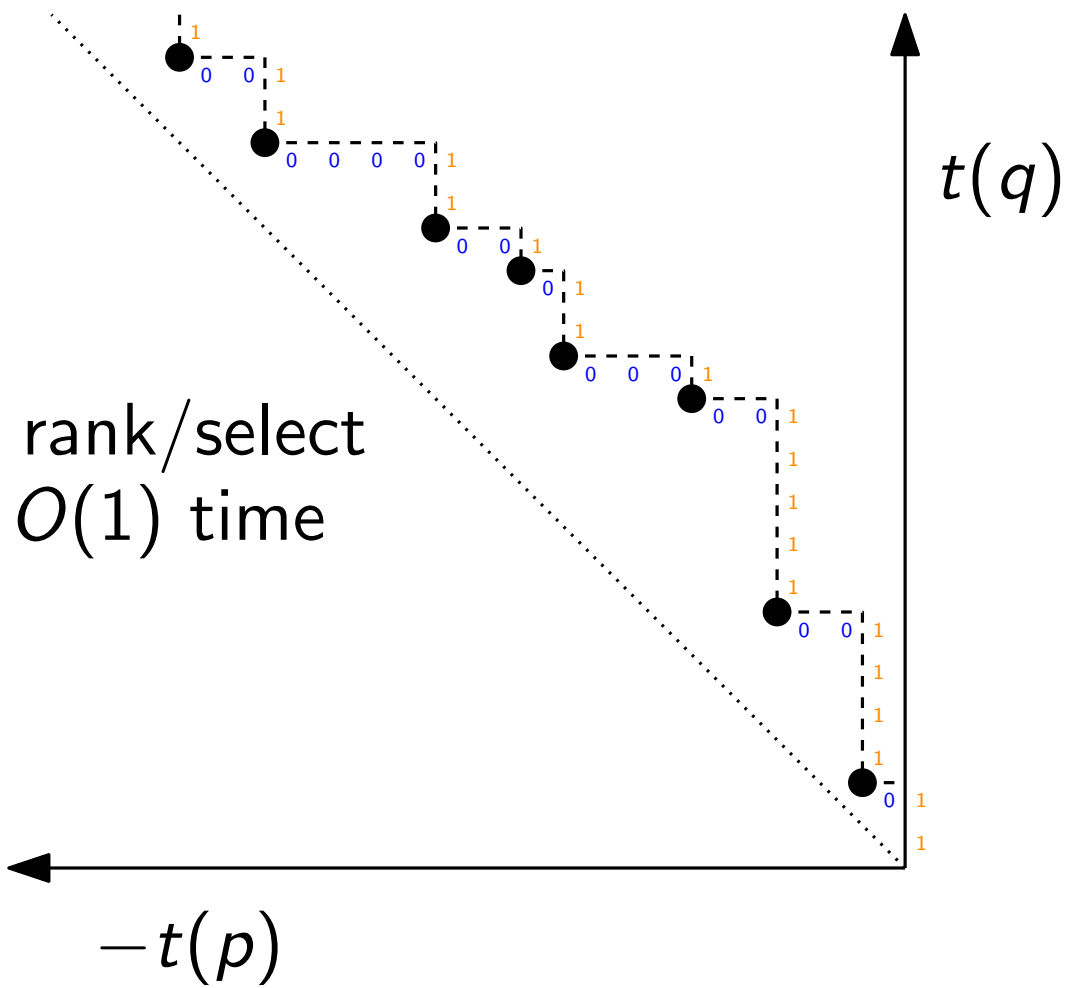


Solve query $\rightarrow$ rank/select

Succinct Representation

- Monotone chain
- Starts at $0, 0$
- Each step moves:
  - ◇ Horizontally 0
  - ◇ Vertically 1



Solve query $\rightarrow$ rank/select
$O(1)$ time

Time-Windowed Closest Pair

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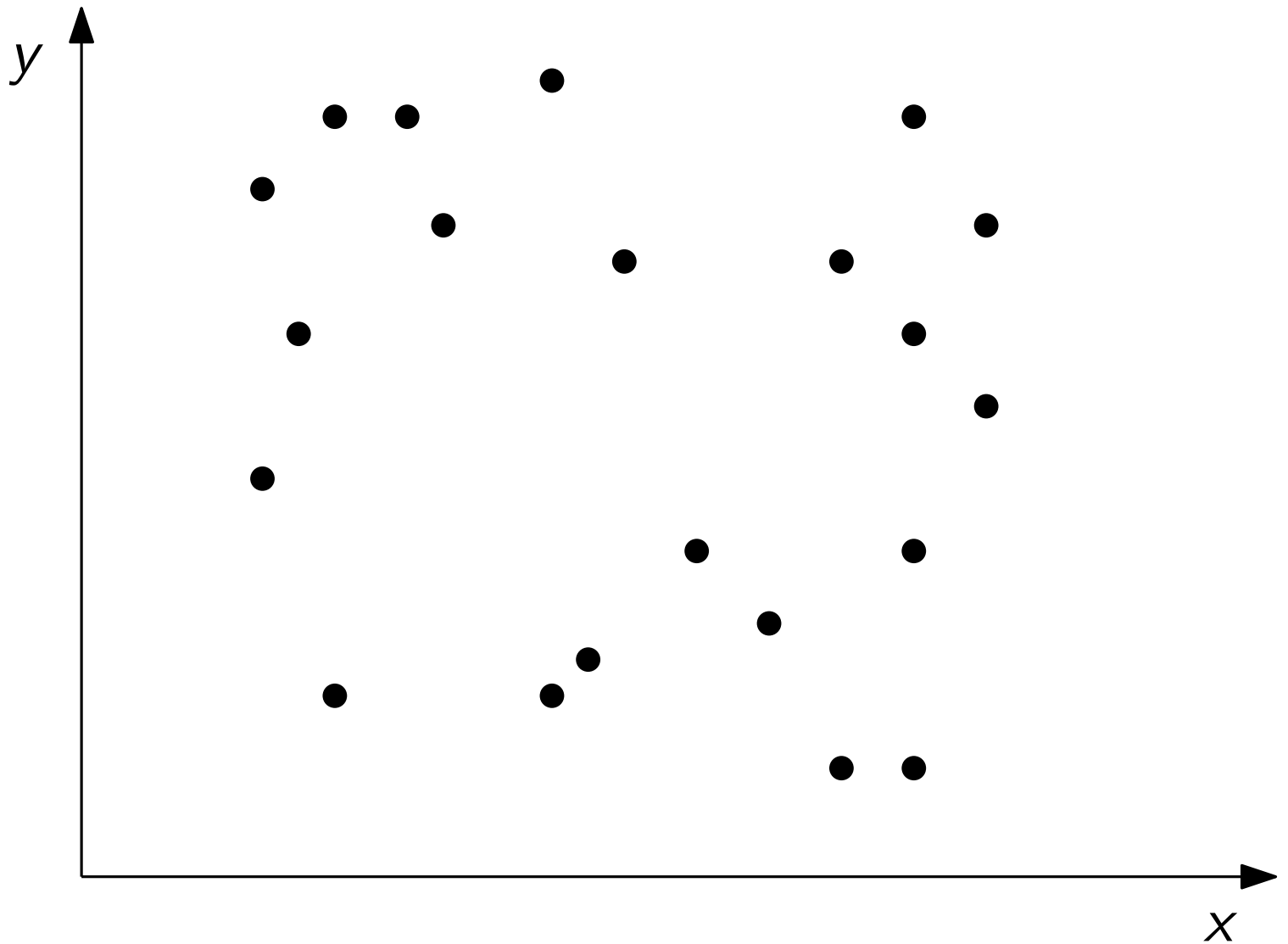

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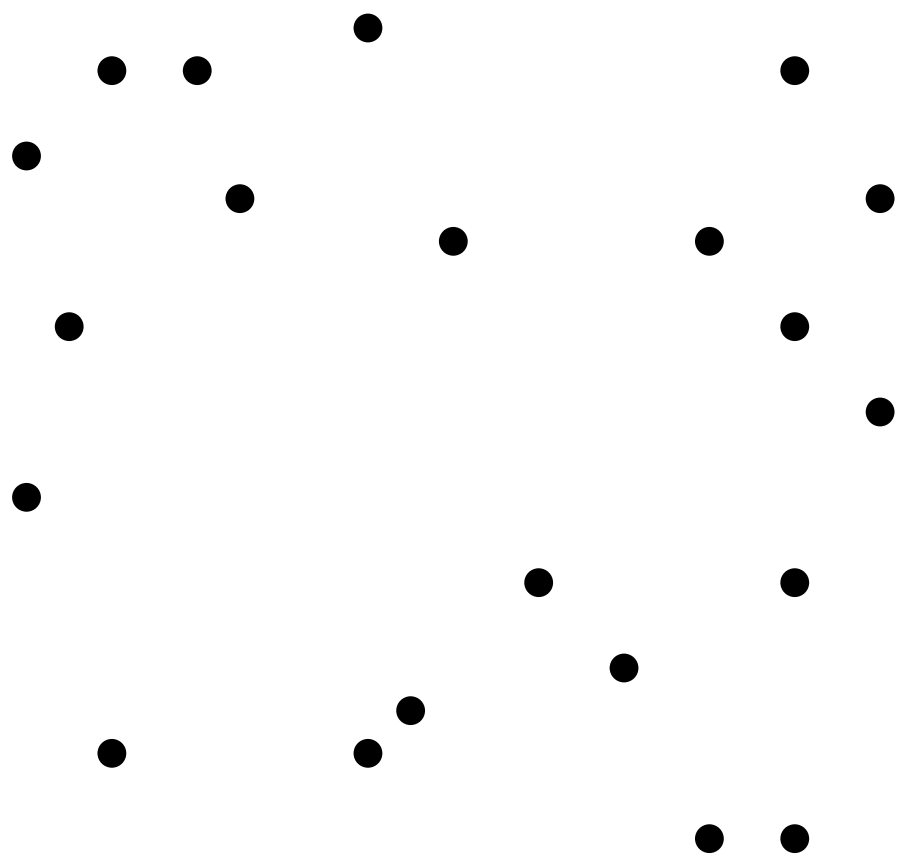

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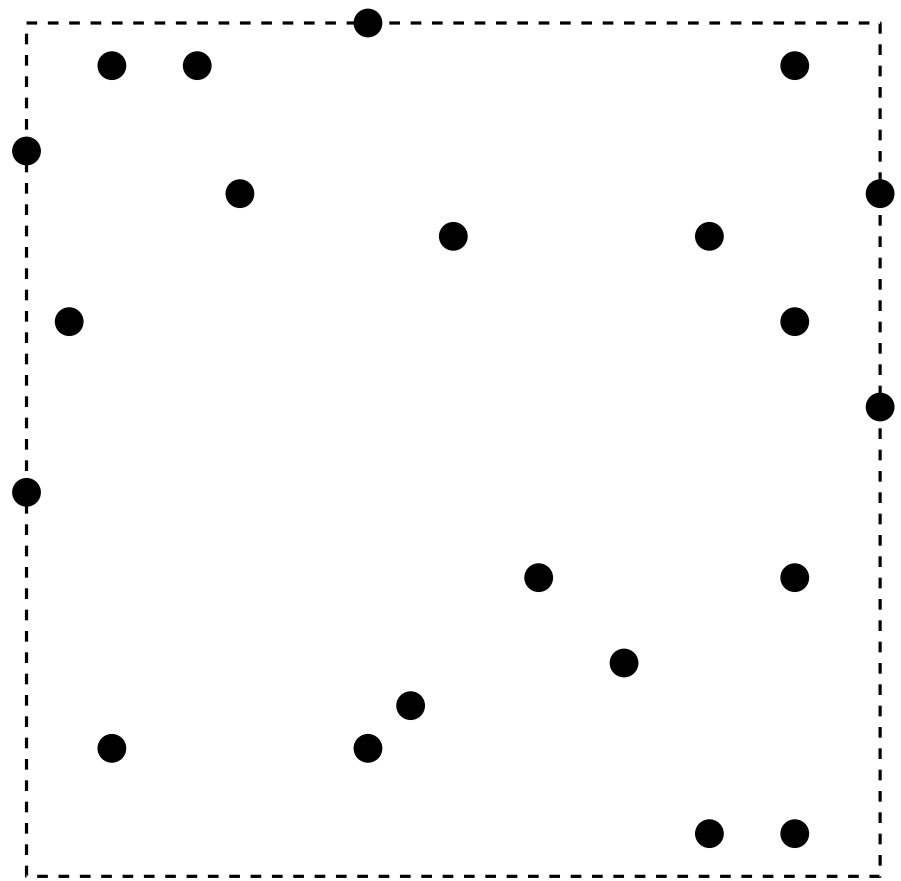

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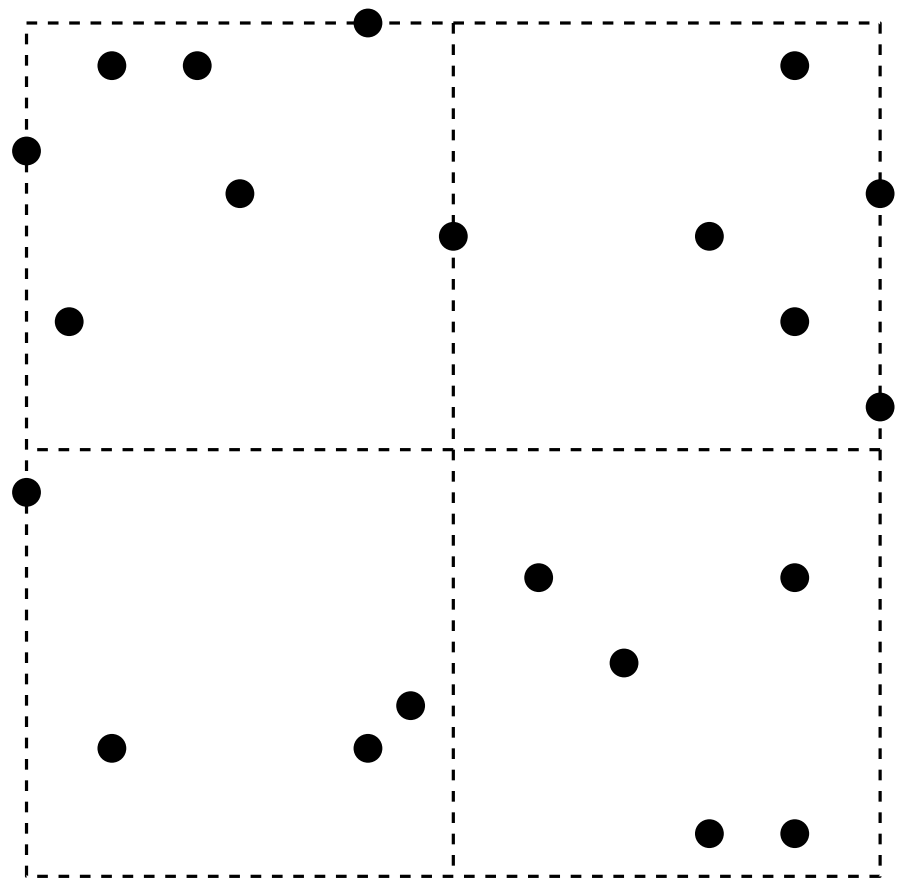

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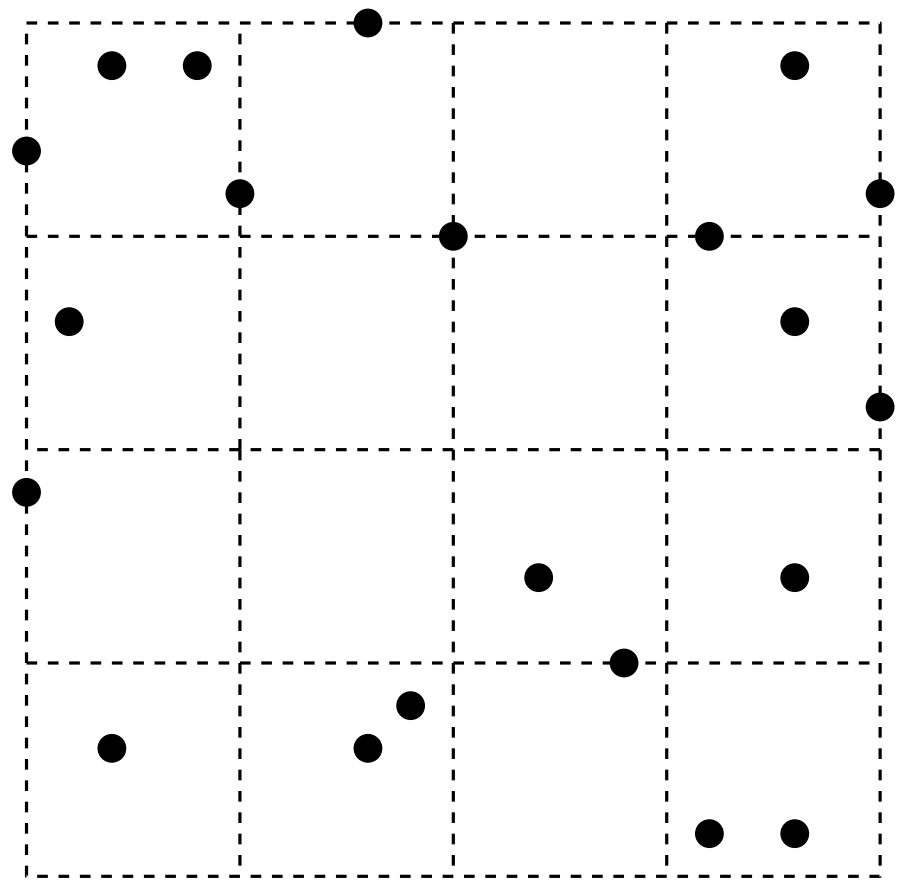

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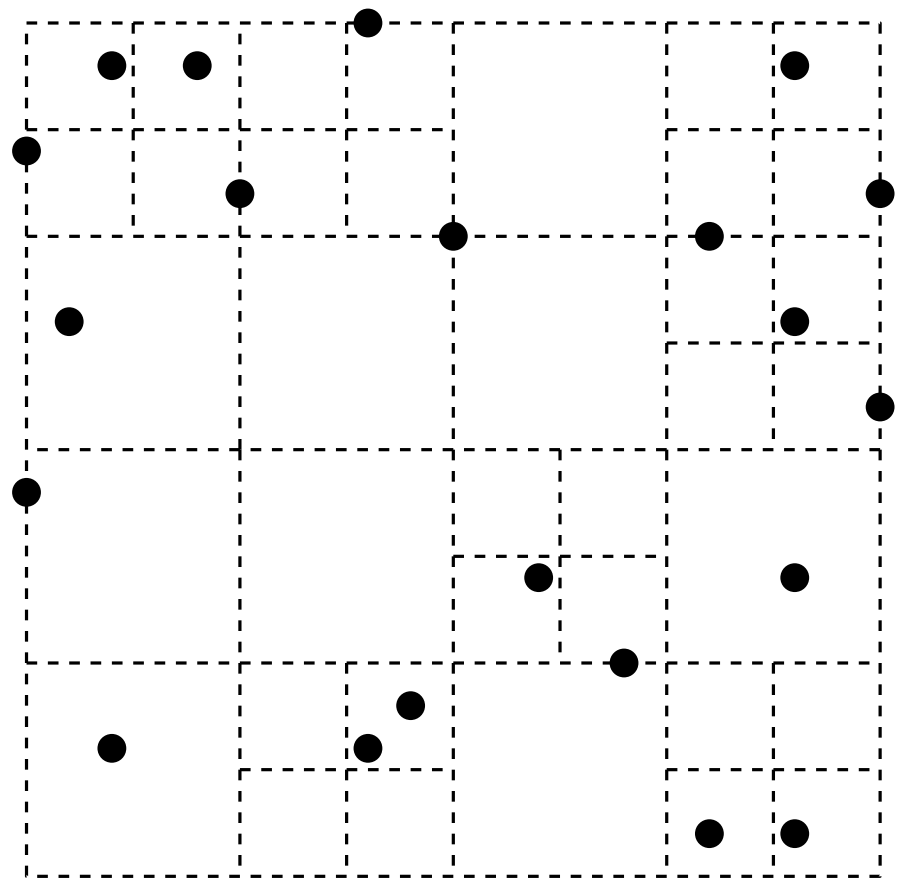

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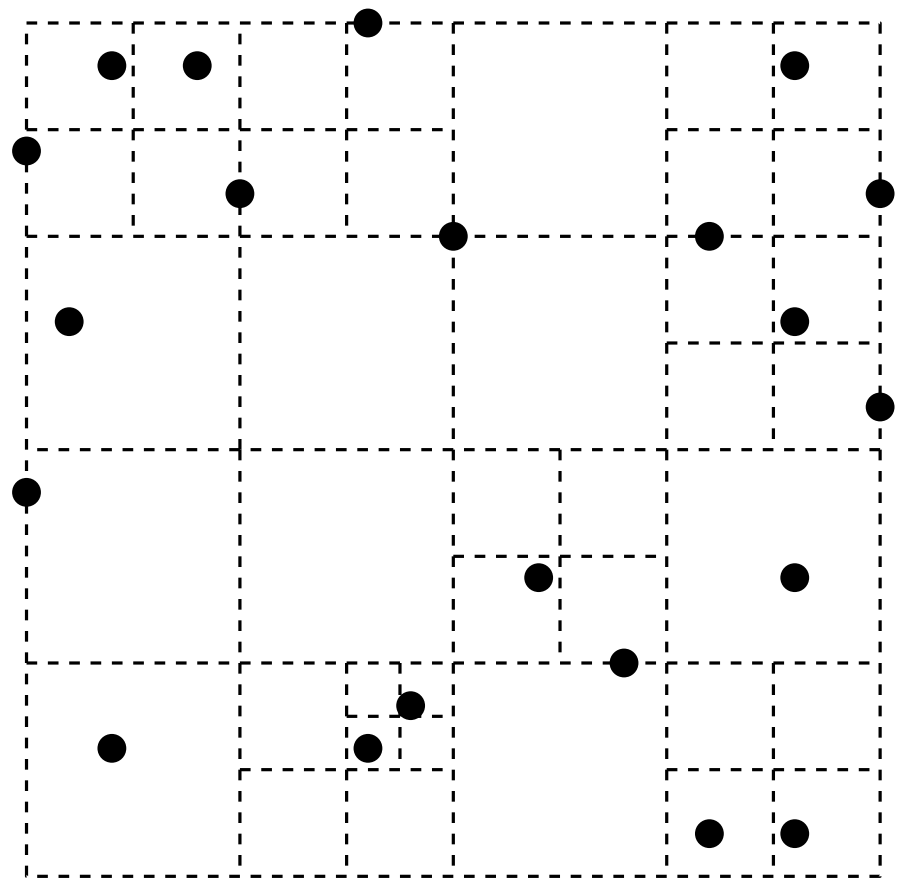

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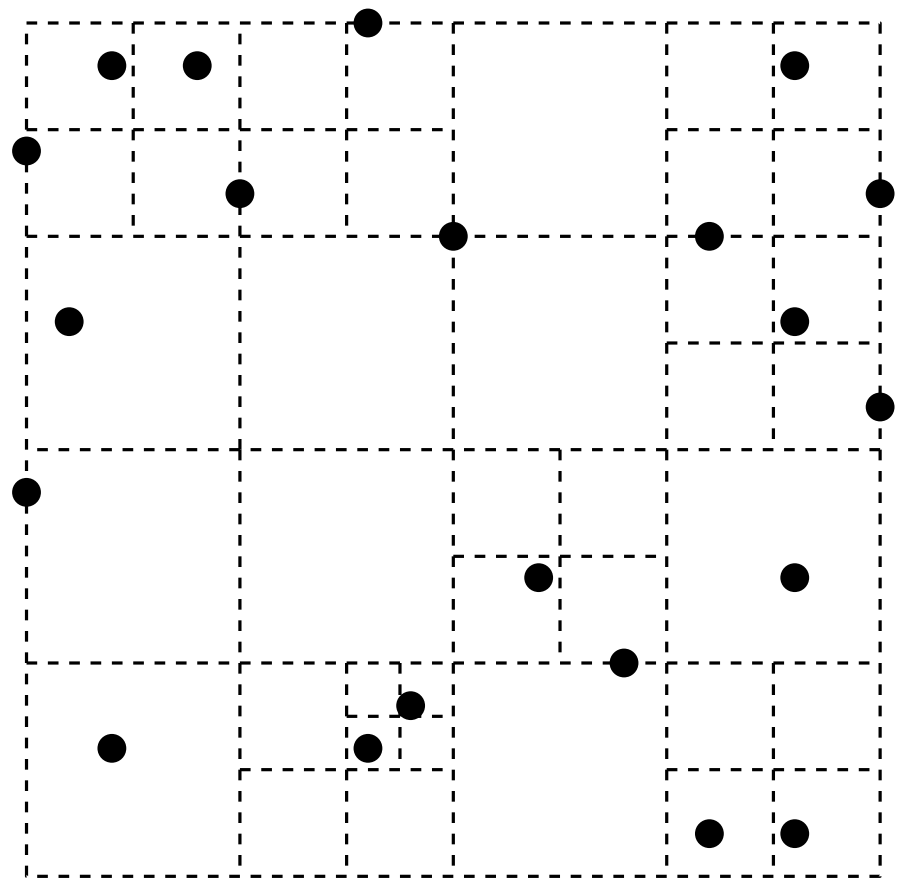
- Problem: quadtree height unbounded w.r.t. #points



Quadtree Centroid

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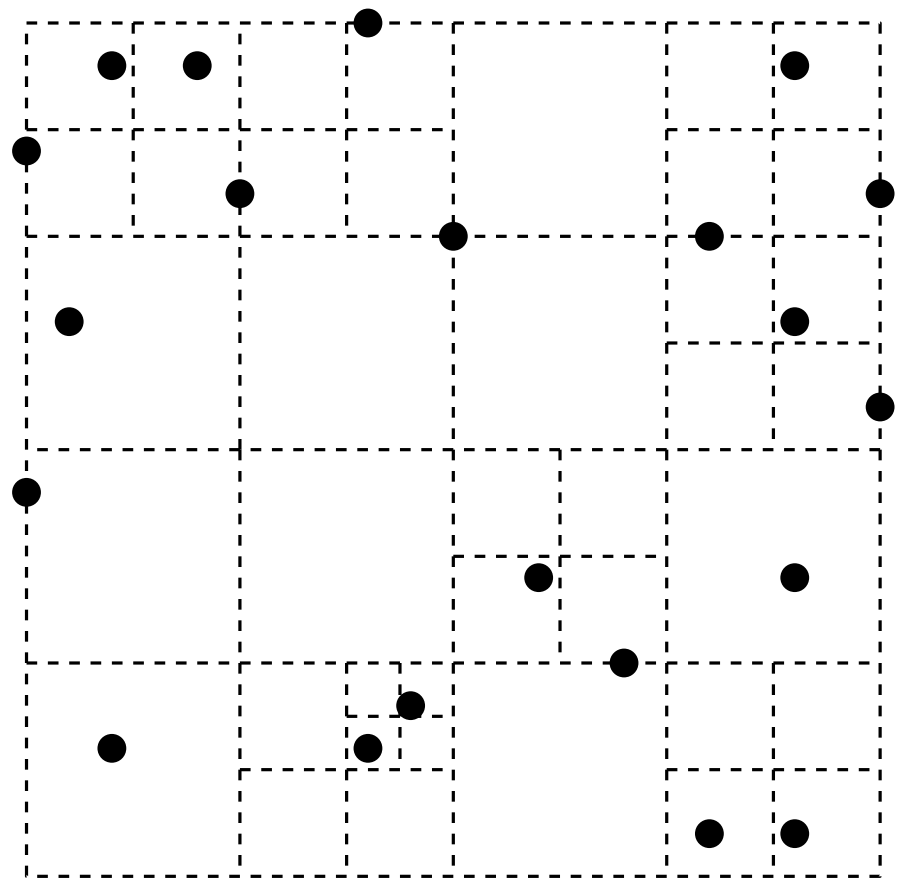

Quadtree Centroid

- For a quadtree on n points P
- $\exists$ cell B s.t.
  - $\diamond$ $|P \cap B| \leq \alpha n$
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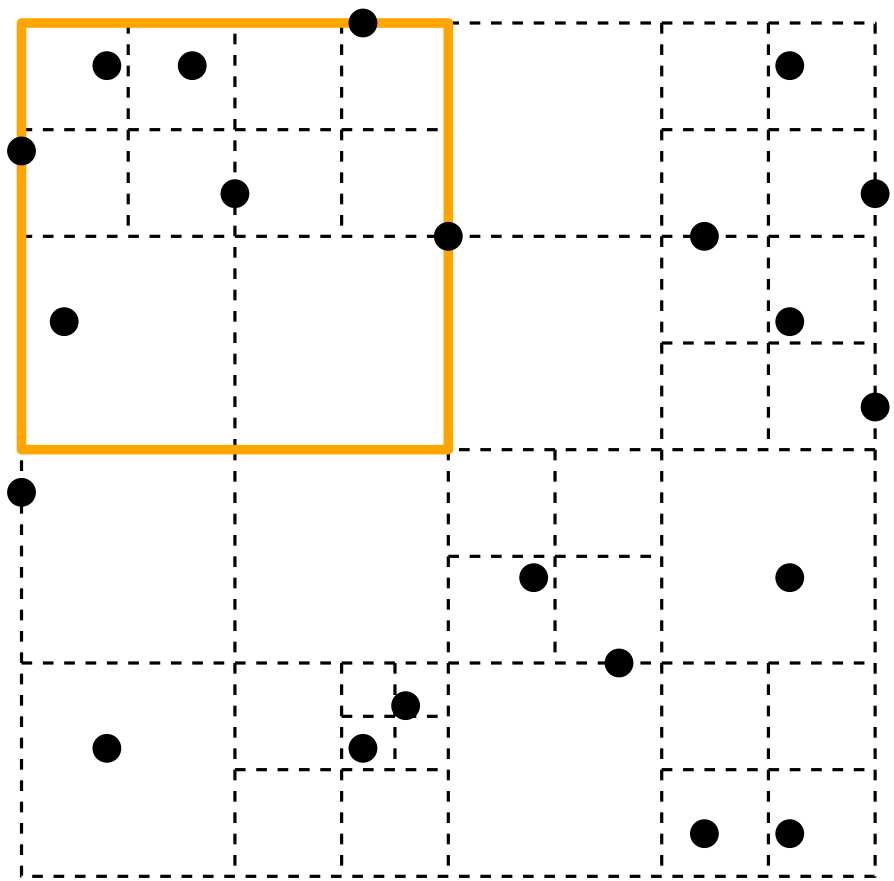

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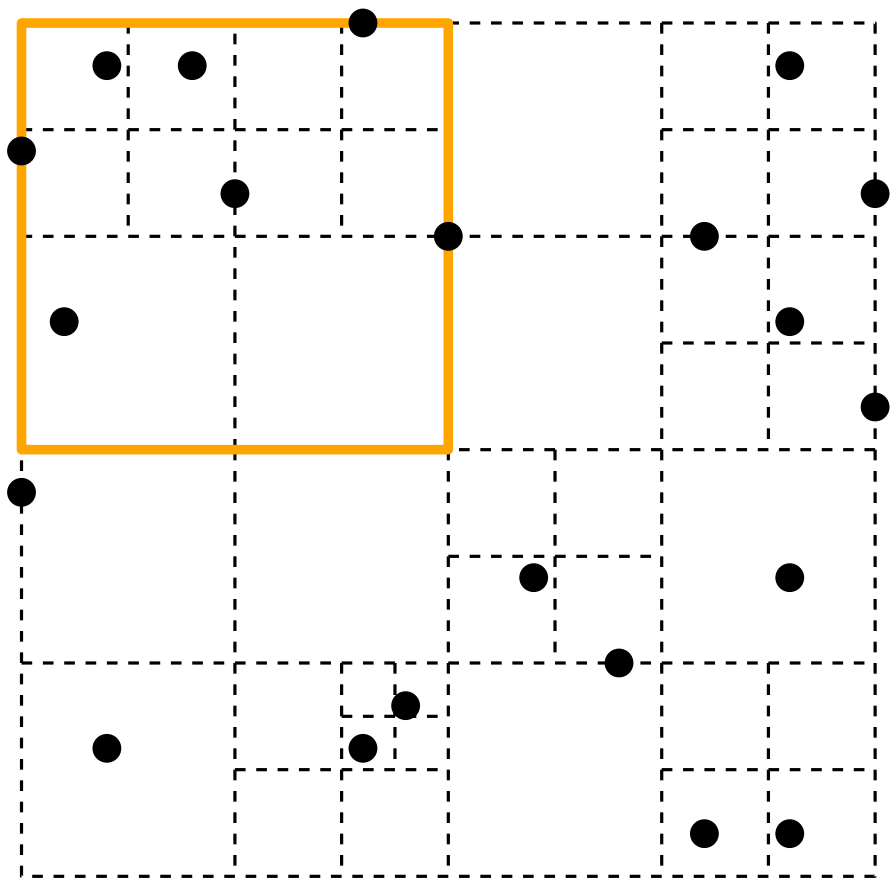

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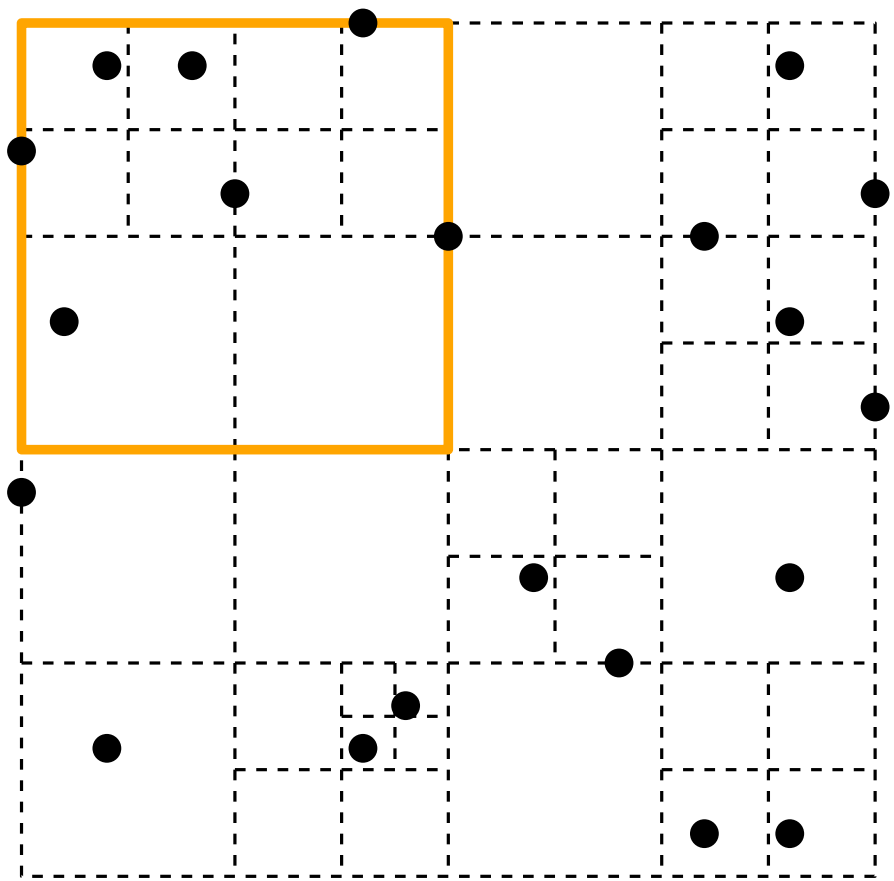

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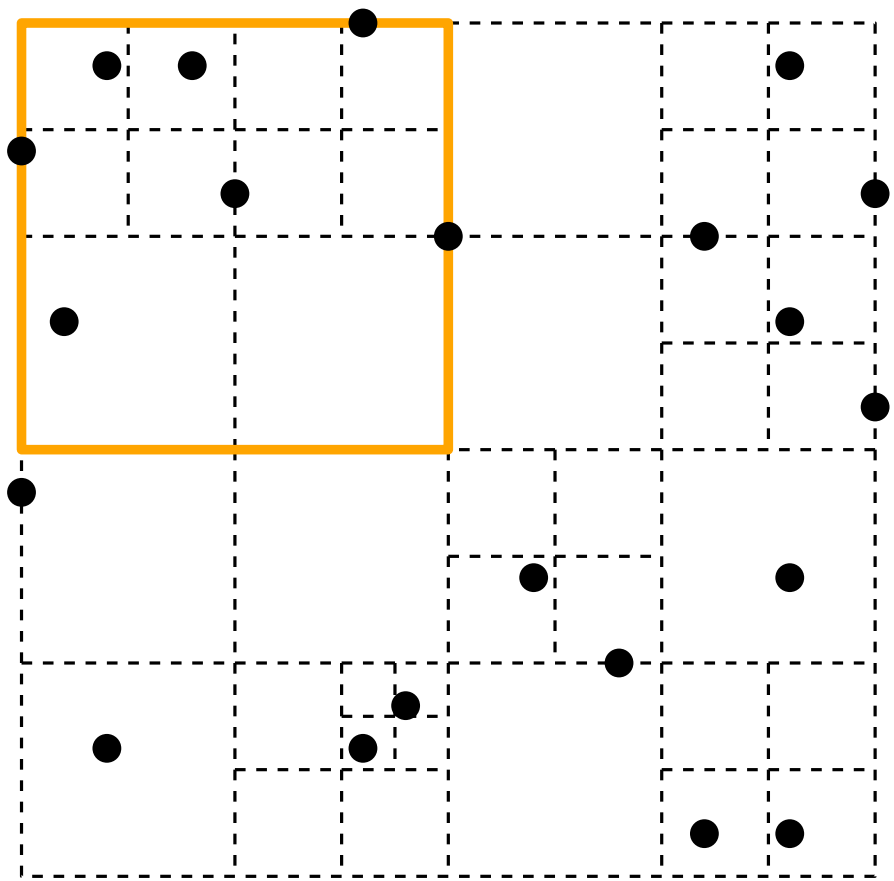

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- At each level, each $p \notin B$.

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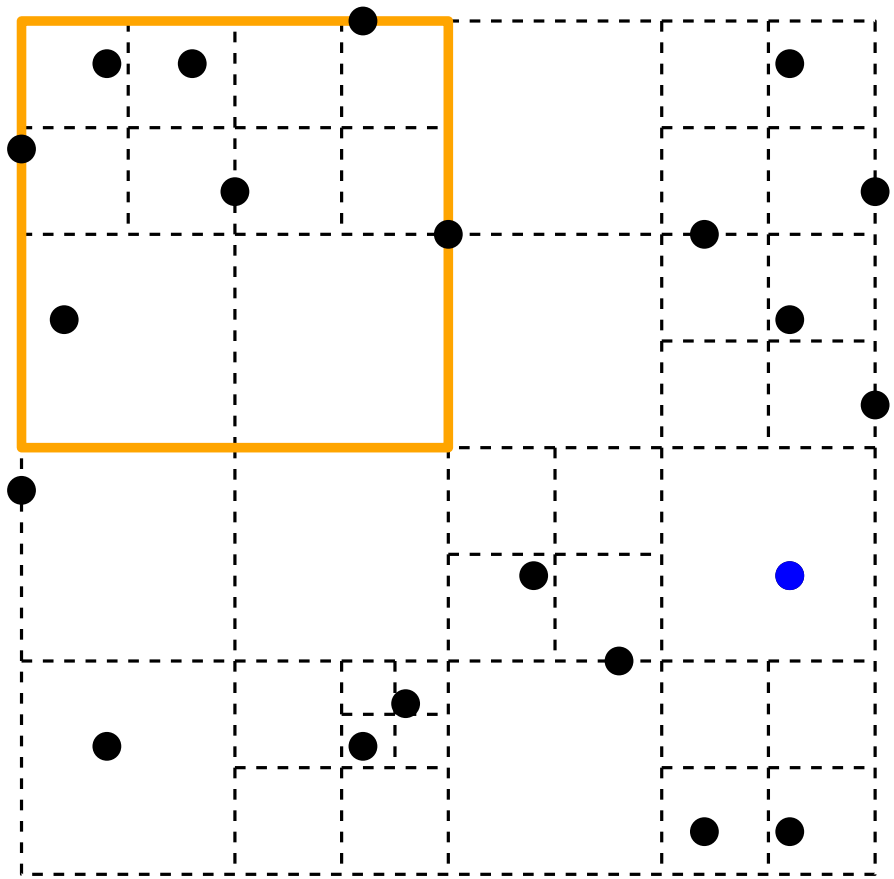

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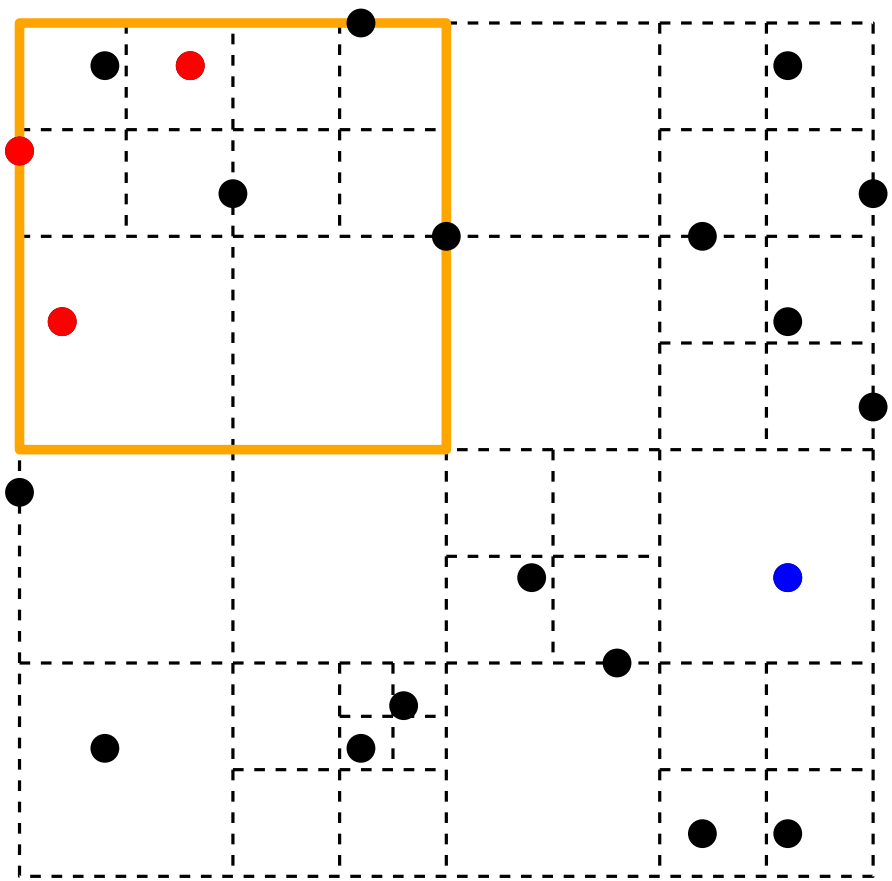

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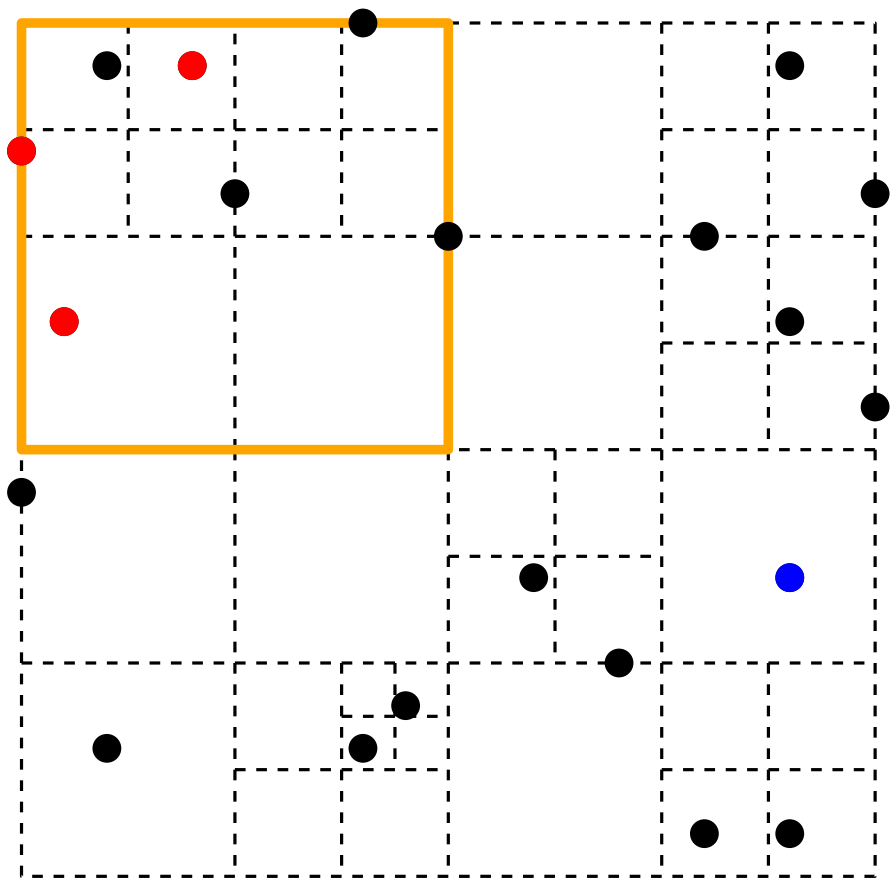

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Reduction to 2D Dominance Range Minimum

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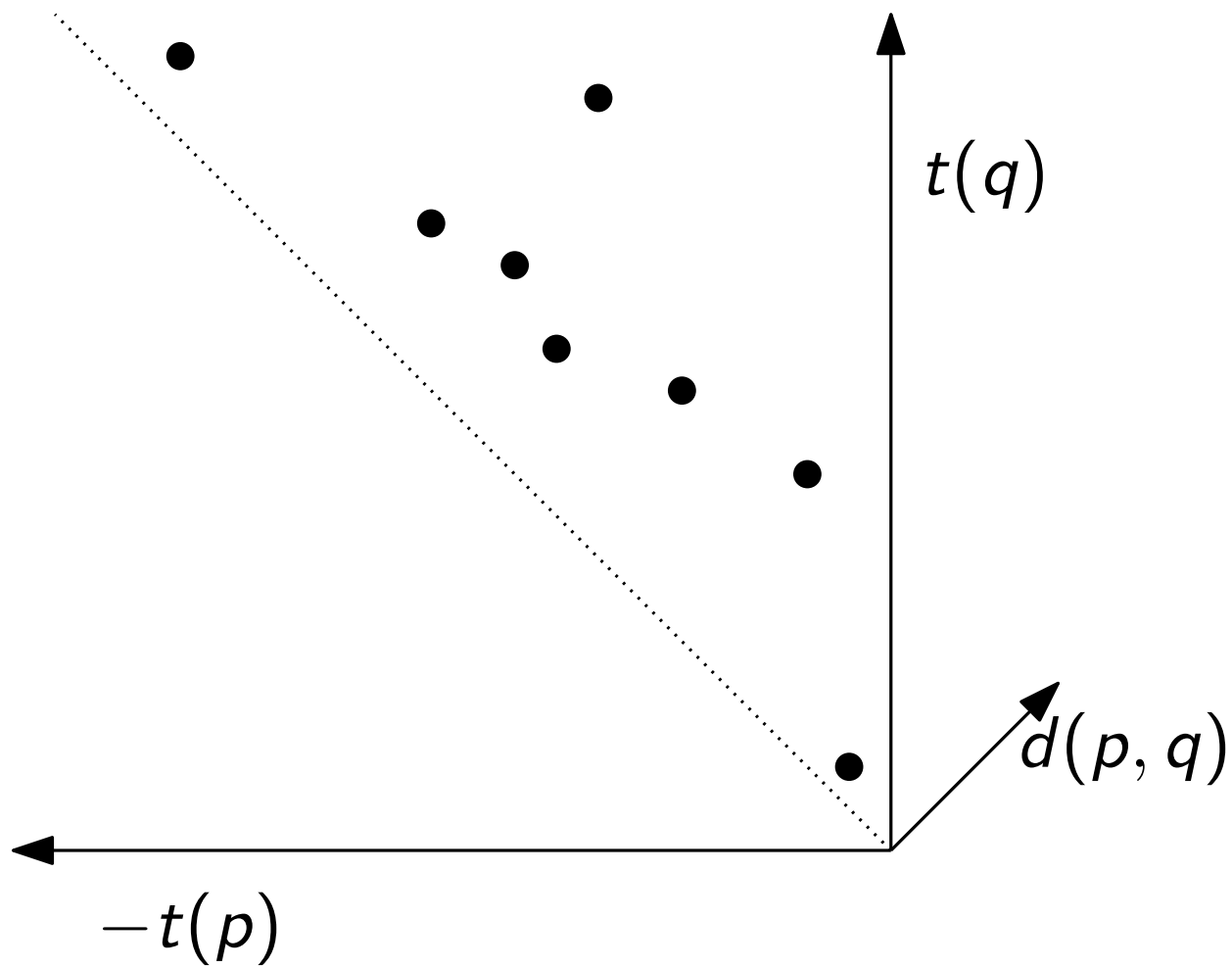

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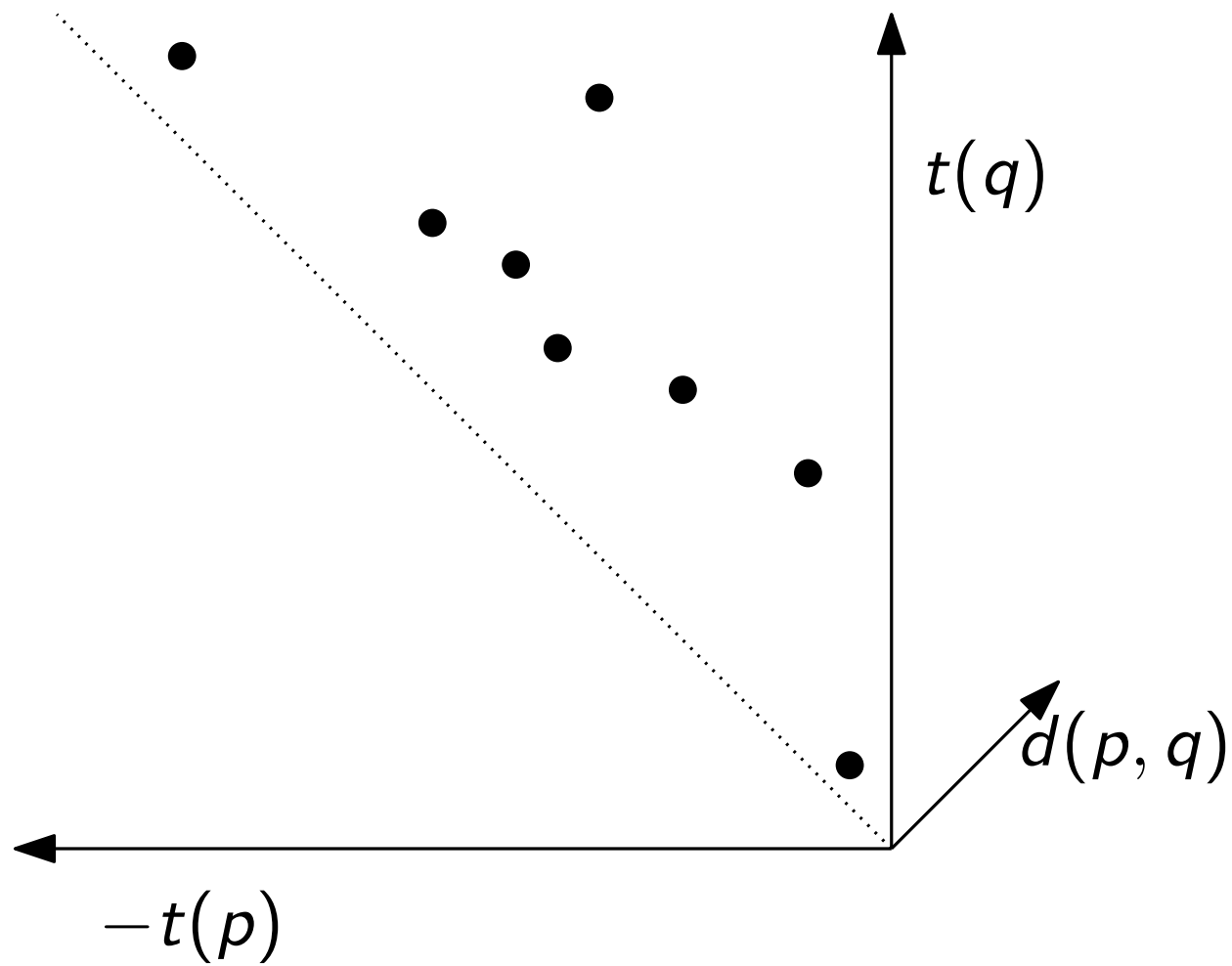

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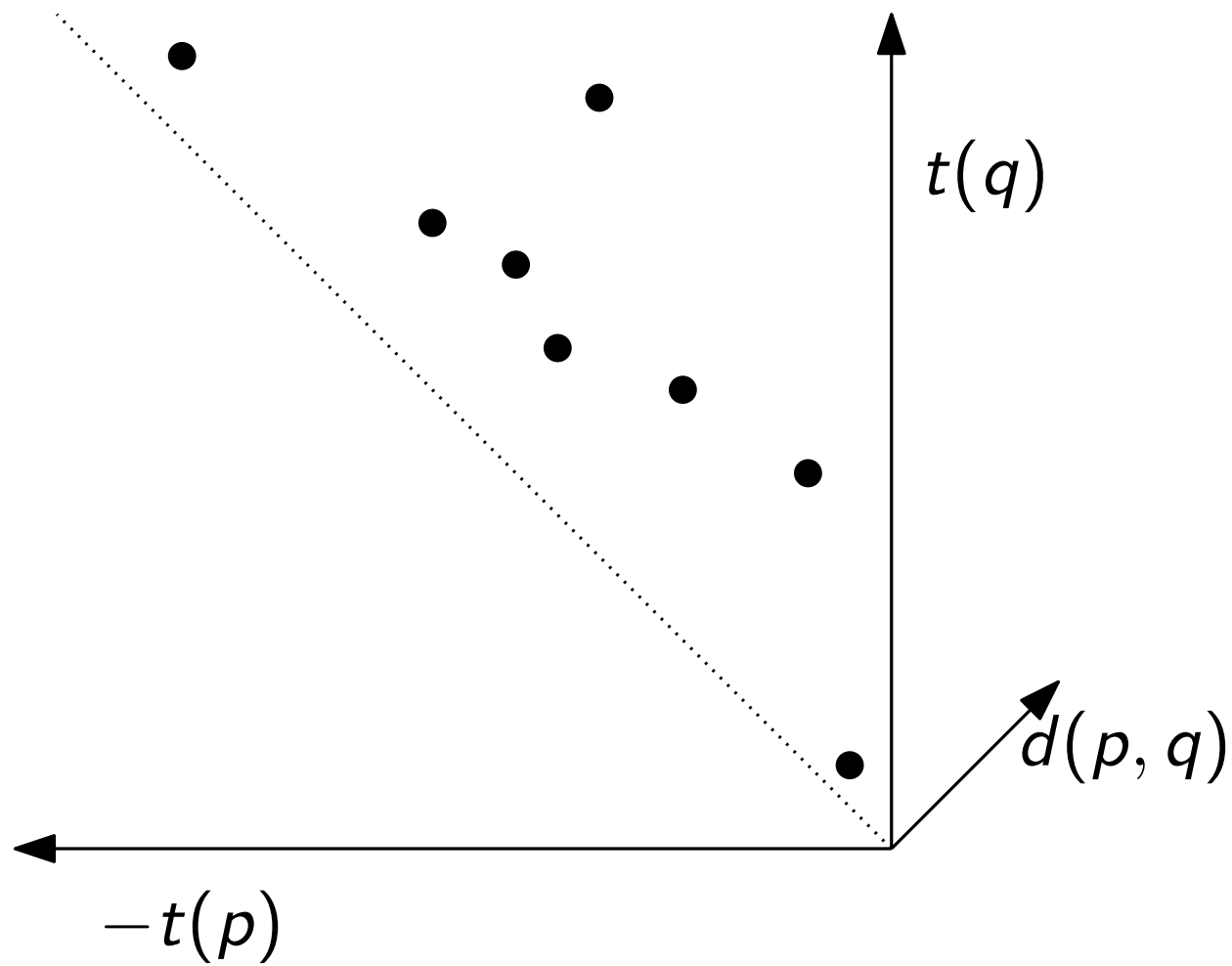

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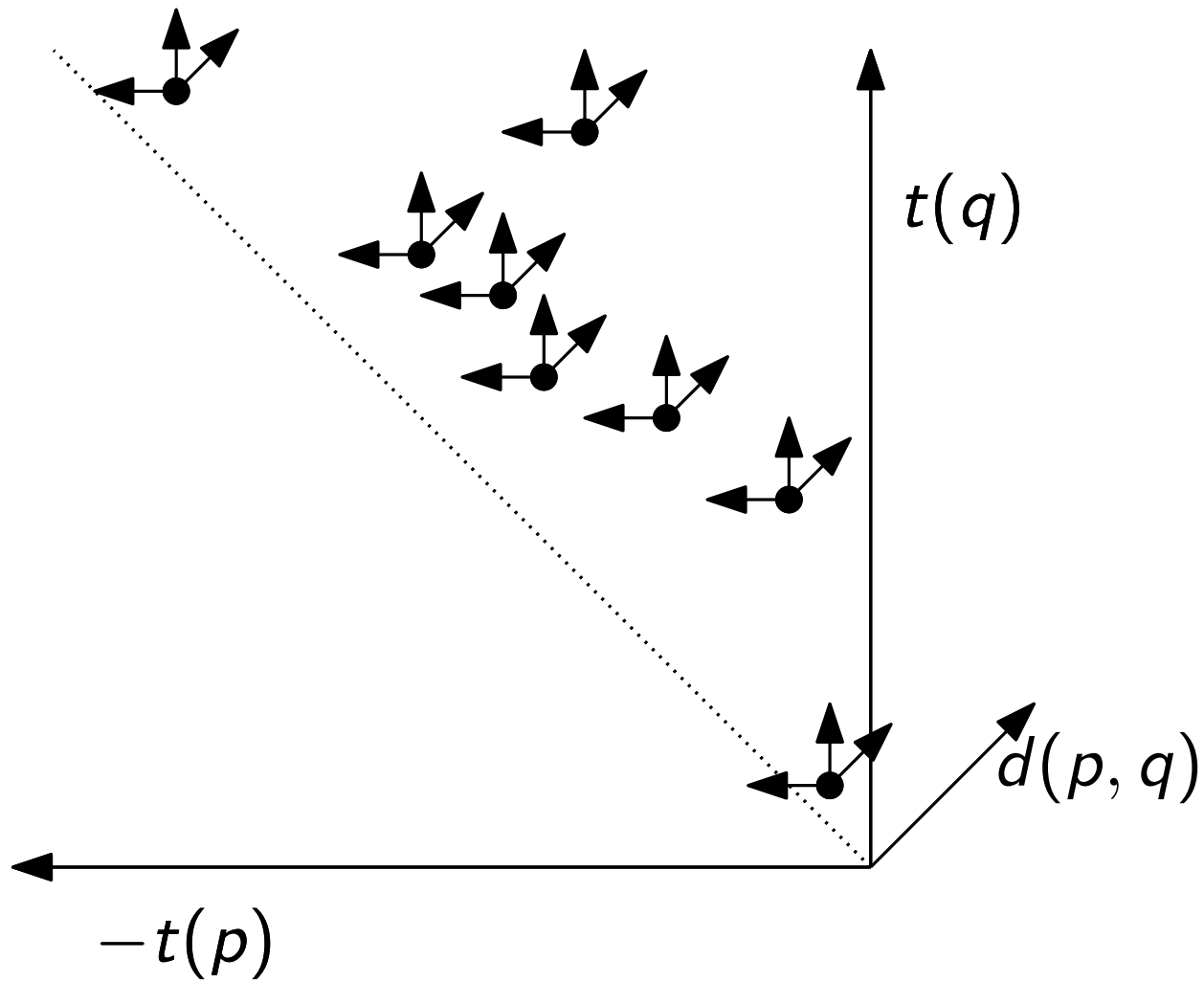

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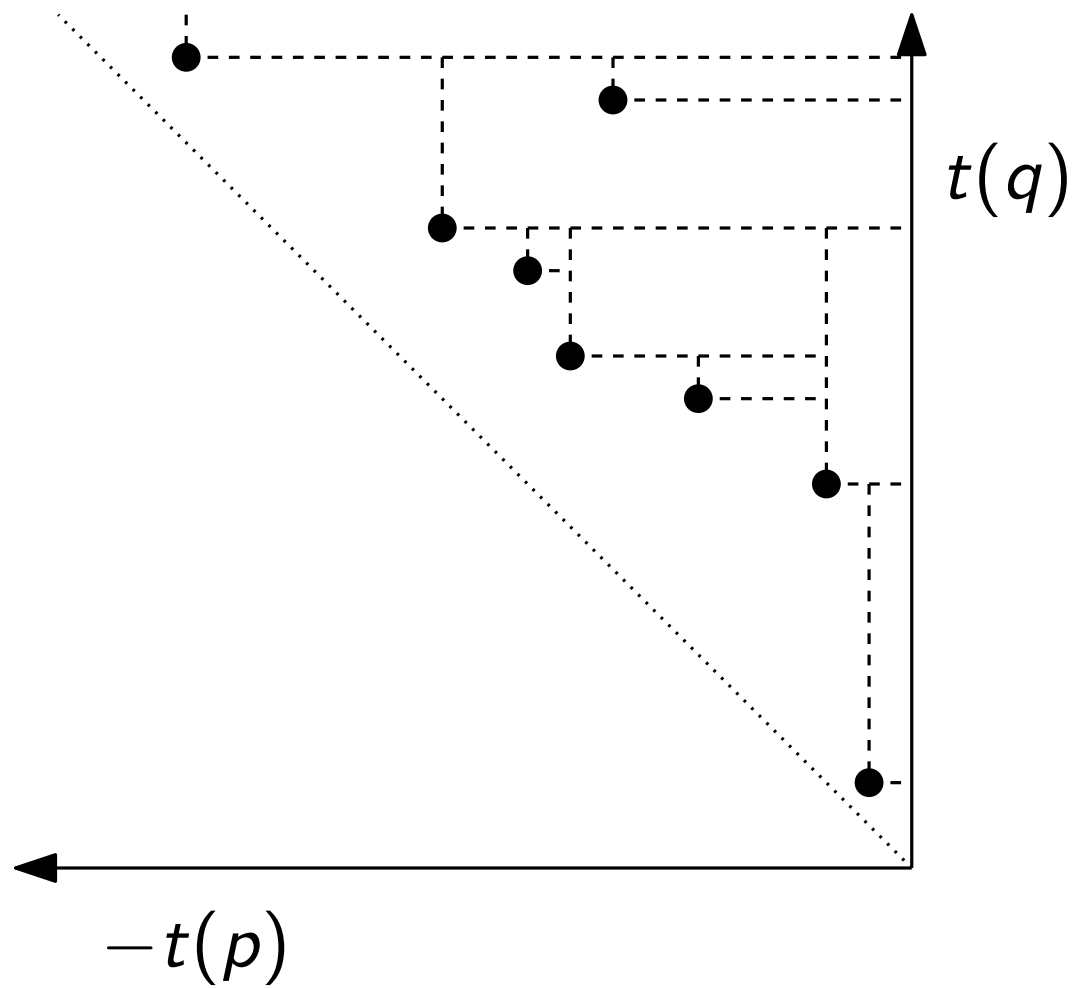

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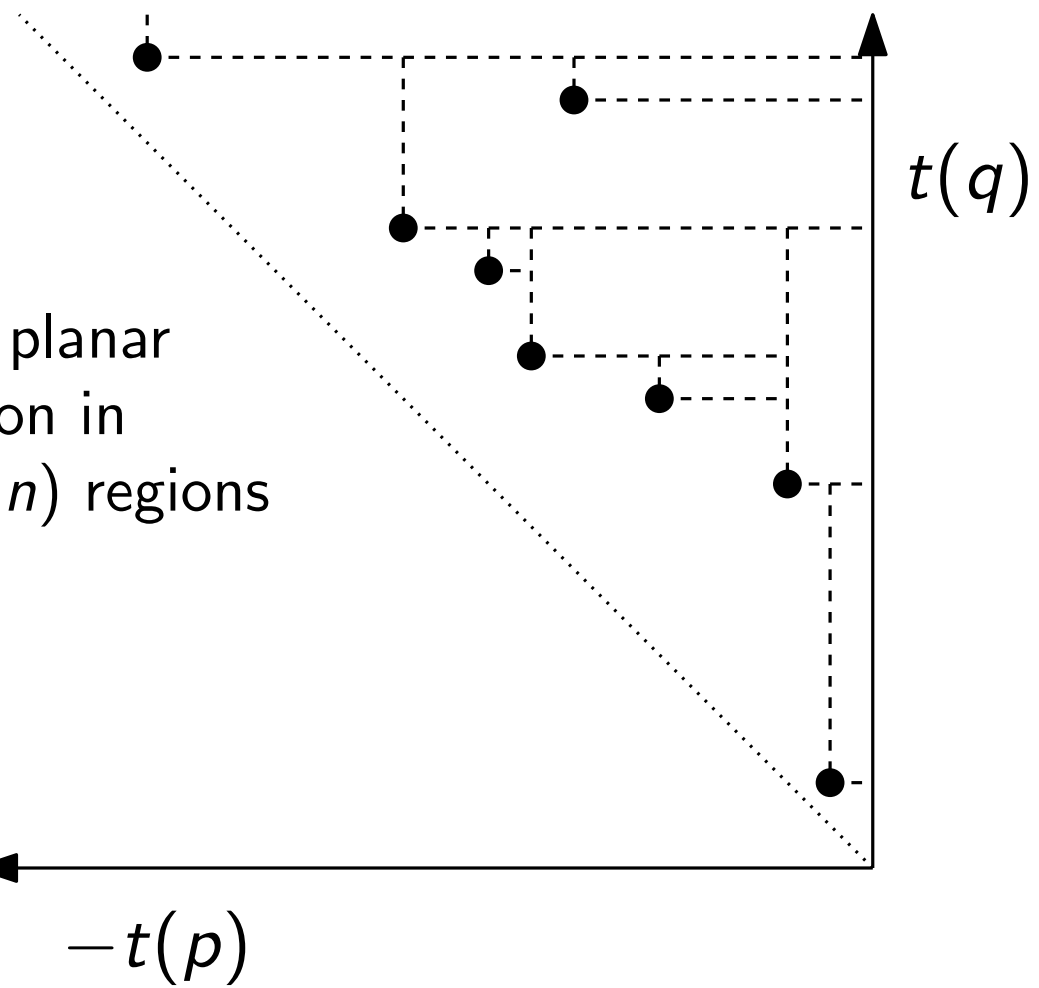

- Orthogonal planar point location in $N = O(n \lg n)$ regions

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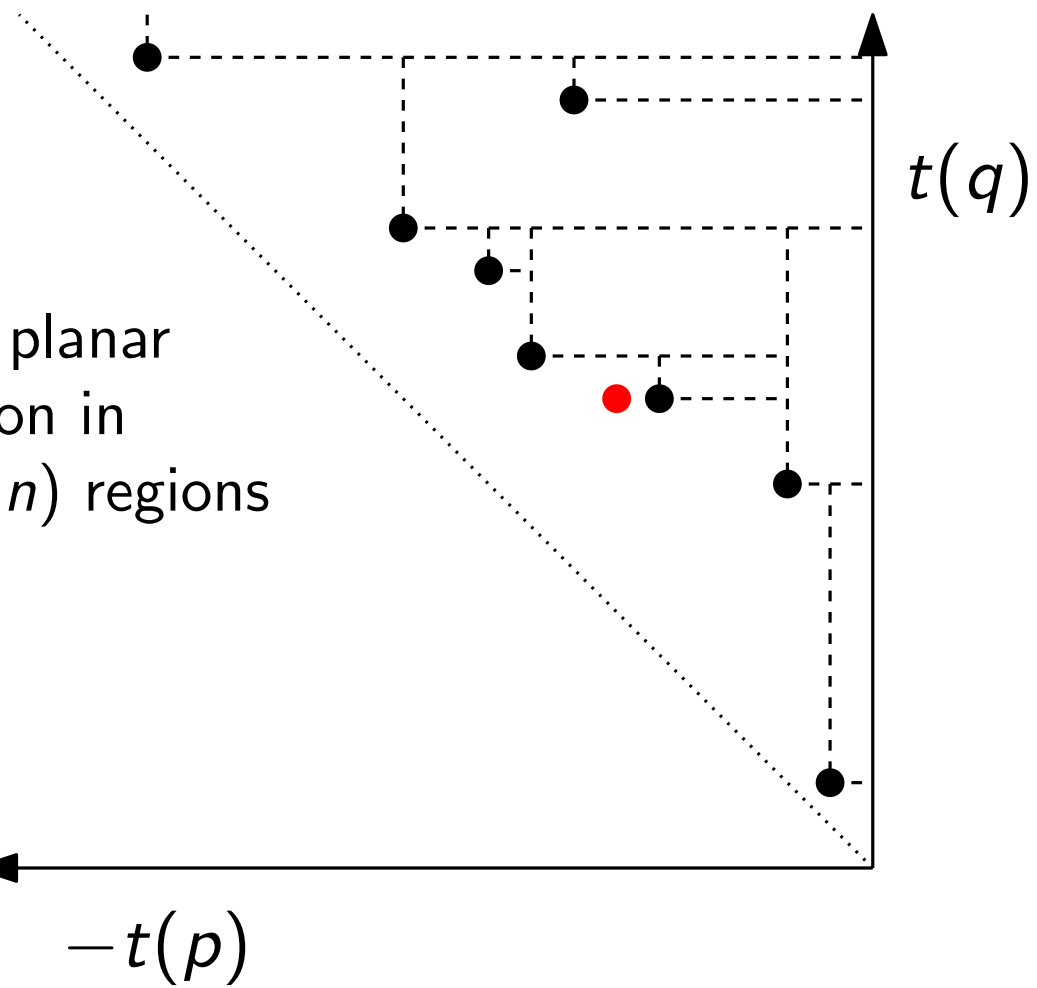

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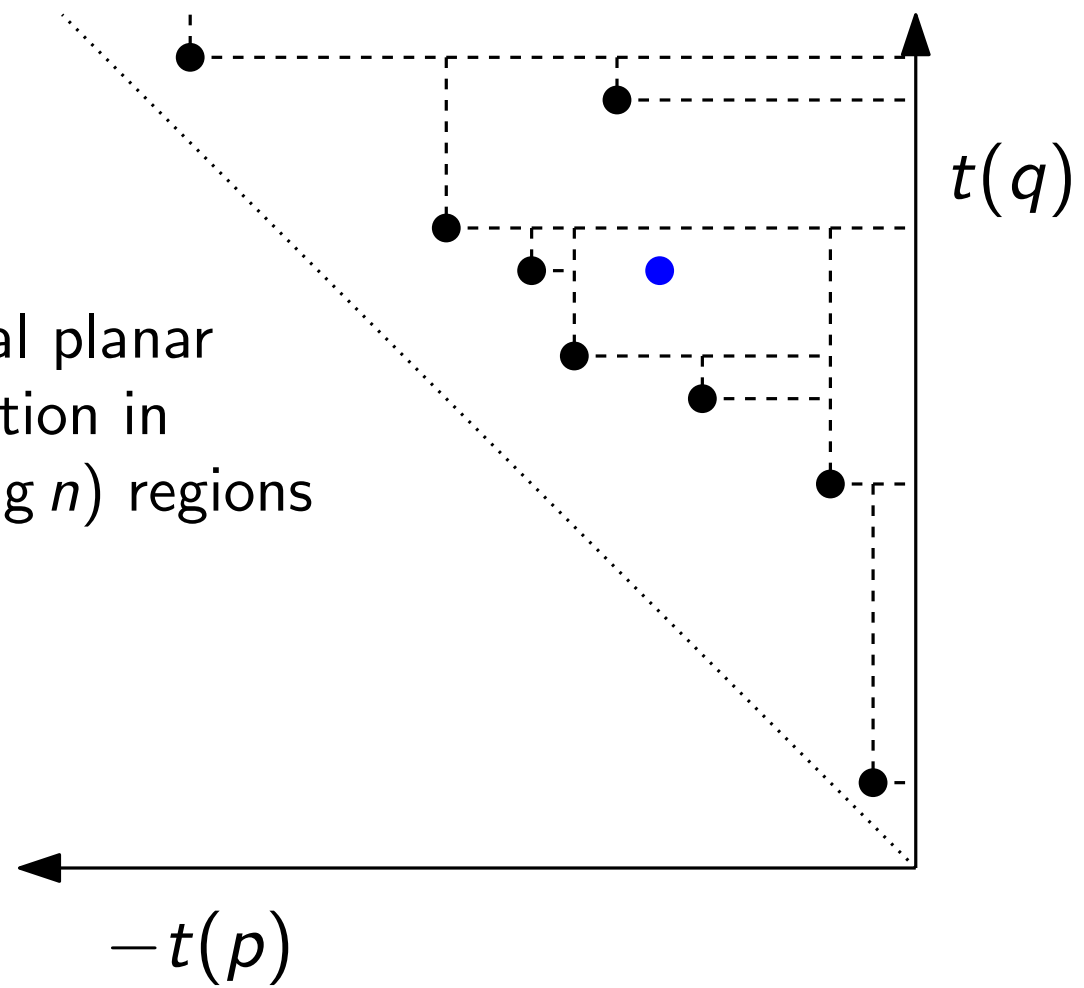

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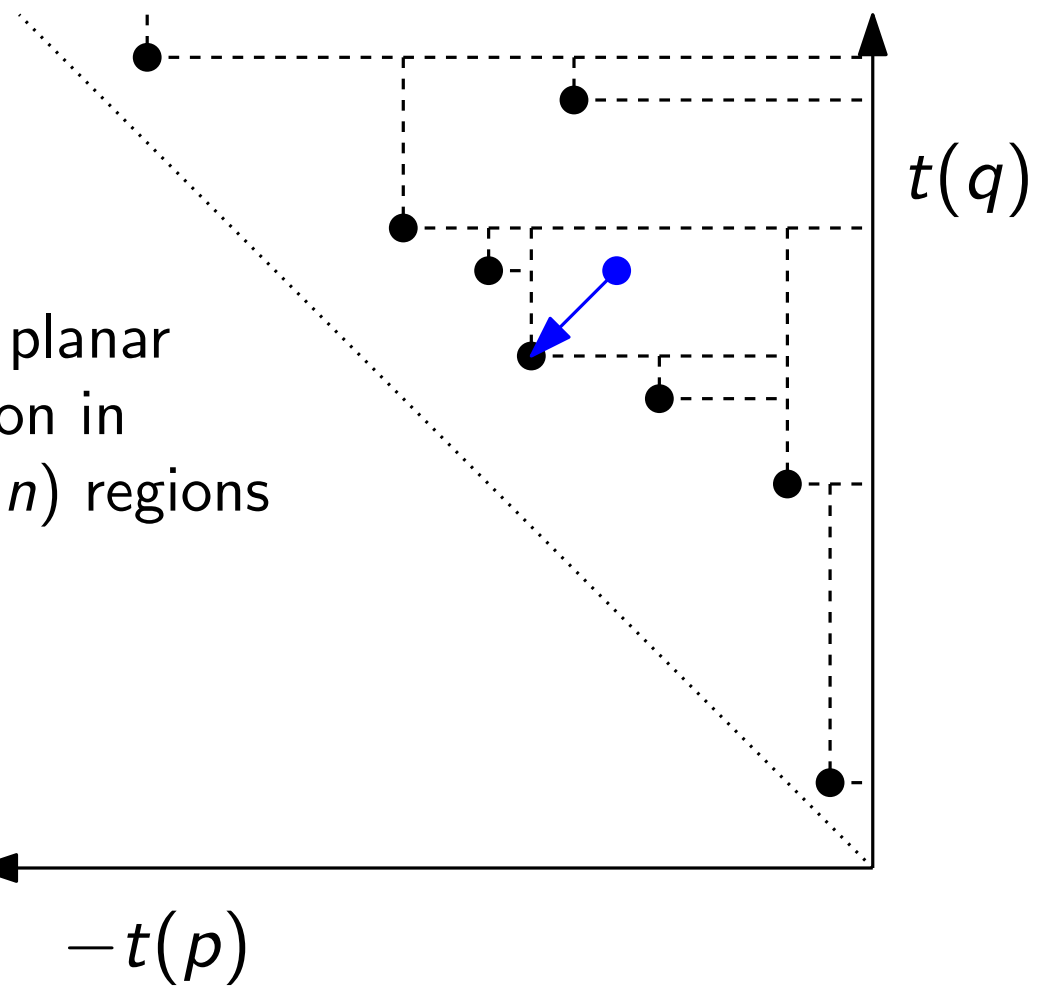

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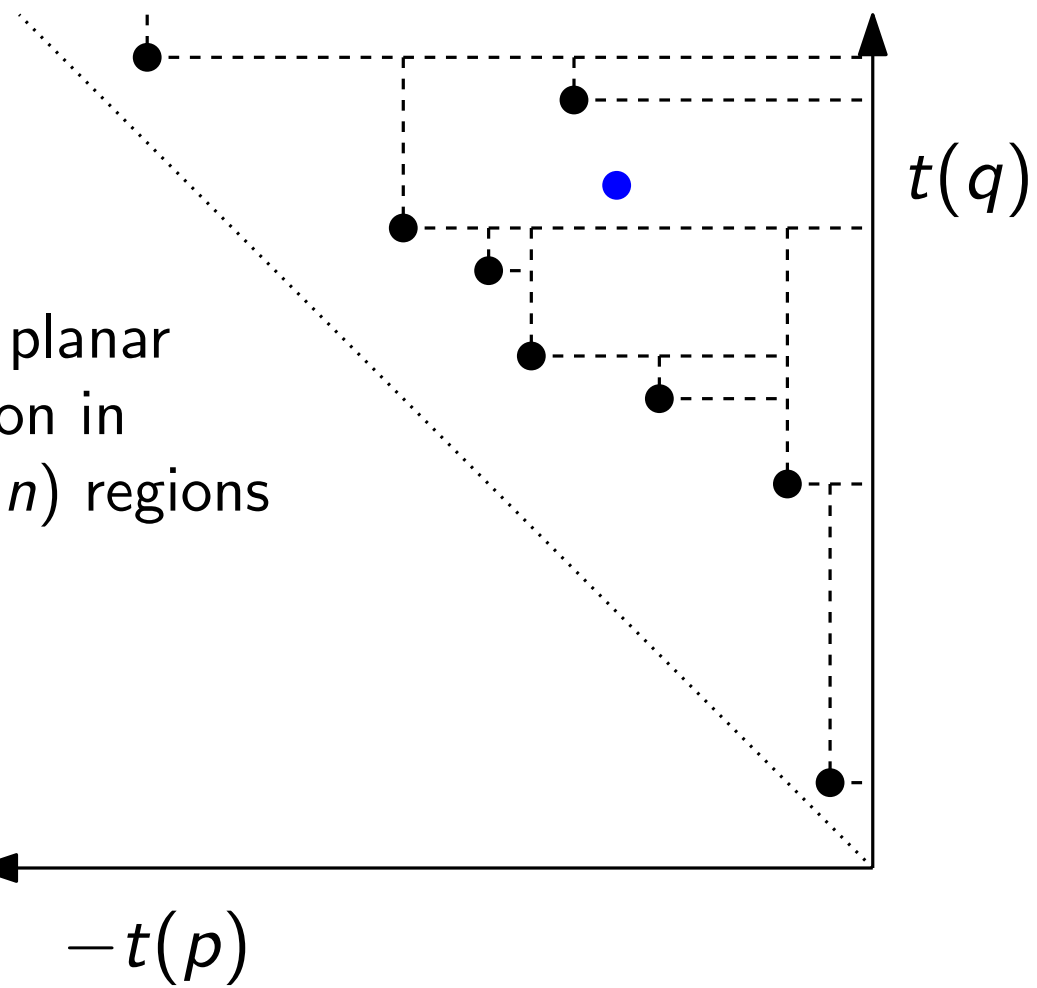

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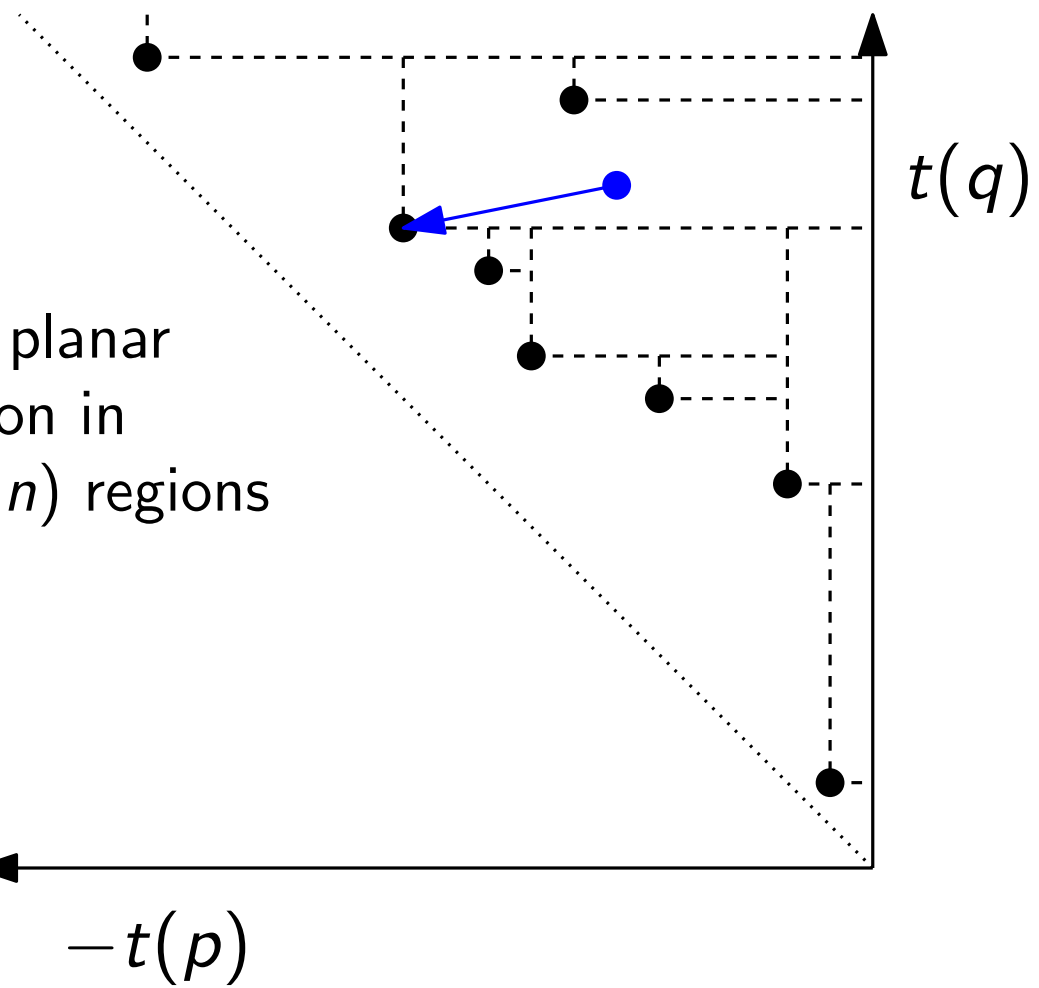

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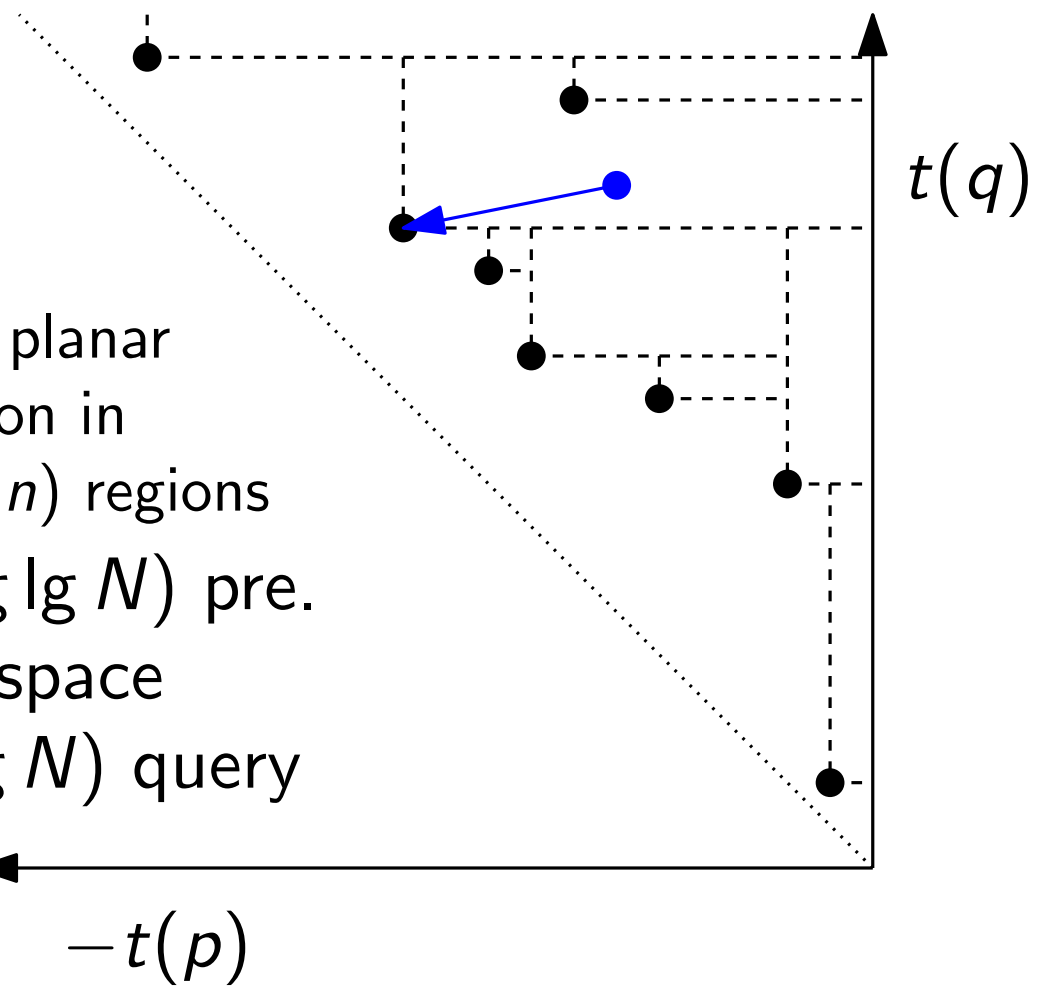

- Orthogonal planar point location in $N = O(n \lg n)$ regions
  - $O(N \lg \lg N)$ pre.
  - $O(N)$ space
  - $O(\lg \lg N)$ query

Grids & Quadtrees: Conclusion

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Closest Pair Decision

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- Grids with unit side-length, shifting, succinct rank/select
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Closest Pair

- Quadtrees + centroid
- Reduction to 2D dominance range min.
- $O(n \log n \log \log n)$ preprocessing time
- $O(n \log n)$ space
- $O(\log \log n)$ query time

New Results

Approach #1: Grids & Quadtrees

Problem	Preprocessing	Space	Query
CP Decision	$O(n)$ exp.	$O(n)$ bits	$O(1)$
Closest Pair	$O(n \lg n \lg \lg n)$	$O(n \lg n)$	$O(\lg \lg n)$

Approach #2: Generalized Range Successor

Problem	Preprocessing Time
2D diameter decision	$O(n \lg n)$
3D diameter decision	$O(n \lg^2 n)$
2D orth. seg. intersection detection	$O(n \lg n \lg \lg n)$

Approach #3: FIFO Update Sequence

Problem	Preprocessing Time
2D convex hull area decision	$O(n\alpha(n) \lg n)$
2D width decision	$O(n \lg^7 n)$

New Results

Approach #1: Grids & Quadtrees

Problem	Preprocessing	Space	Query
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Closest Pair	$O(n \lg n \lg \lg n)$	$O(n \lg n)$	$O(\lg \lg n)$

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New Results

BUT FIRST!

Approach #1: Grids & Quadtrees

Problem	Preprocessing	Space	Query
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Closest Pair	$O(n \lg n \lg \lg n)$	$O(n \lg n)$	$O(\lg \lg n)$

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Time-Windowed Decision Problems

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From here on,
only time-windowed decision problems!

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. . . with *hereditary* properties!

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If a set S has a *hereditary* property $\mathcal{P}$, then any superset $S' \supseteq S$ has $\mathcal{P}$.

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Examples:

Problem	Property
diameter decision	diameter > 1
convex hull area decision	area of convex hull > 1
orthogonal segment inters. detect.	$\exists$ intersecting segments
width decision	width > 1

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Problem	Property
diameter decision	diameter > 1
convex hull area decision	area of convex hull > 1
orthogonal segment inters. detect.	$\exists$ intersecting segments
width decision	width > 1

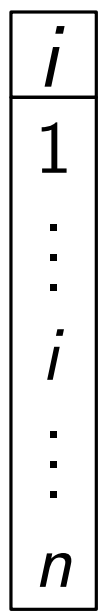
Intuition:

Adding more points won't remove these properties.

Hereditary Properties

If a set S has a *hereditary* property $\mathcal{P}$, then any superset $S' \supseteq S$ has $\mathcal{P}$.

For each time i:



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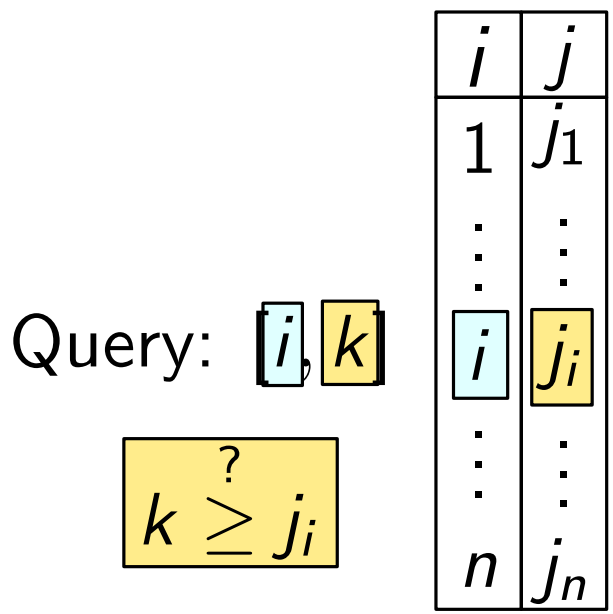

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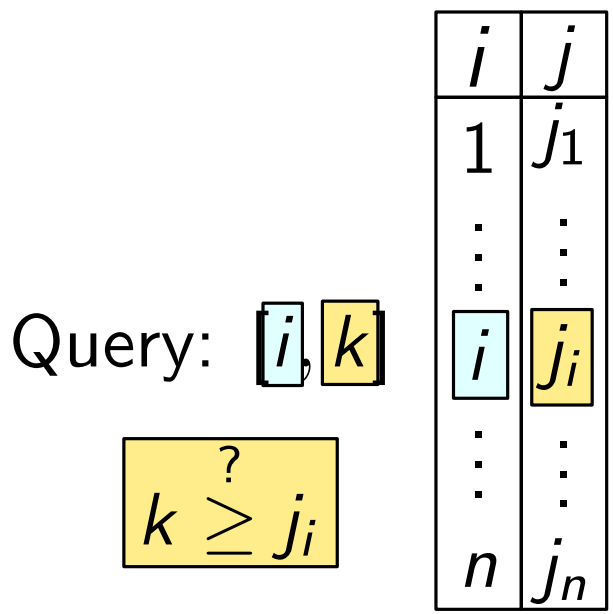

Monotone chain

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$O(n)$ bits
$O(1)$ query time
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Henceforth, only concerned with preprocessing.

More Related Work

Bokal, Cabello, Eppstein; *SoCG 2015*:

- Time-windowed decision problems w/ hereditary properties
  - ◇ 2D diameter
    - · Preprocessing: $O(n \log^2 n)$
  - ◇ 2D convex hull area
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New Results

Approach #1: Grids & Quadtrees

Problem	Preprocessing	Space	Query
CP Decision	$O(n)$ exp.	$O(n)$ bits	$O(1)$
Closest Pair	$O(n \lg n \lg\lg n)$	$O(n \lg n)$	$O(\lg\lg n)$

Approach #2: Generalized Range Successor

Problem	Preprocessing Time
2D diameter decision	$O(n \lg^2 n)$
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Approach #3: FIFO Update Sequence

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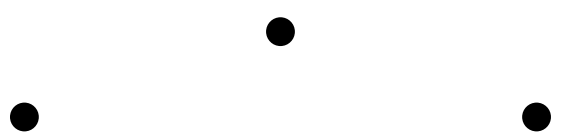
Example problem: is there a pair of points p, q s.t. $d(p, q) > 1$?



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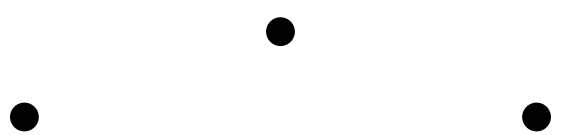

Range of a point p: the complement of a unit disk centered at p

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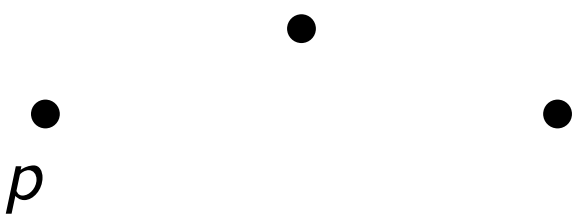

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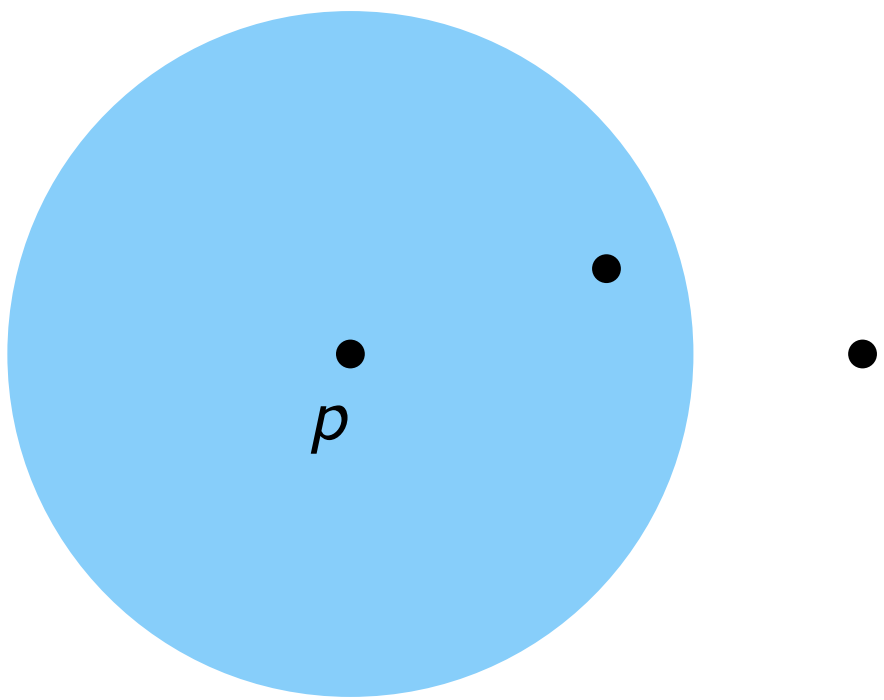

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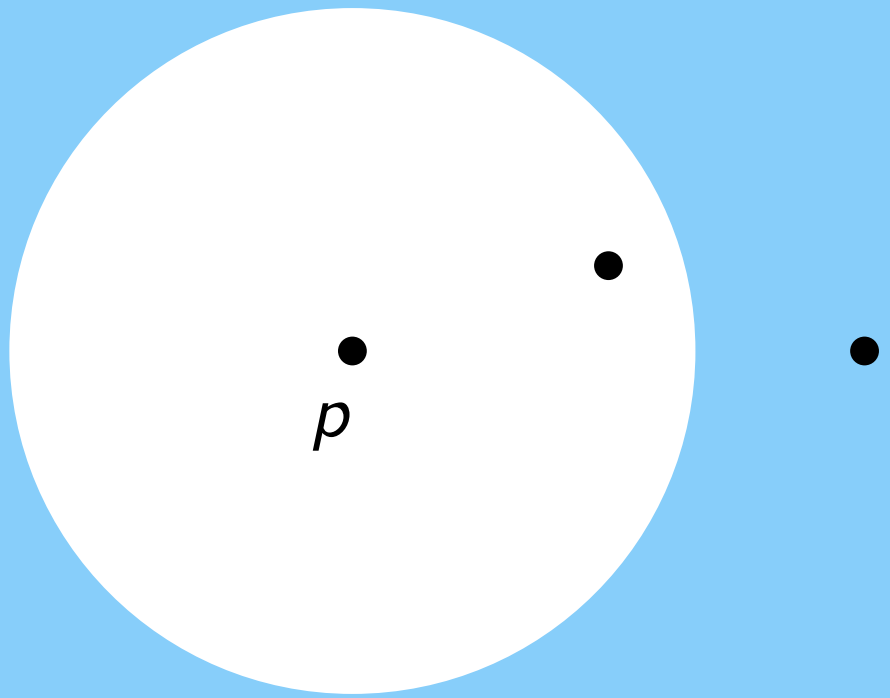

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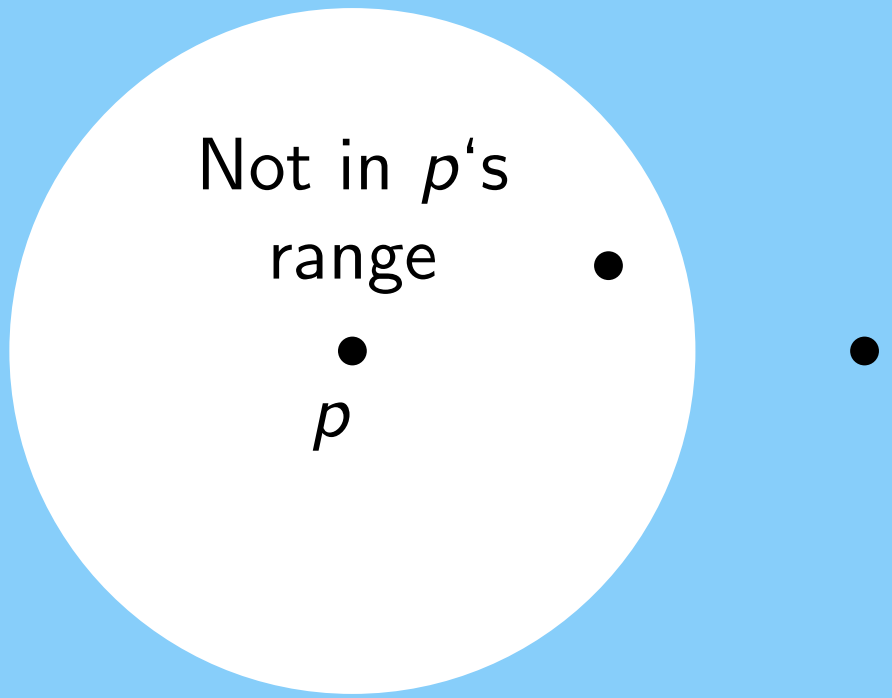

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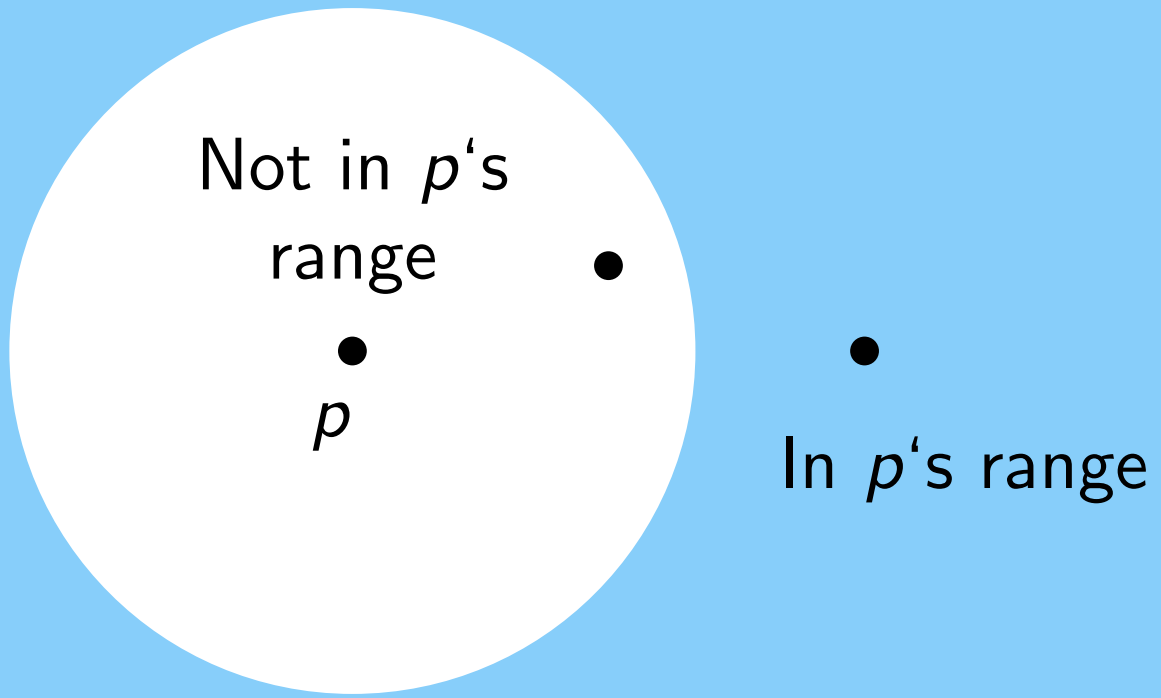

Why Generalized Range Successor?

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Recall this table

i	j
1	j_1
$\vdots$	$\vdots$
i	j_i
$\vdots$	$\vdots$
n	j_n

Why Generalized Range Successor?

Recall this table

Given time

i	j
1	j_1
⋮	⋮
i	j_i
⋮	⋮
n	j_n

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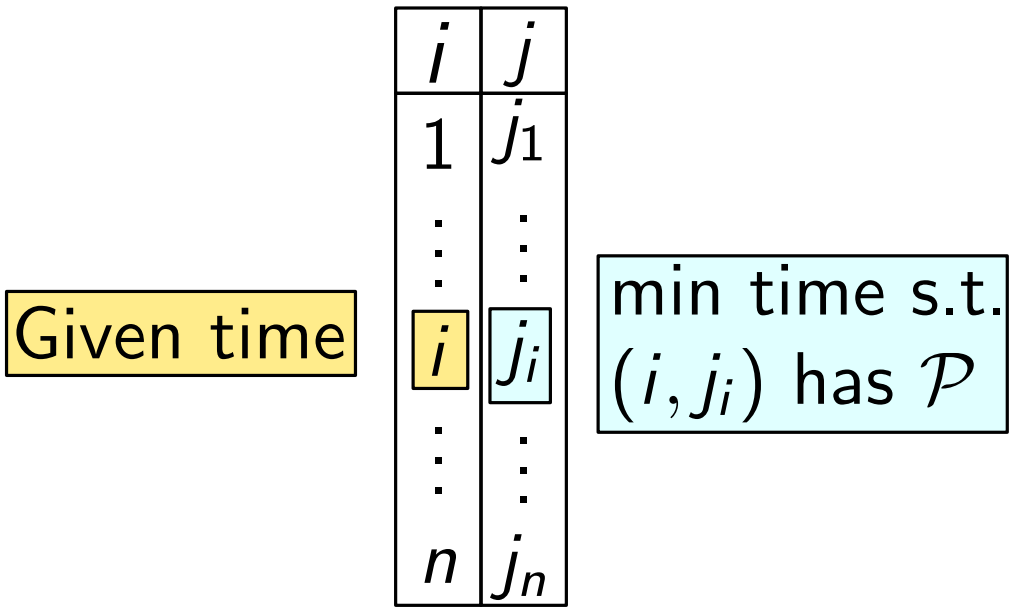
Recall this table



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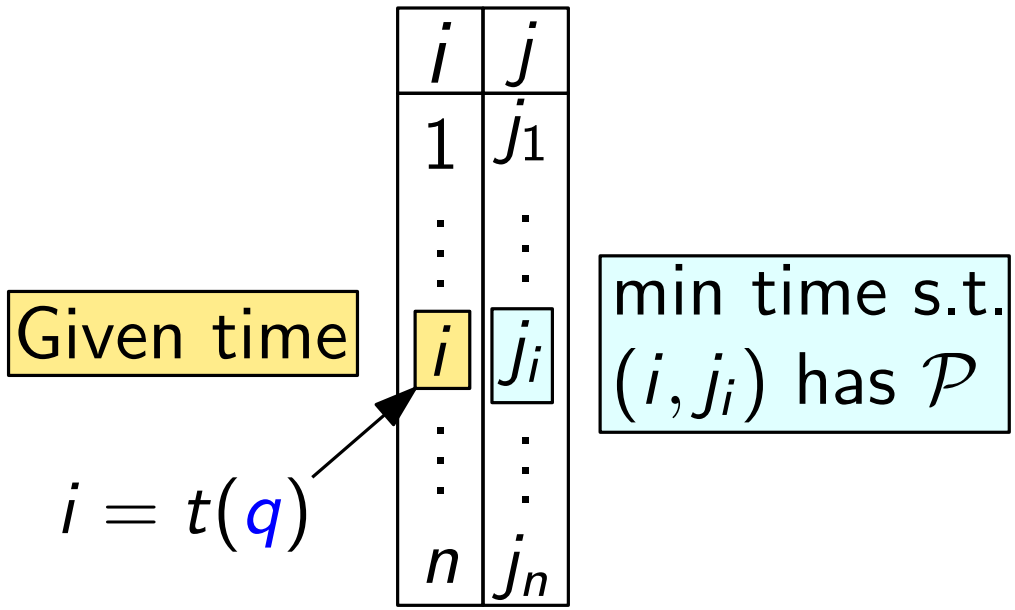

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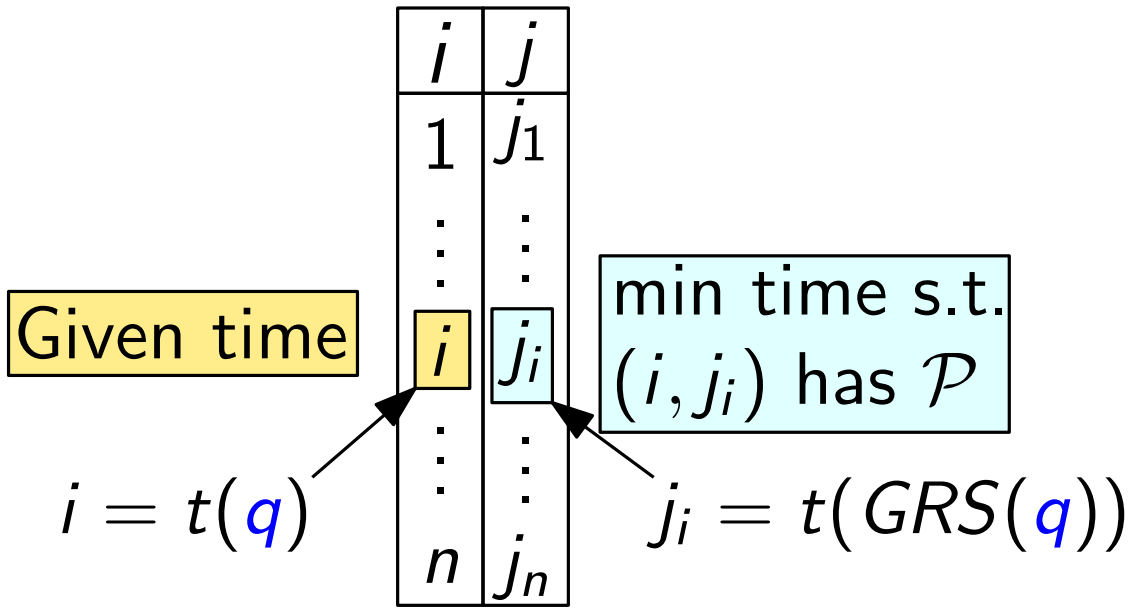

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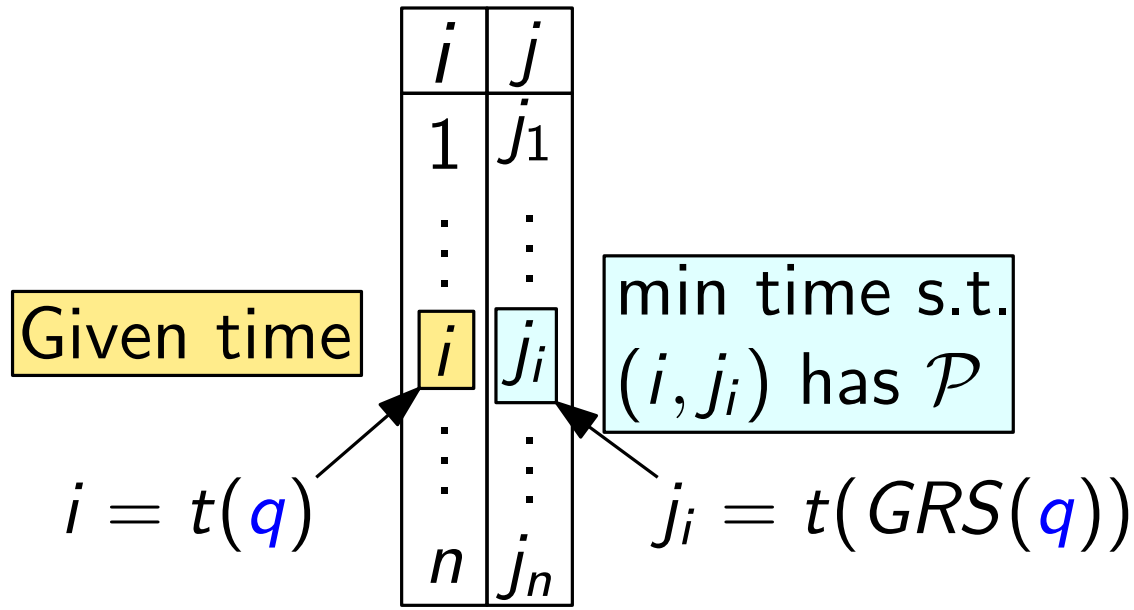
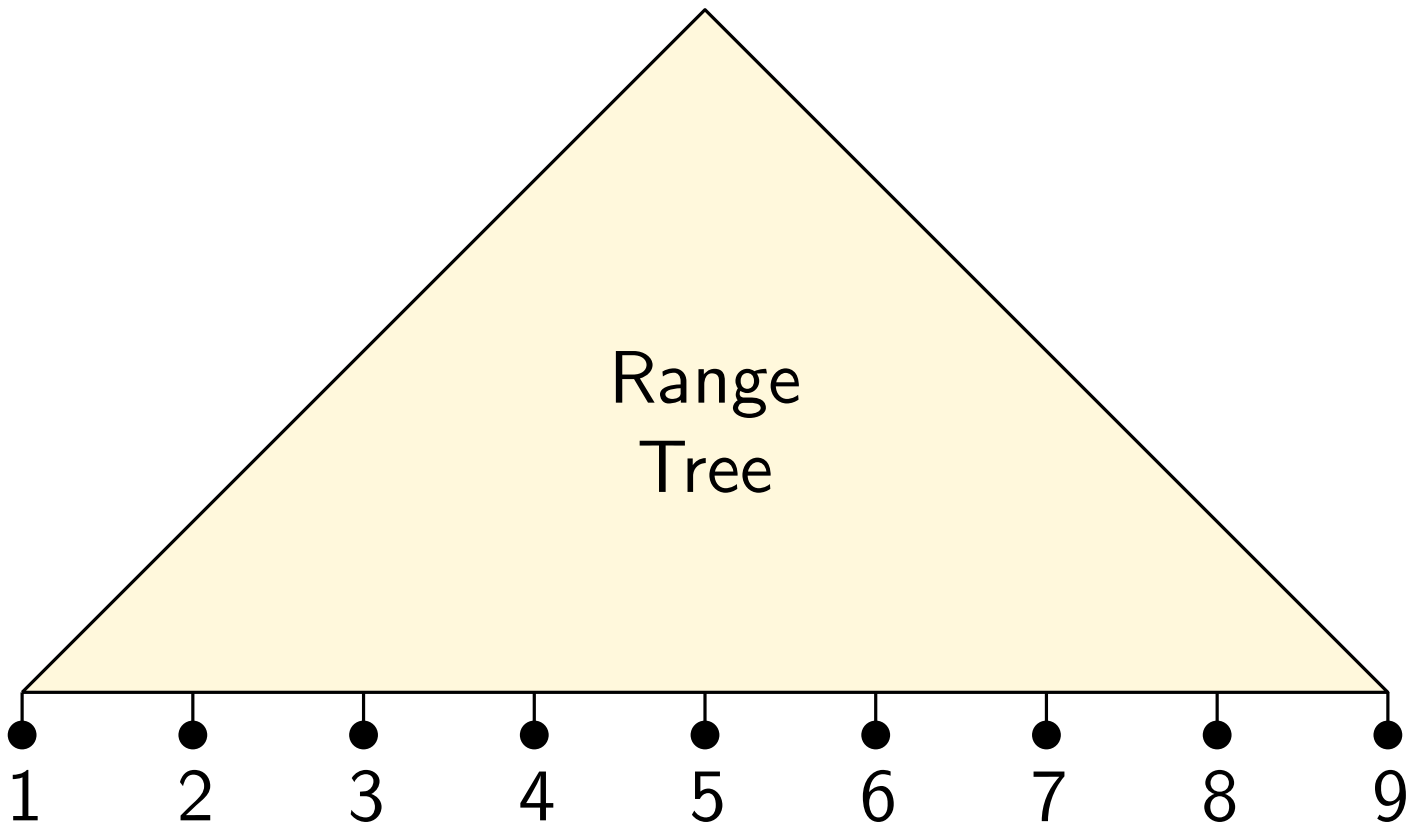


$\therefore$ can build table by finding $GRS(q)$ for all q

Generalized Range Successor

Consider the objects in time-order

Generalized Range Successor



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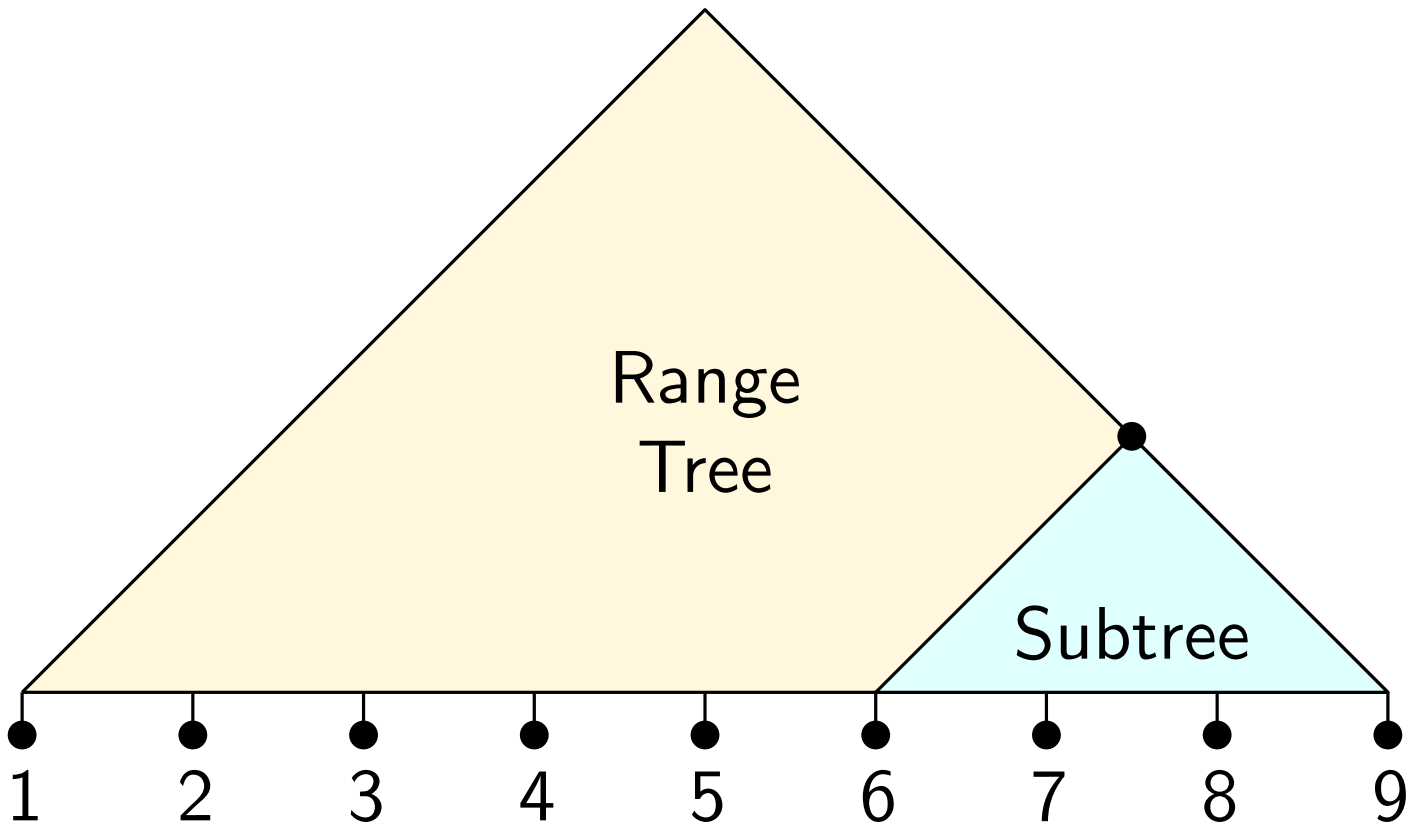

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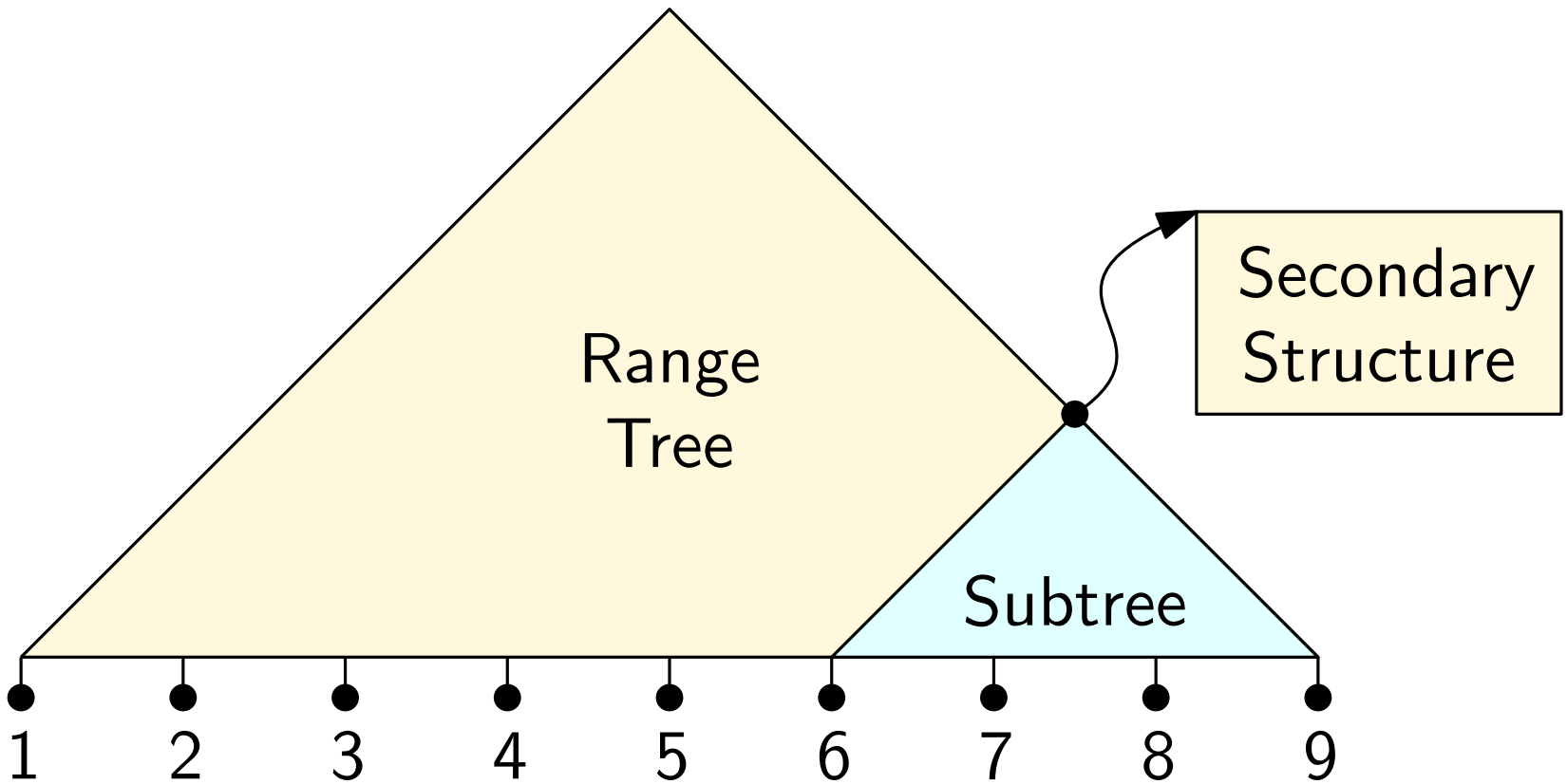

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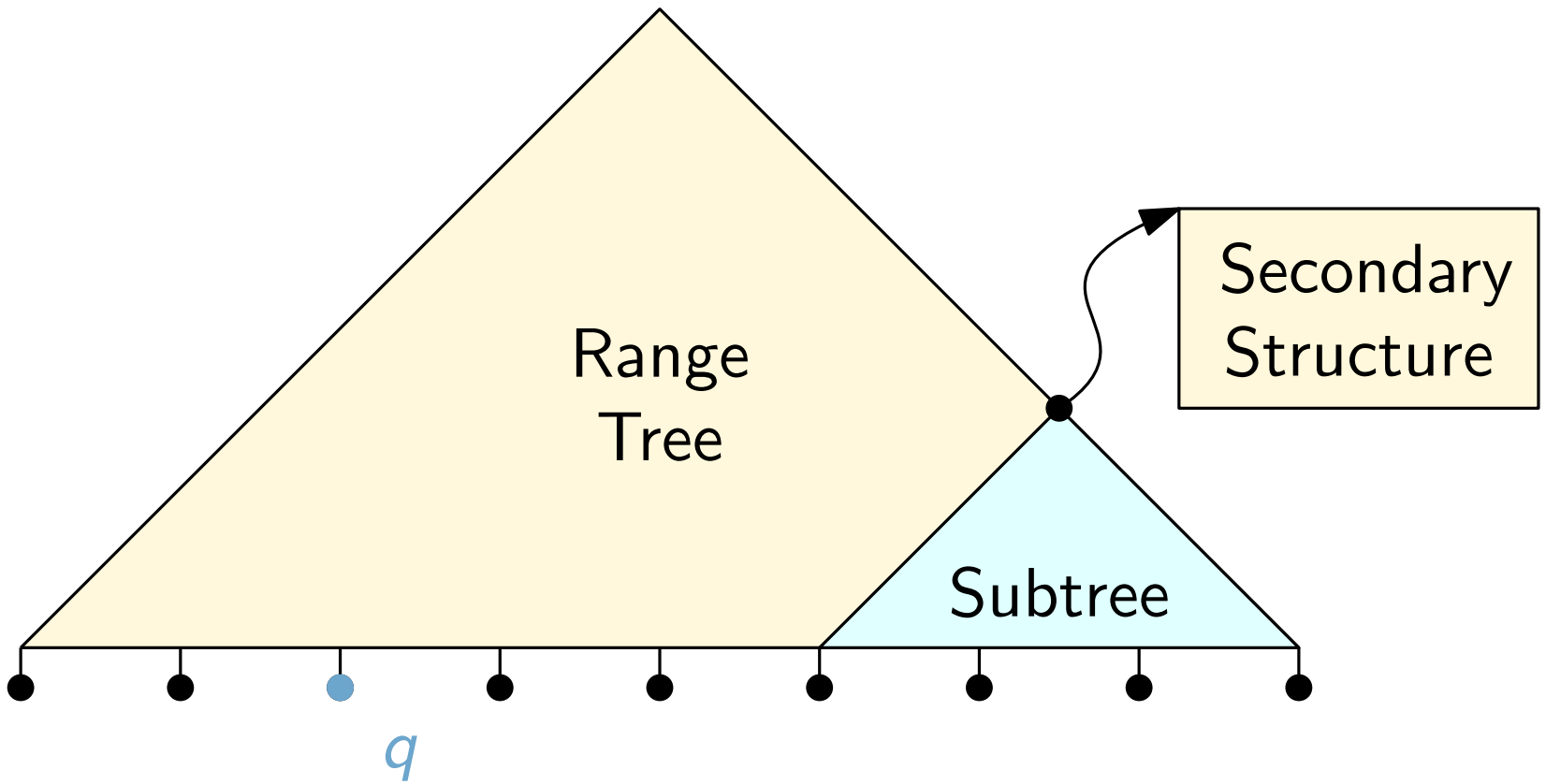

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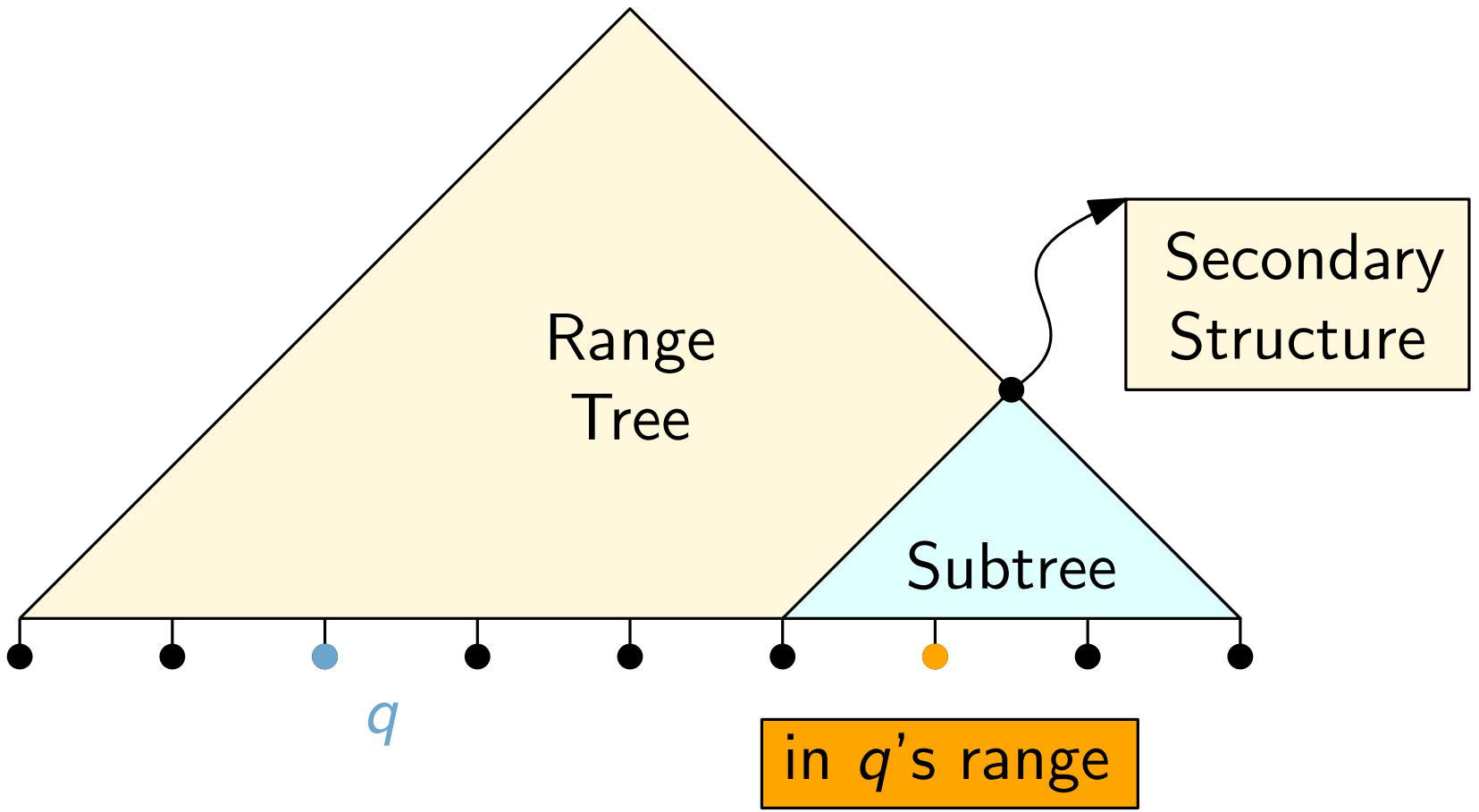
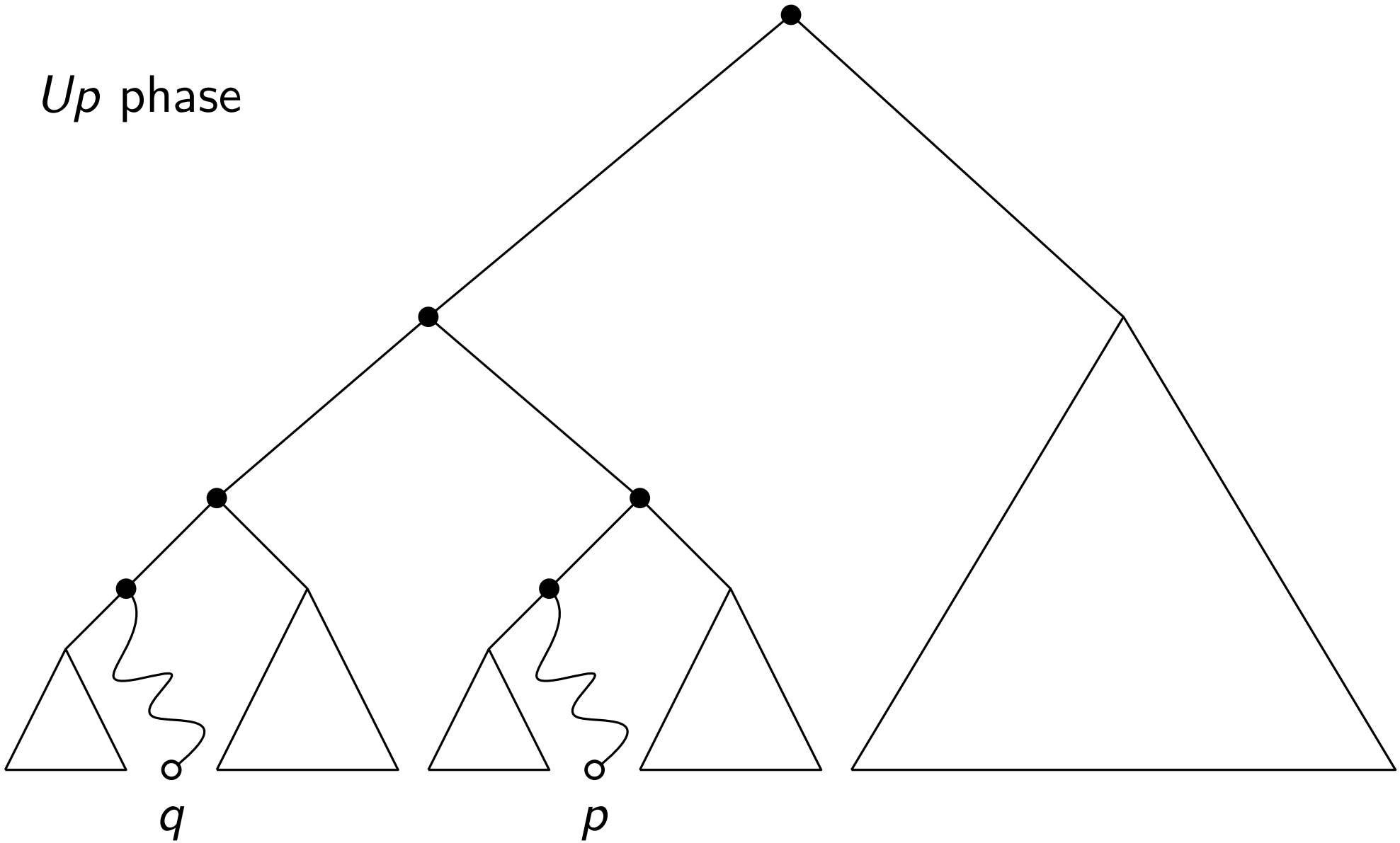


Avoiding Binary Search

Up phase



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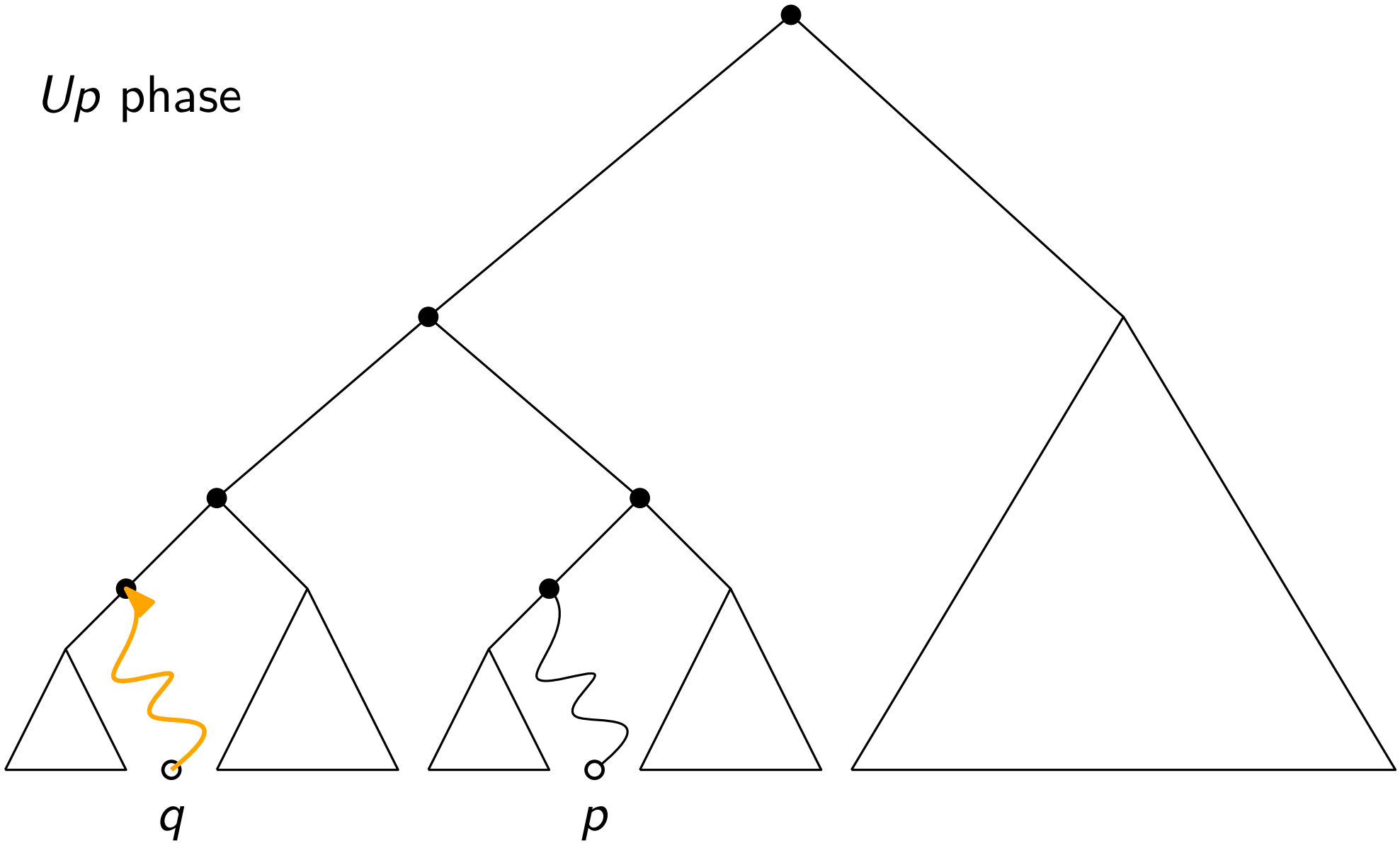

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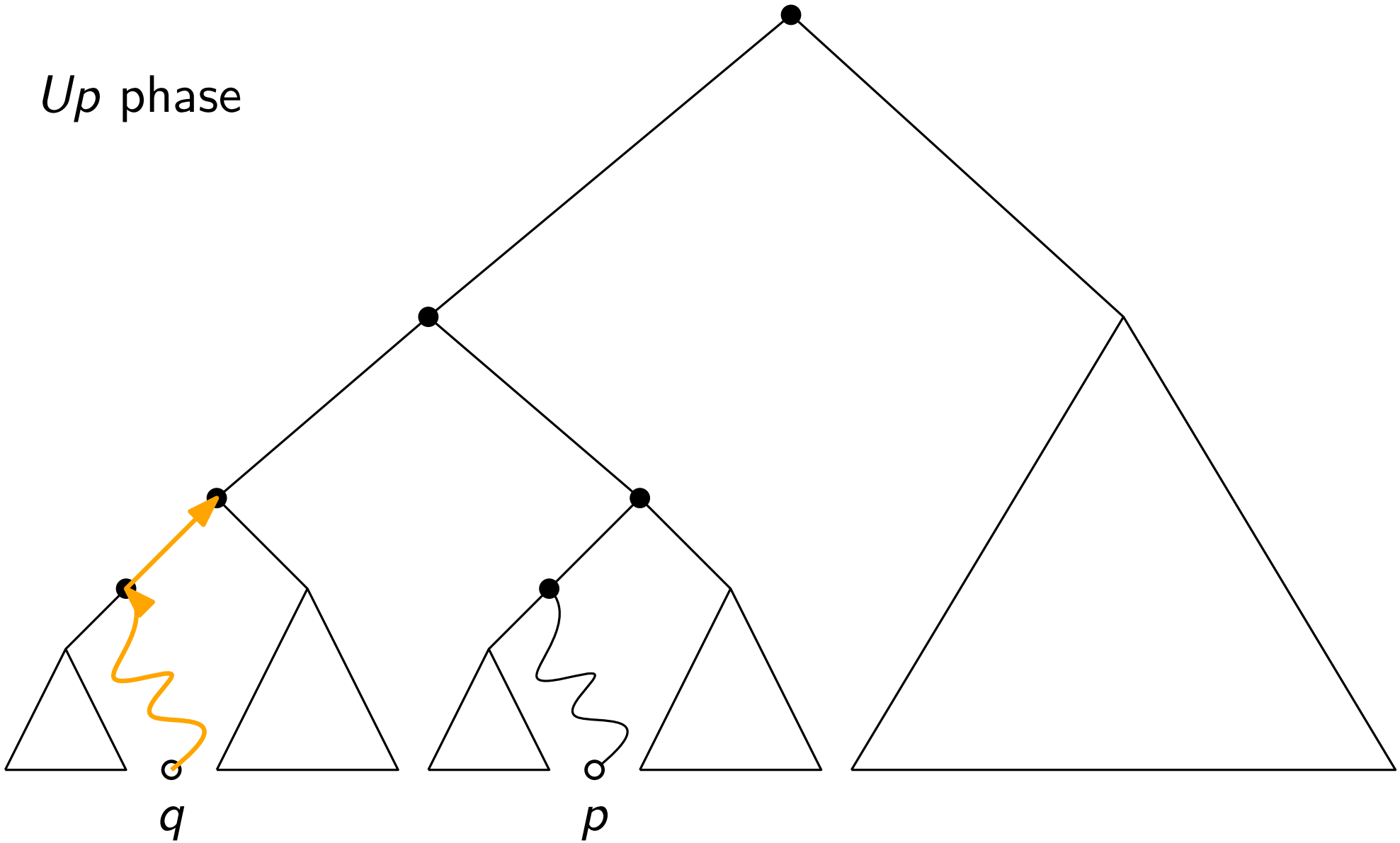

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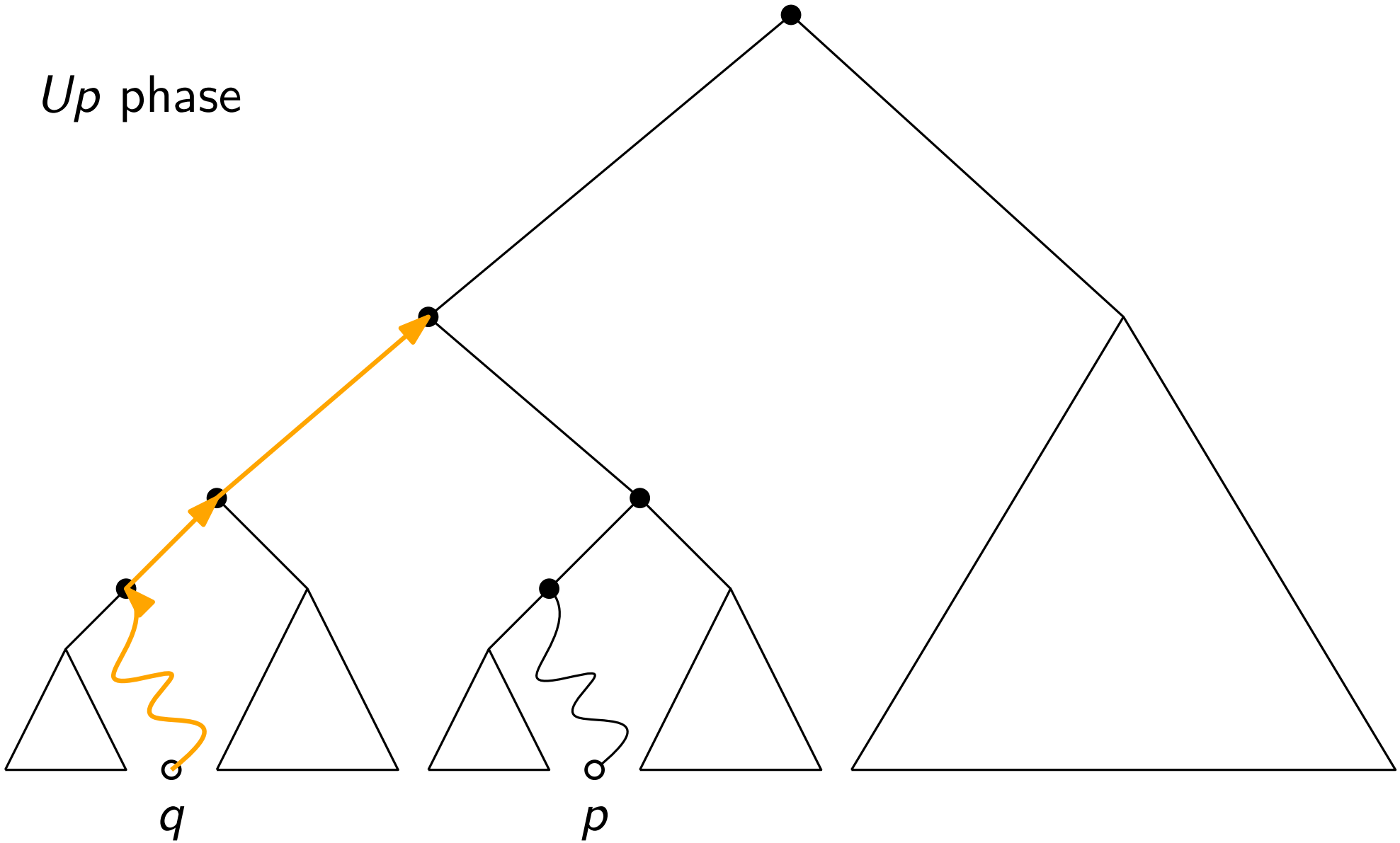

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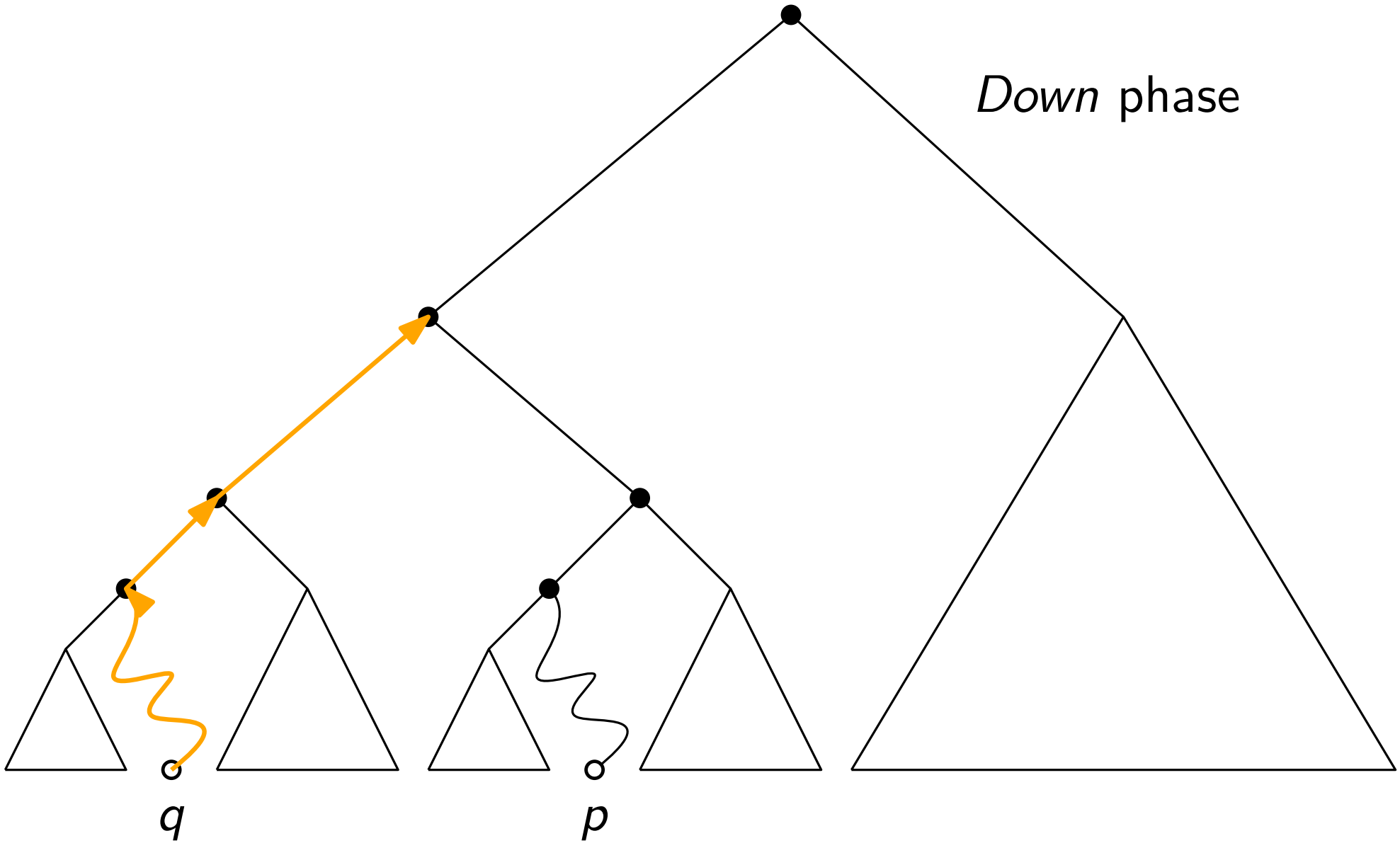

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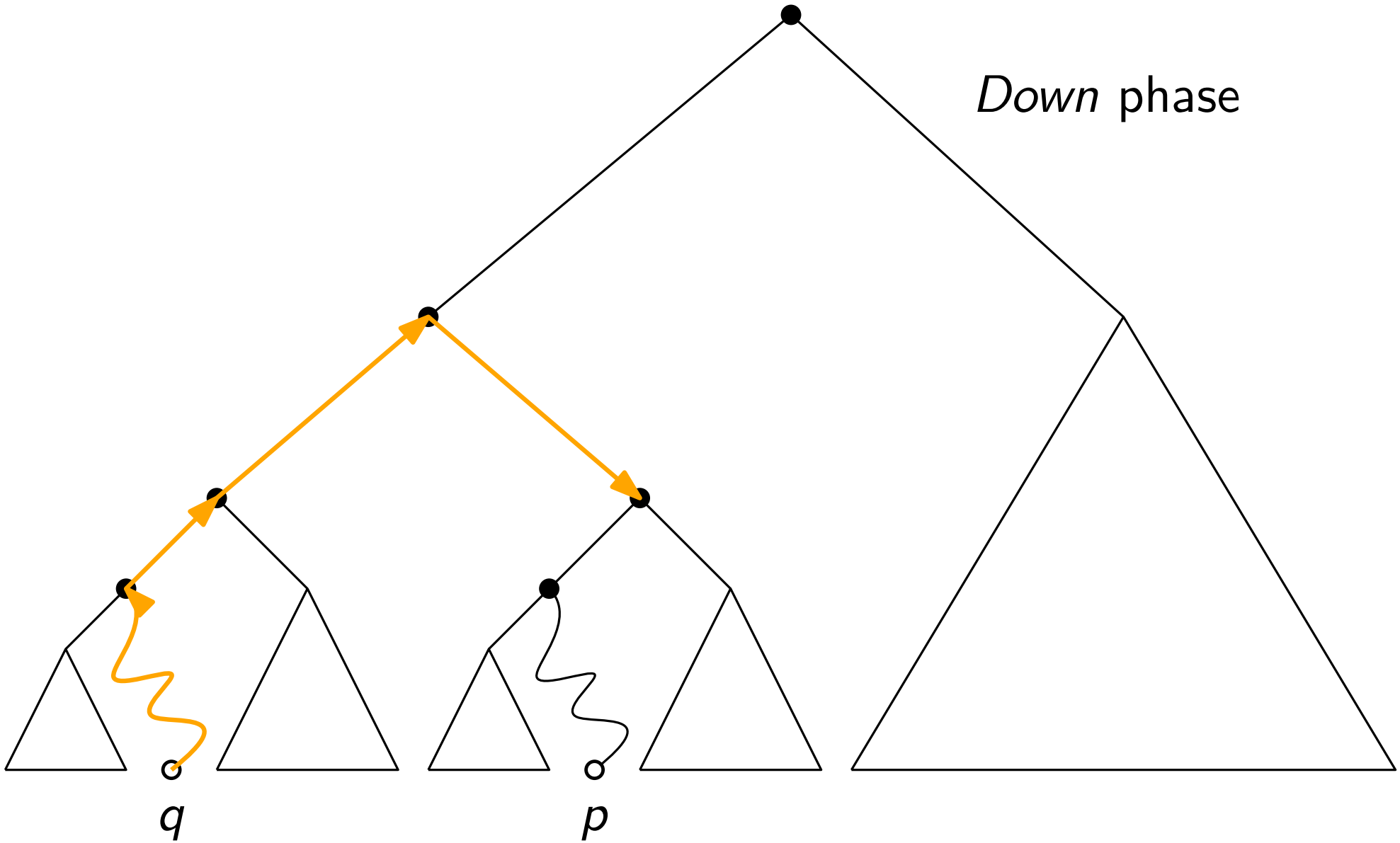

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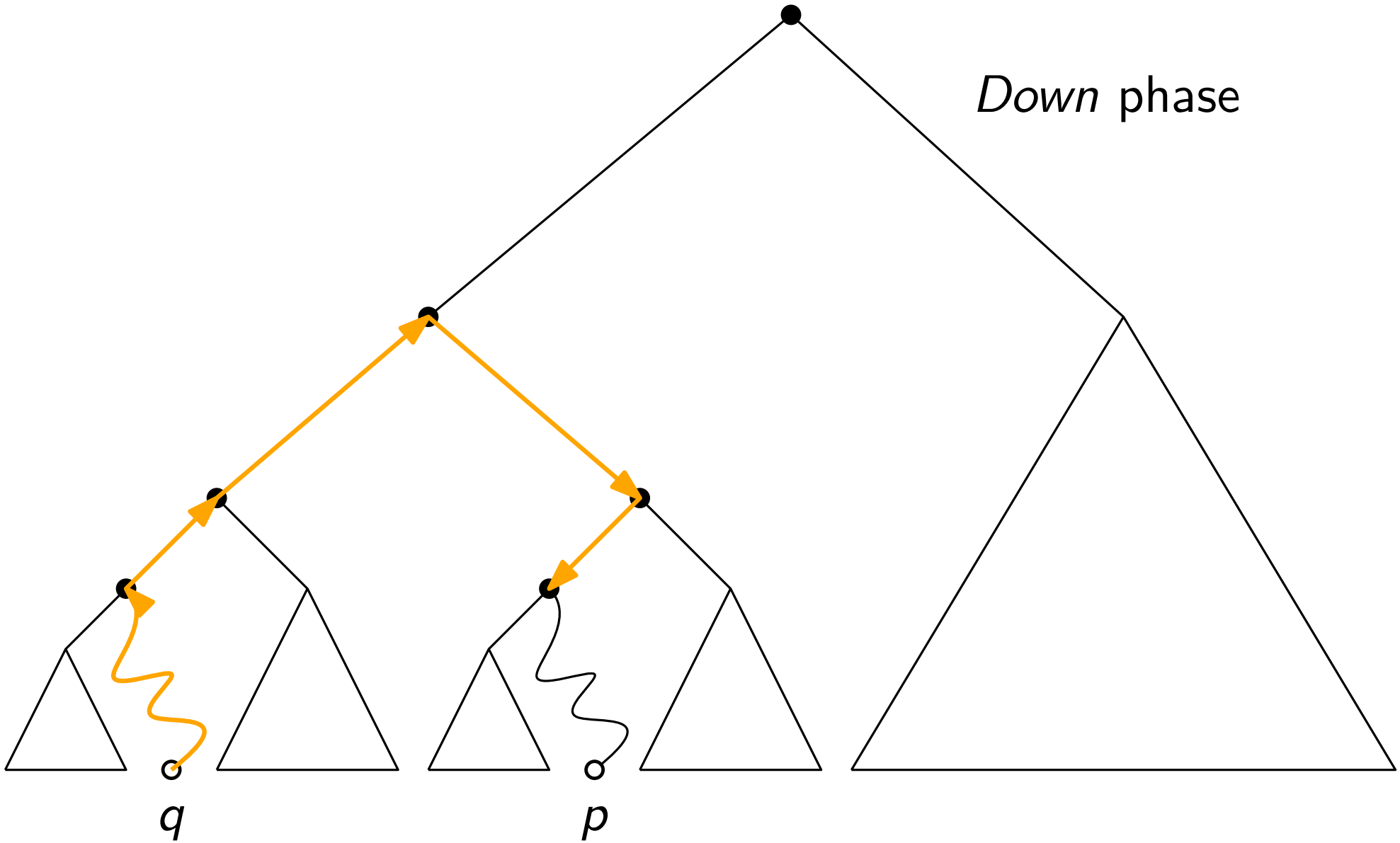

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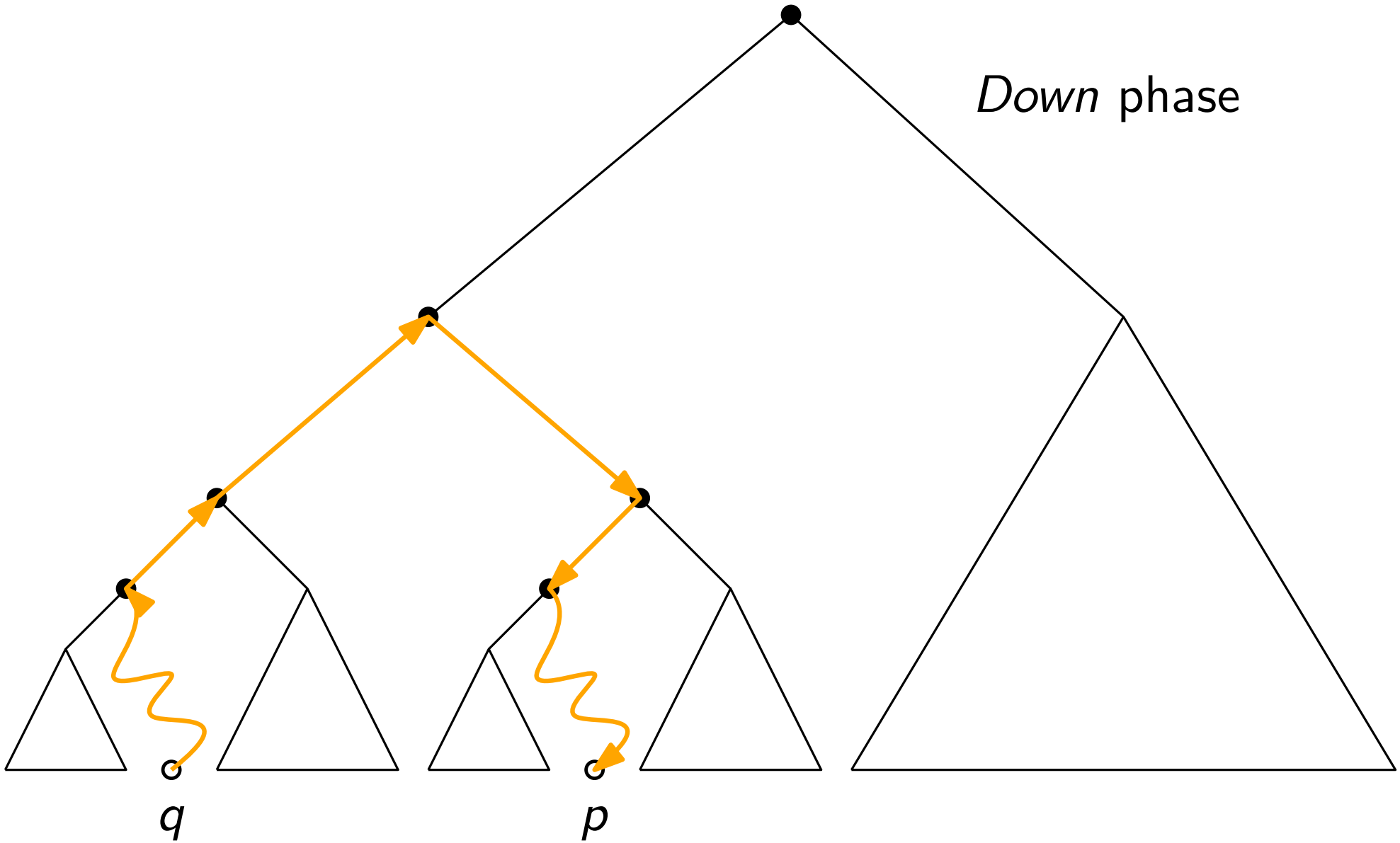
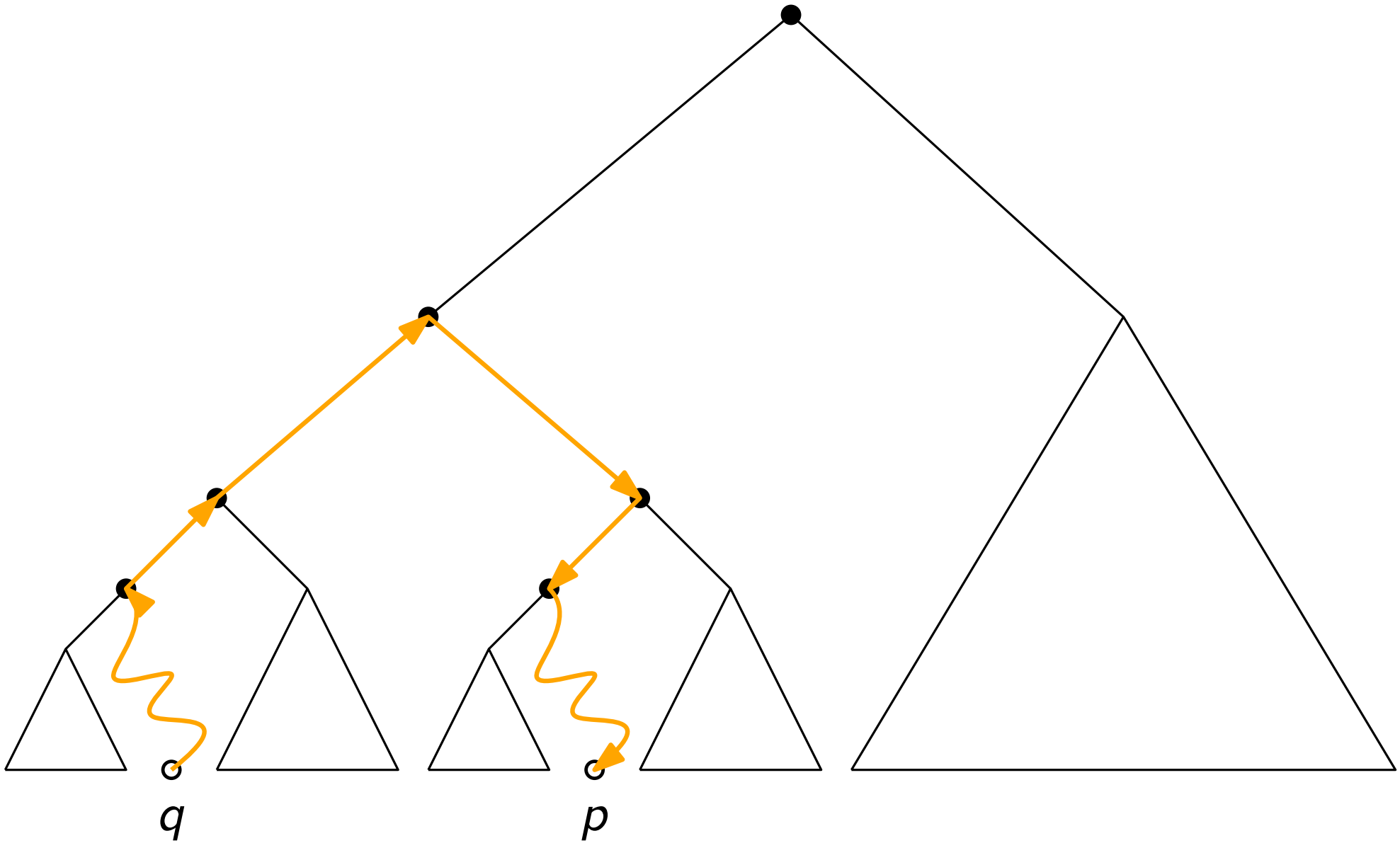


Quick Analysis



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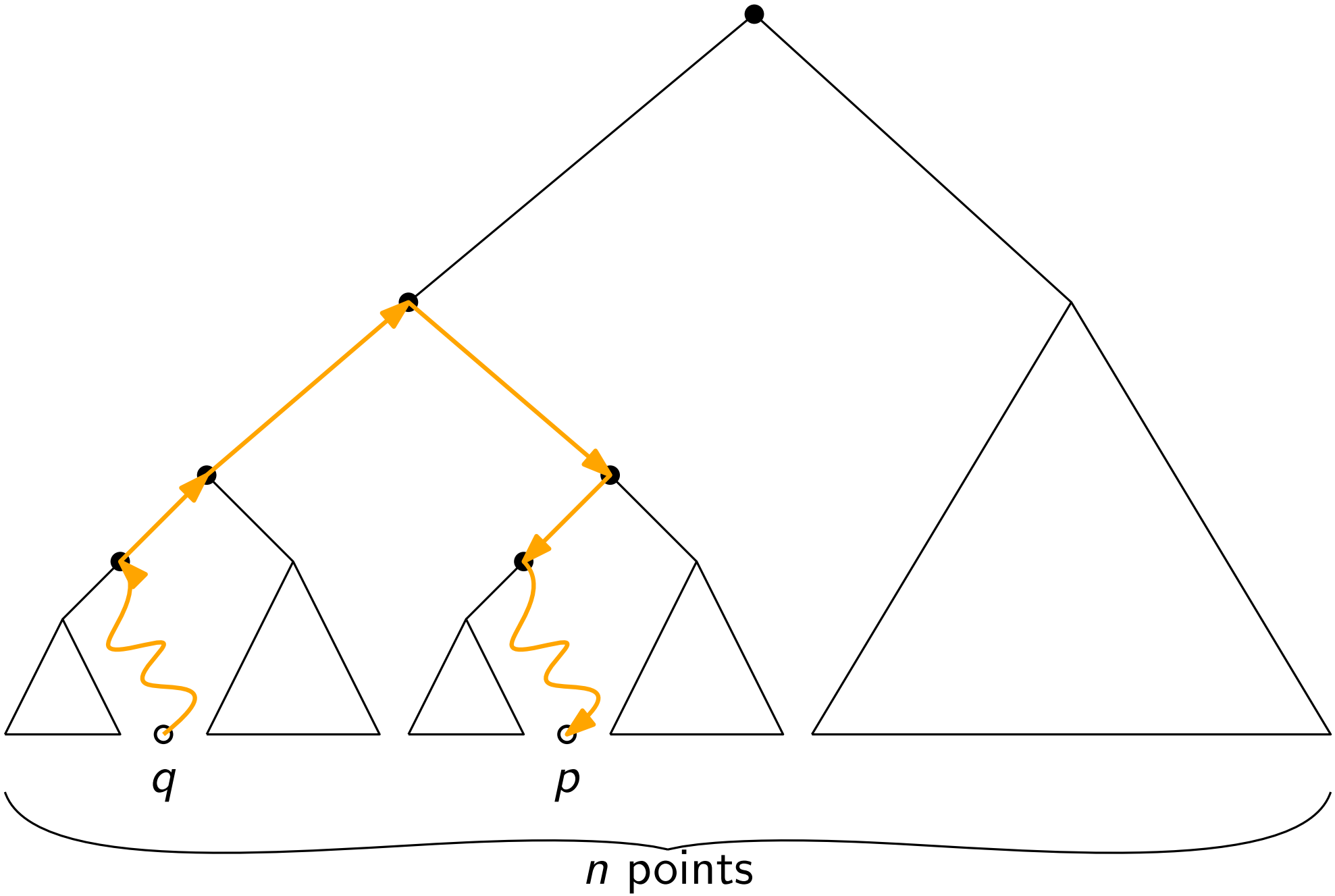

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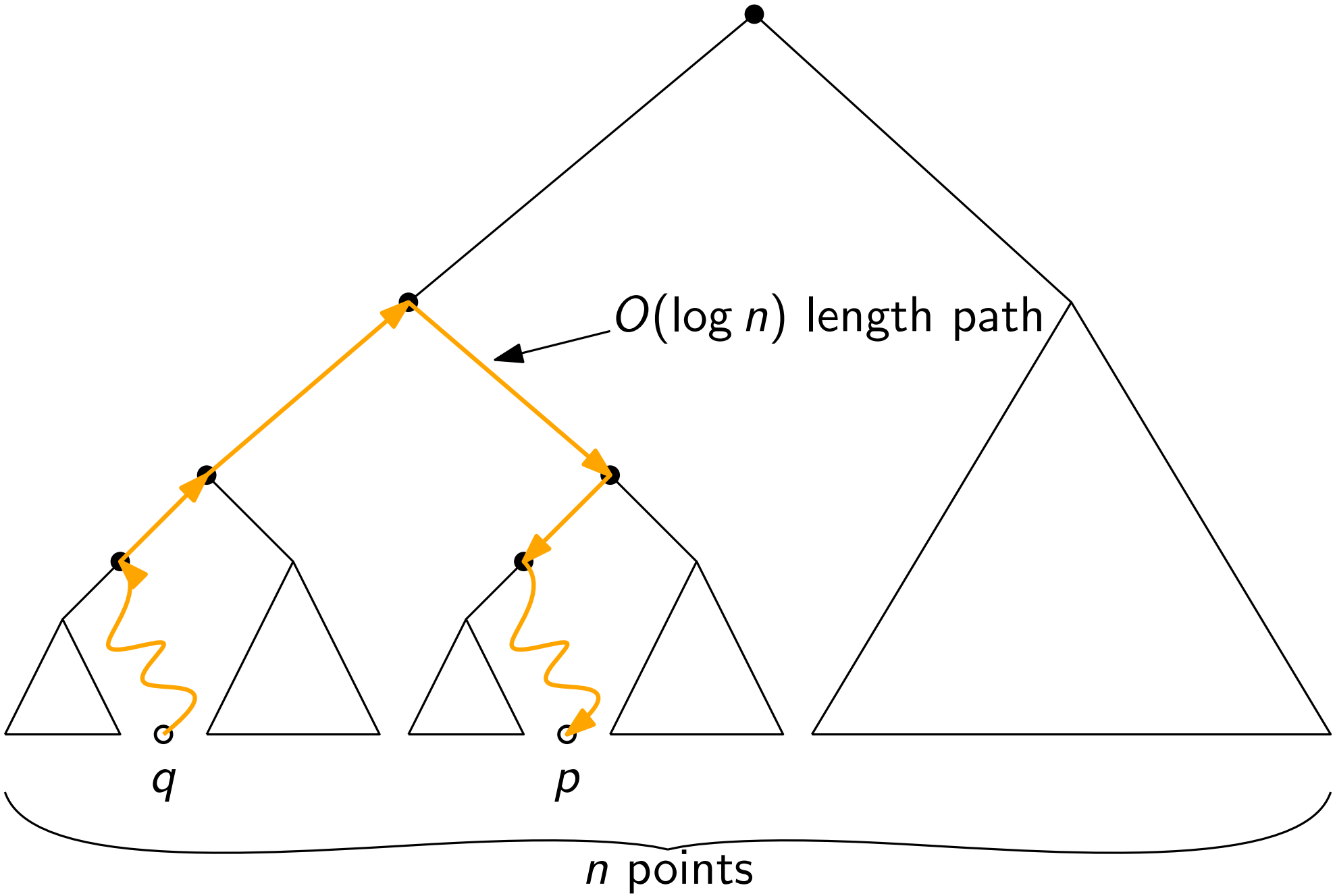
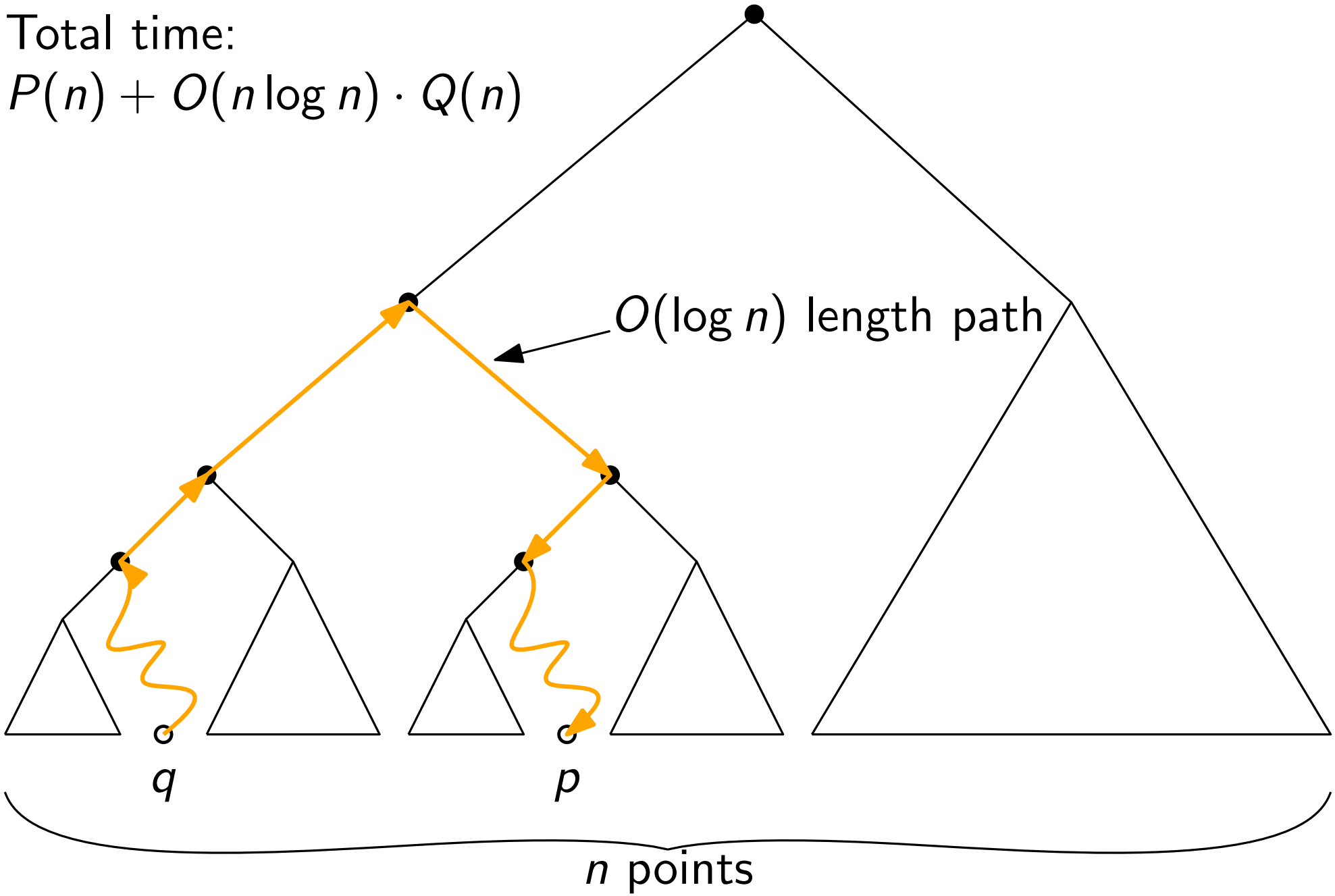


Quick Analysis

Total time:
$P(n) + O(n \log n) \cdot Q(n)$



Generalized Range Successor: Conclusion

2D diameter decision

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Time-Windowed Decision $\rightarrow$ Dynamic

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$\vdots$	$\vdots$
i	j_i
$\vdots$	$\vdots$
n	j_n

Time-Windowed Decision $\rightarrow$ Dynamic

One way to build it:

i	j
1	
2	
3	
4	
$\vdots$	
n	

Time-Windowed Decision $\rightarrow$ Dynamic

One way to build it:

- Use a dynamic DS

	Dynamic DS	
	Yes	No
Property Satisfied:	o	✓

i	j
1	
2	
3	
4	
⋮	
n	

Time-Windowed Decision $\rightarrow$ Dynamic

One way to build it:

- Use a dynamic DS
- Insert until satisfied

Dynamic DS		
	Yes	No
Property Satisfied:	o	✓

i	j
1	
2	
3	
4	
⋮	
n	

Time-Windowed Decision $\rightarrow$ Dynamic

One way to build it:

- Use a dynamic DS
- Insert until satisfied

	Dynamic DS	
	Yes	No
Property Satisfied:	o	✓

Insert 1

i	j
1	
2	
3	
4	
⋮	
n	

Time-Windowed Decision $\rightarrow$ Dynamic

One way to build it:

- Use a dynamic DS
- Insert until satisfied

	Dynamic DS	
	Yes	No
Property Satisfied:	o	✓

Insert 1
Insert 2

i	j
1	
2	
3	
4	
⋮	
n	

Time-Windowed Decision $\rightarrow$ Dynamic

One way to build it:

- Use a dynamic DS
- Insert until satisfied

	Dynamic DS	
	Yes	No
Property Satisfied:	✓	o

Insert 1
Insert 2
Insert 3

i	j
1	
2	
3	
4	
⋮	
n	

Time-Windowed Decision $\rightarrow$ Dynamic

One way to build it:

- Use a dynamic DS
- Insert until satisfied

	Dynamic DS	
	Yes	No
Property Satisfied:	✓	o

Insert 1
Insert 2
Insert 3

i	j
1	3
2	
3	
4	
⋮	
n	

Time-Windowed Decision $\rightarrow$ Dynamic

	Dynamic DS	
	Yes	No
Property Satisfied:	✓	o

One way to build it:

- Use a dynamic DS
- Insert until satisfied
- Delete (from beginning) until not satisfied

Insert 1
Insert 2
Insert 3

i	j
1	3
2	
3	
4	
⋮	
n	

Time-Windowed Decision $\rightarrow$ Dynamic

	Dynamic DS	
	Yes	No
Property Satisfied:	✓	o

One way to build it:

- Use a dynamic DS
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- Delete (from beginning) until not satisfied

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Insert 2
Insert 3

i	j
1	3
2	
3	
4	
⋮	
n	

Time-Windowed Decision $\rightarrow$ Dynamic

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	Yes	No
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- Use a dynamic DS
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Insert 1 Delete
Insert 2
Insert 3

i	j
1	3
2	3
3	
4	
⋮	
n	

Time-Windowed Decision $\rightarrow$ Dynamic

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Insert 3

i	j
1	3
2	3
3	
4	
⋮	
n	

Time-Windowed Decision $\rightarrow$ Dynamic

	Dynamic DS	
	Yes	No
Property Satisfied:	o	✓

One way to build it:

- Use a dynamic DS
- Insert until satisfied
- Delete (from beginning) until not satisfied
- Repeat until all points deleted

Insert 1 Delete
Insert 2 Delete
Insert 3

i	j
1	3
2	3
3	
4	
⋮	
n	

Time-Windowed Decision $\rightarrow$ Dynamic

	Dynamic DS	
	Yes	No
Property Satisfied:	✓	o

One way to build it:

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Insert 2 Delete
Insert 3
Insert 4

i	j
1	3
2	3
3	
4	
⋮	
n	

Time-Windowed Decision $\rightarrow$ Dynamic

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	Yes	No
Property Satisfied:	✓	o

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Insert 4

i	j
1	3
2	3
3	4
4	
⋮	
n	

Time-Windowed Decision $\rightarrow$ Dynamic

	Dynamic DS	
	Yes	No
Property Satisfied:	o	✓

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Insert 2 Delete
Insert 3 Delete
Insert 4

i	j
1	3
2	3
3	4
4	
⋮	
n	

Time-Windowed Decision $\rightarrow$ Dynamic

	Dynamic DS	
	Yes	No
Property Satisfied:	o	✓

One way to build it:

- Use a dynamic DS
- Insert until satisfied
- Delete (from beginning) until not satisfied
- Repeat until all points deleted

Insert 1 Delete
Insert 2 Delete
Insert 3 Delete
Insert 4

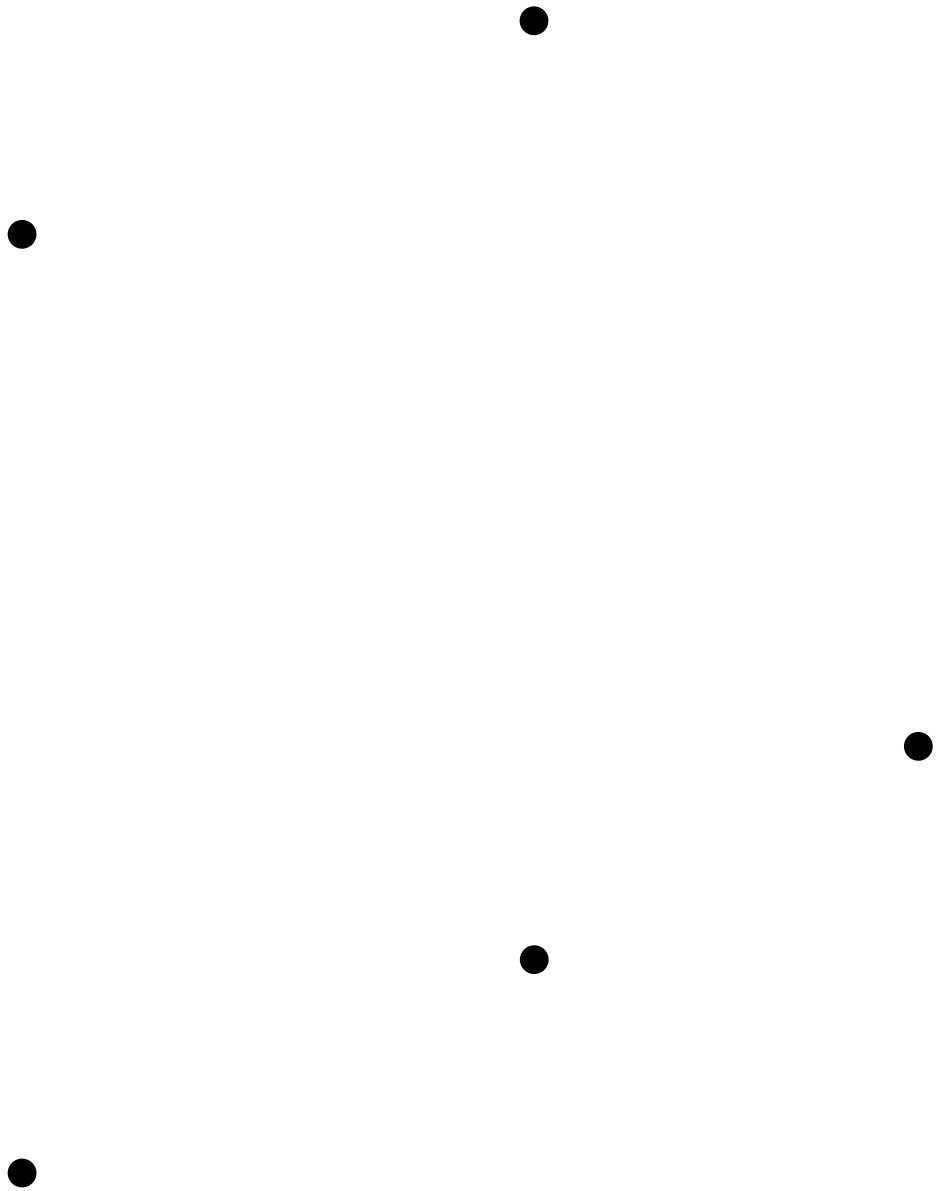
i	j
1	3
2	3
3	4
4	⋮
⋮	
n	

Time-Windowed $\rightarrow$ Dynamic

CH area > 1?

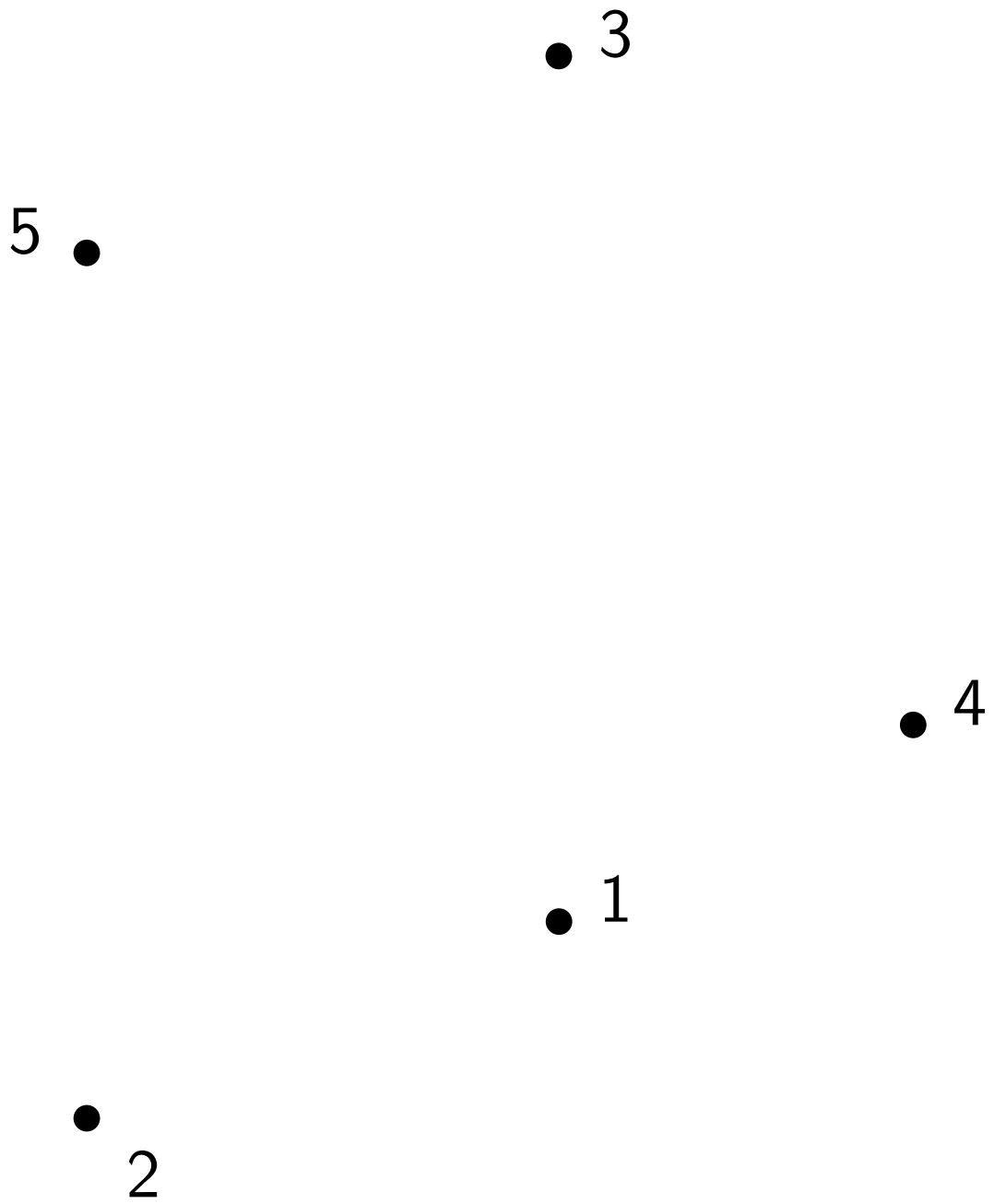
Time-Windowed $\rightarrow$ Dynamic

CH area > 1?



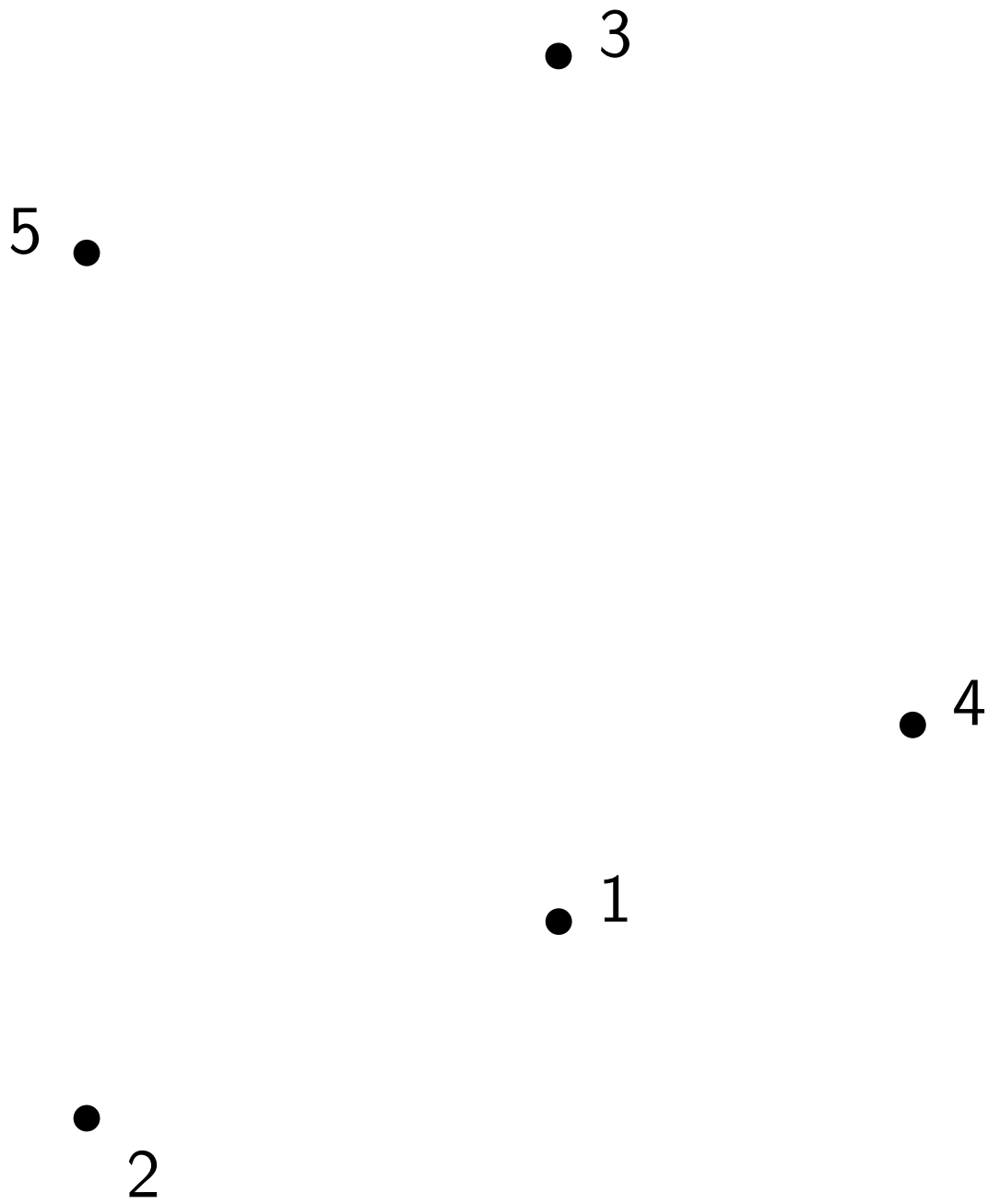
Time-Windowed $\rightarrow$ Dynamic

CH area > 1?



Time-Windowed $\rightarrow$ Dynamic

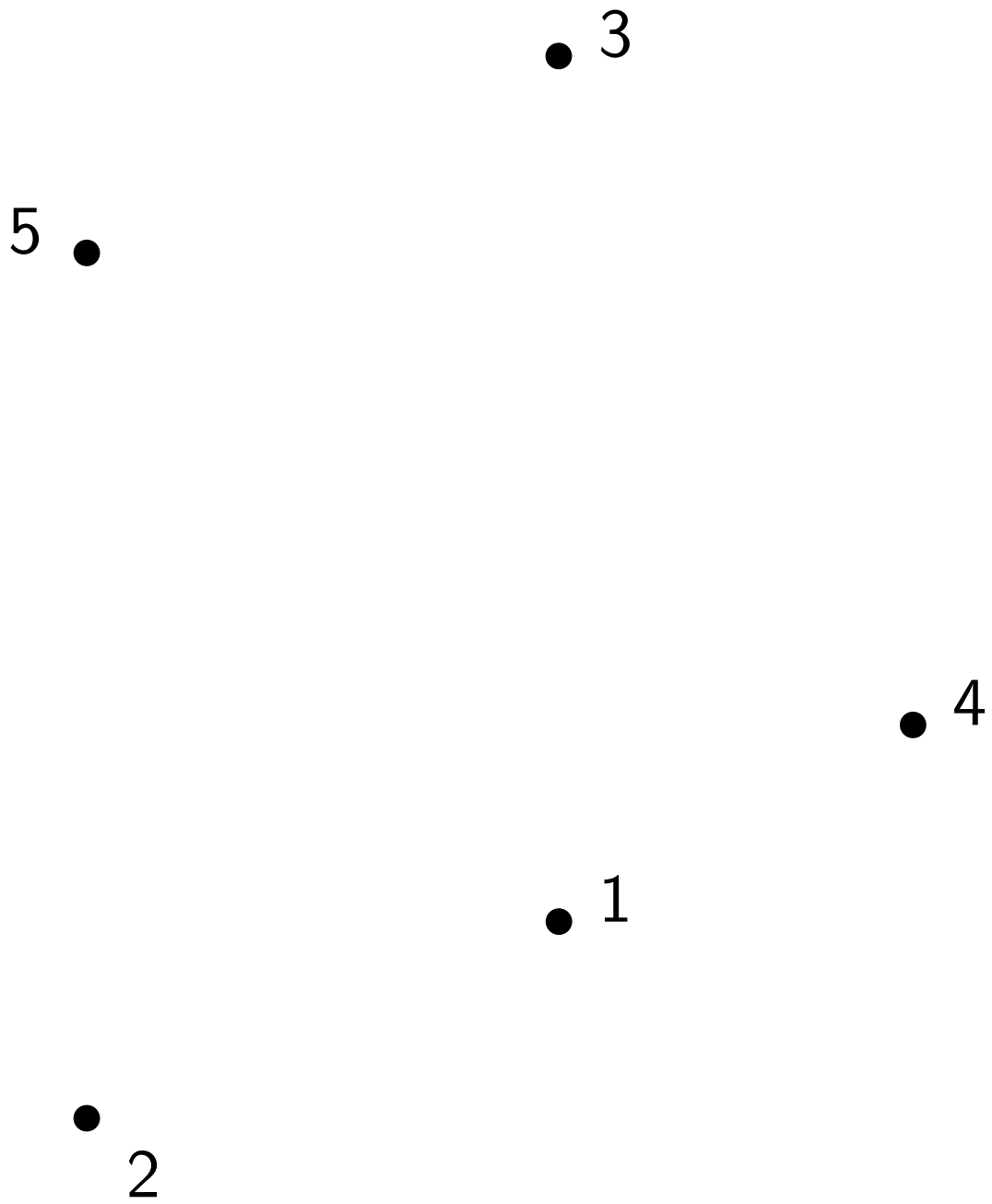
CH area > 1?



i
1
2
3
4
5

Time-Windowed $\rightarrow$ Dynamic

CH area > 1?

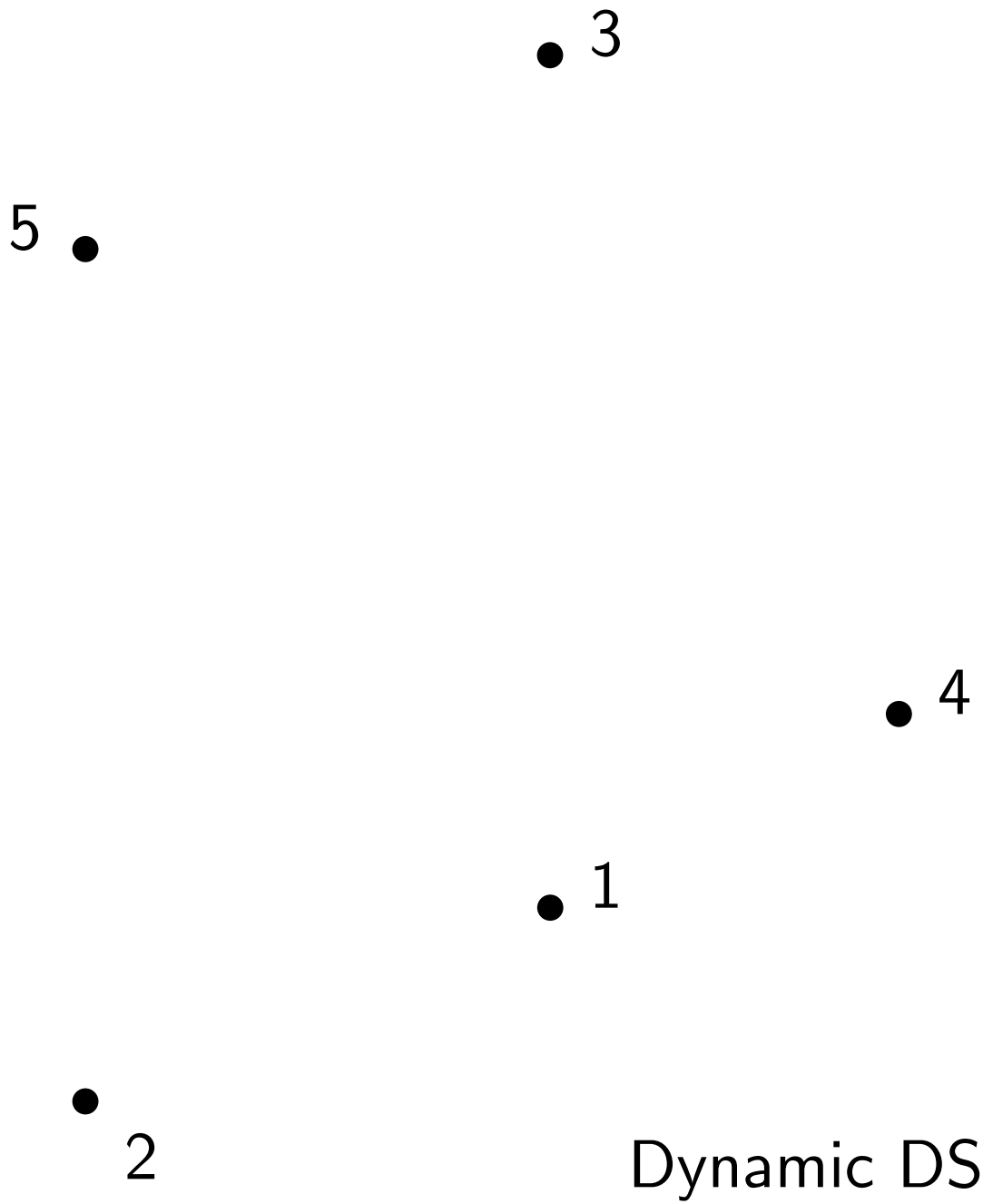


i	j
1	
2	
3	
4	
5	

Time-Windowed $\rightarrow$ Dynamic

CH area > 1?

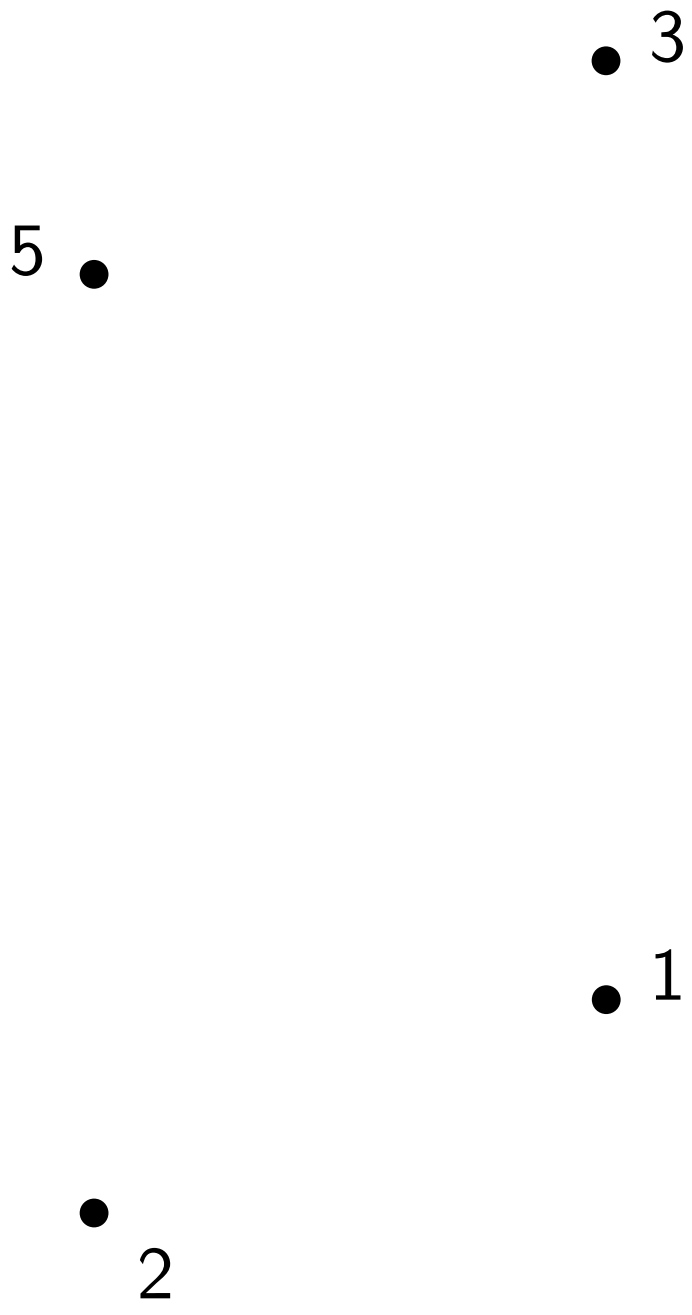
i	j
1	
2	
3	
4	
5	



Dynamic DS

Time-Windowed $\rightarrow$ Dynamic

CH area > 1?



i	j
1	
2	
3	
4	
5	

Dynamic DS : Has property? No

Time-Windowed $\rightarrow$ Dynamic

CH area > 1?

i	j
1	
2	
3	
4	
5	

Dynamic DS : Has property? No

Time-Windowed → Dynamic

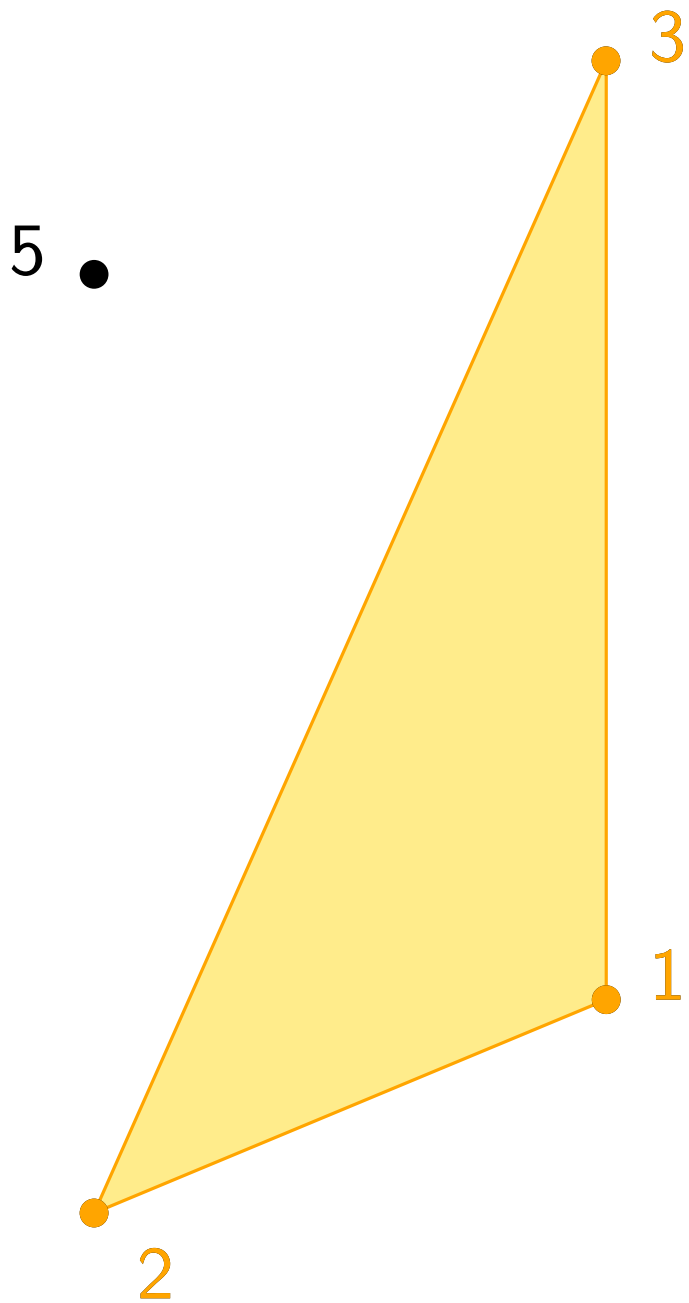
CH area > 1?

i	j
1	
2	
3	
4	
5	

Dynamic DS : Has property? No

Time-Windowed → Dynamic

CH area > 1?

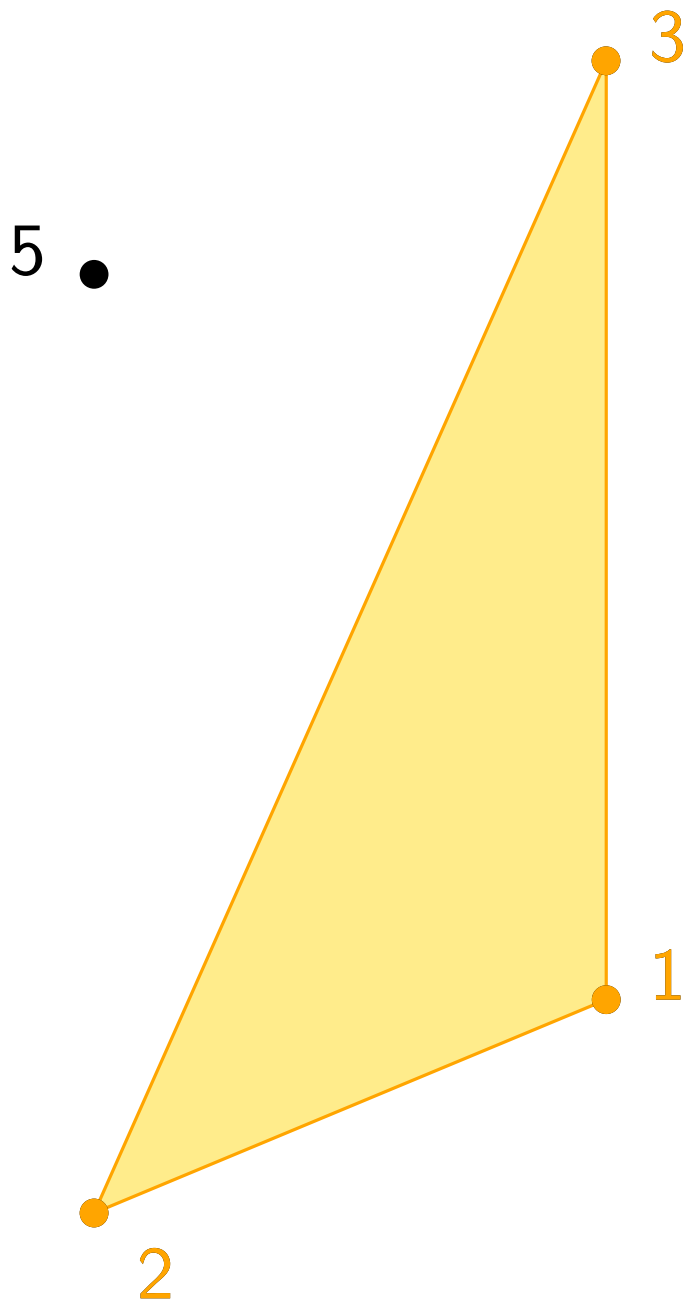


i	j
1	
2	
3	
4	
5	

Dynamic DS : Has property? Yes

Time-Windowed → Dynamic

CH area > 1?

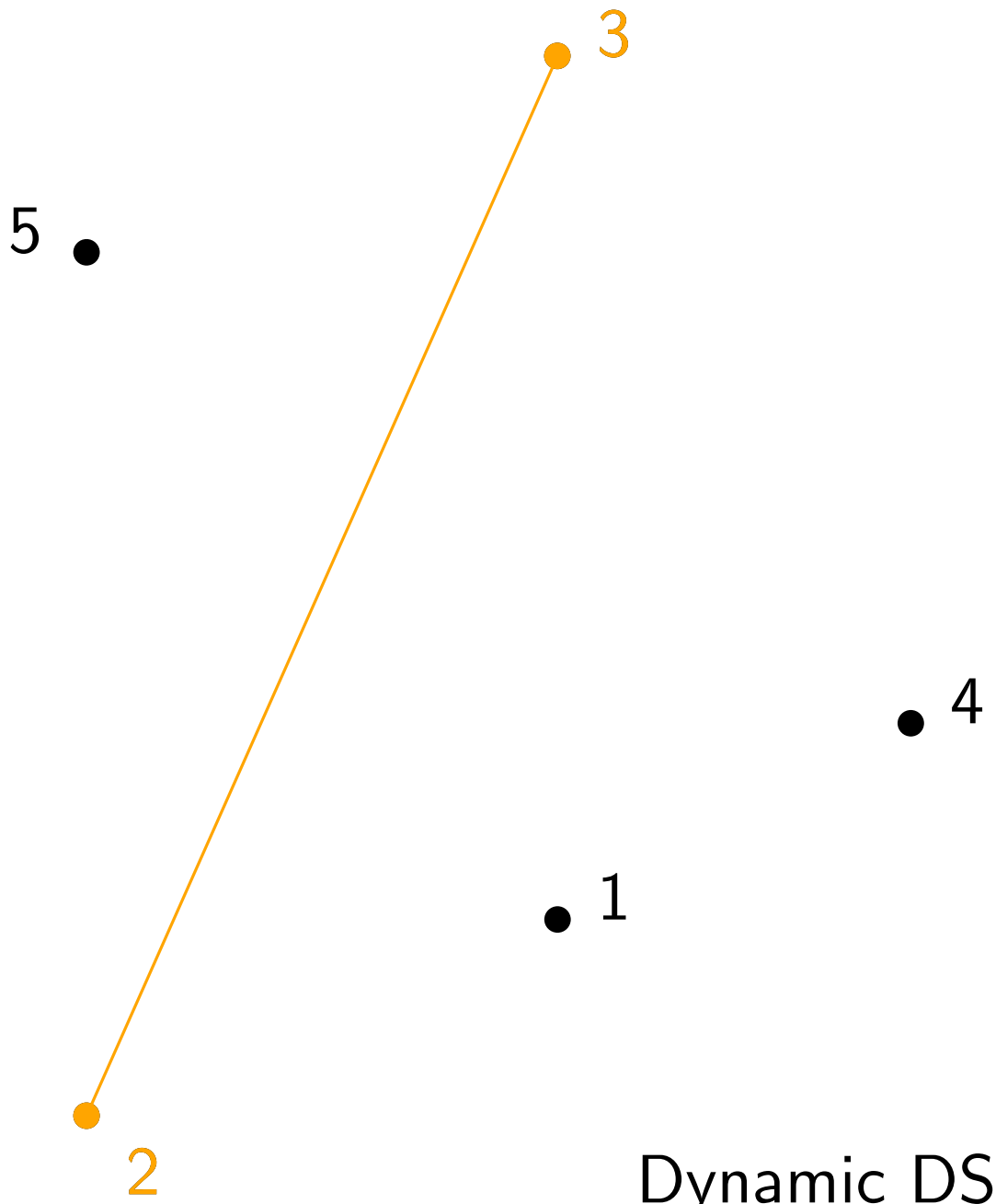


i	j
1	3
2	
3	
4	
5	

Dynamic DS : Has property? Yes

Time-Windowed $\rightarrow$ Dynamic

CH area > 1?

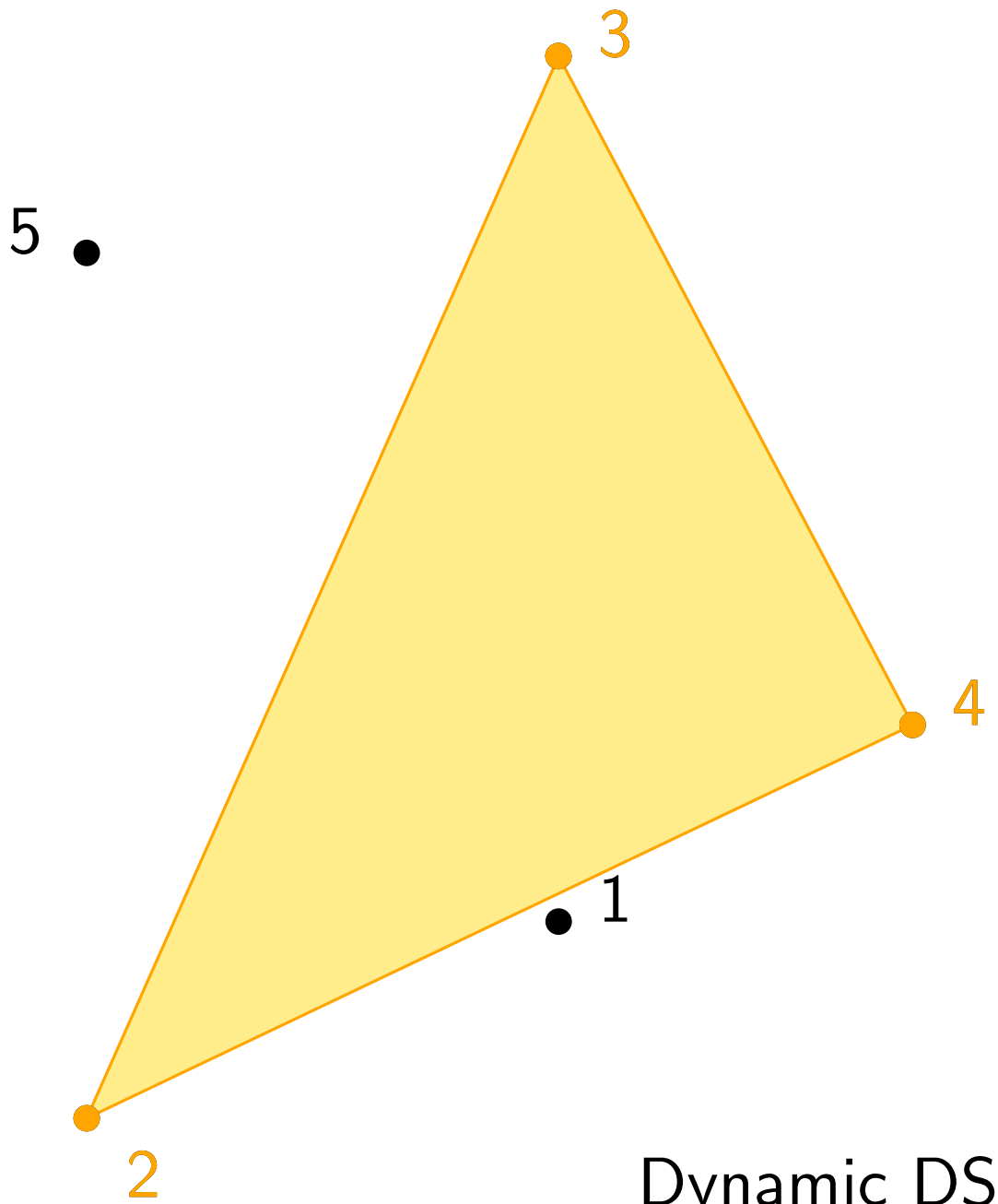


i	j
1	3
2	
3	
4	
5	

Dynamic DS : Has property? No

Time-Windowed $\rightarrow$ Dynamic

CH area > 1?

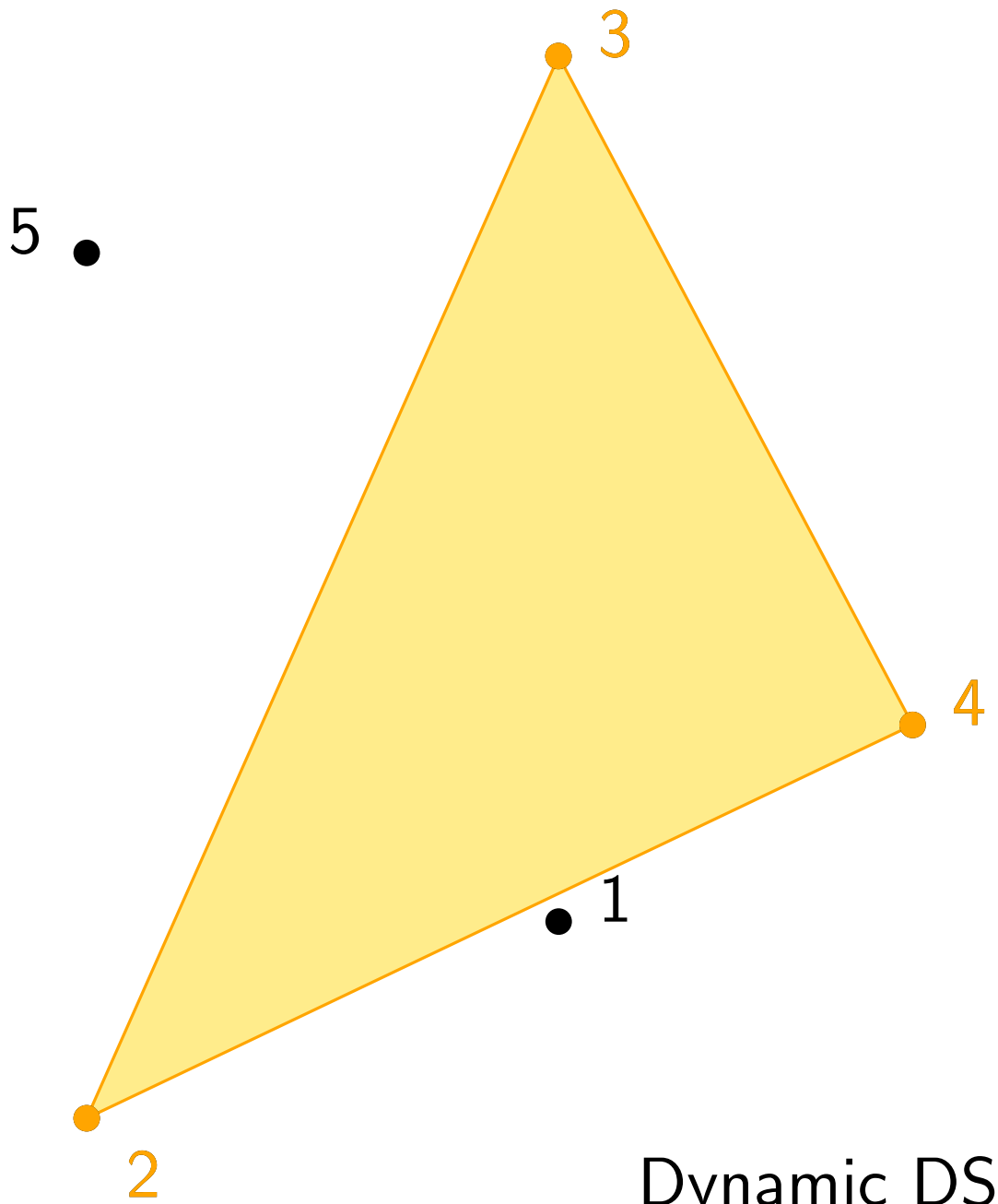


i	j
1	3
2	
3	
4	
5	

Dynamic DS : Has property? Yes

Time-Windowed → Dynamic

CH area > 1?

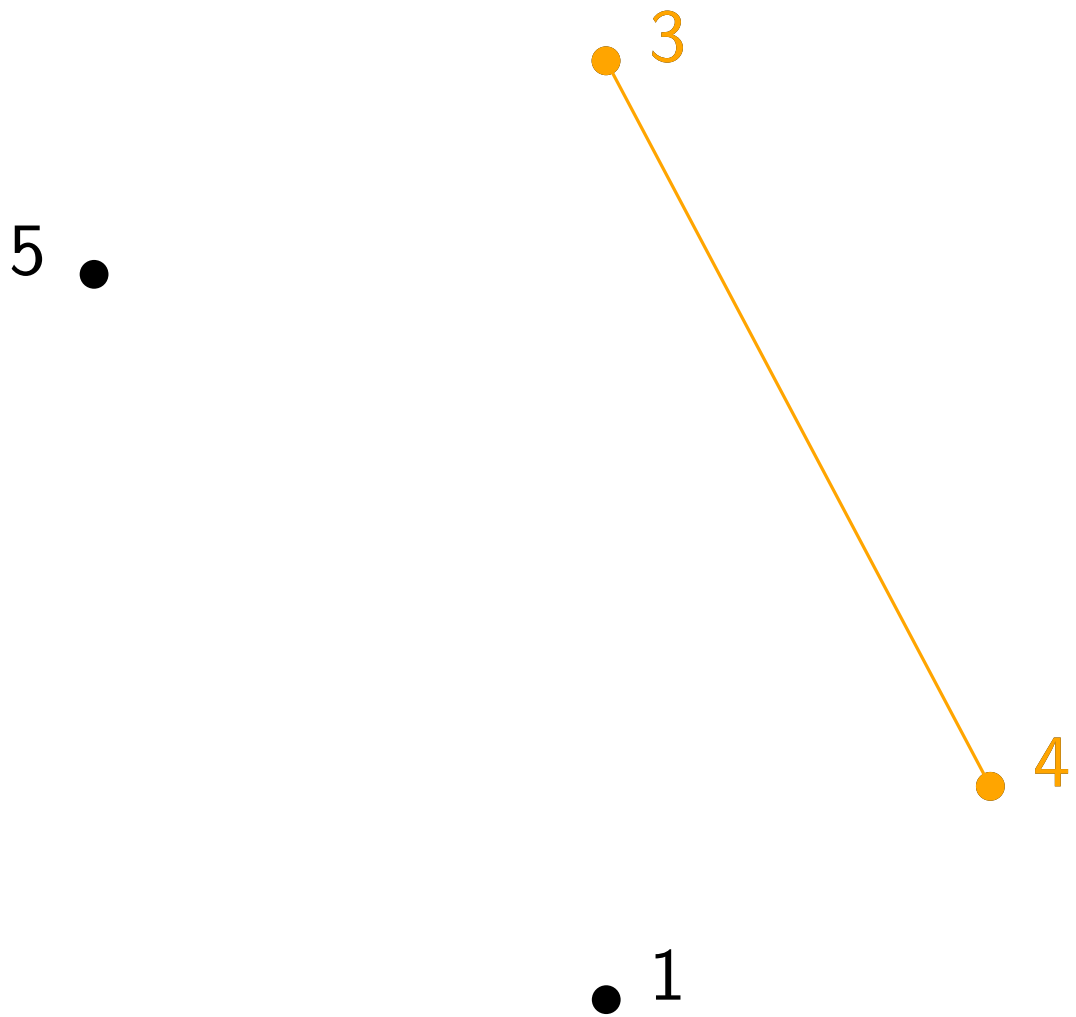


i	j
1	3
2	4
3	
4	
5	

Dynamic DS : Has property? Yes

Time-Windowed → Dynamic

CH area > 1?

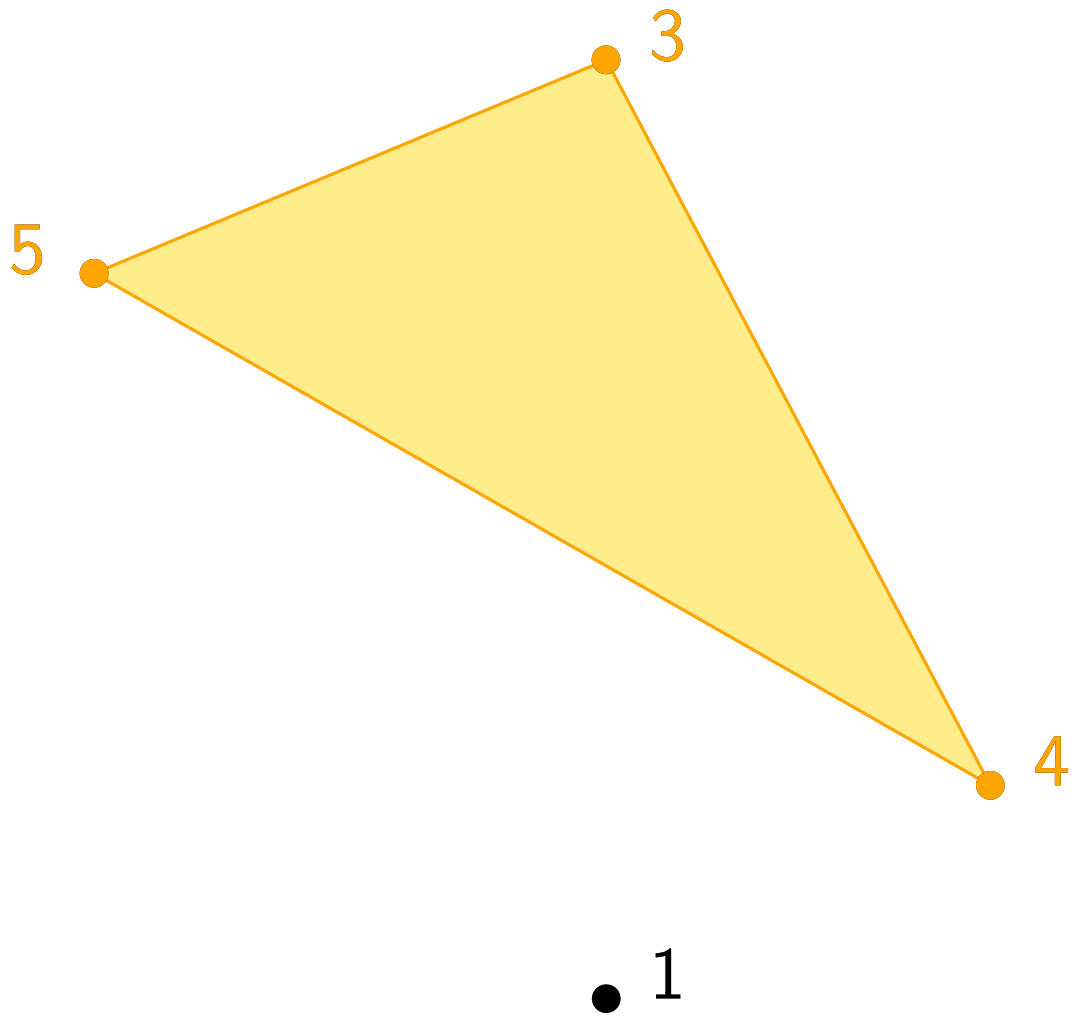


i	j
1	3
2	4
3	
4	
5	

Dynamic DS : Has property? No

Time-Windowed → Dynamic

CH area > 1?

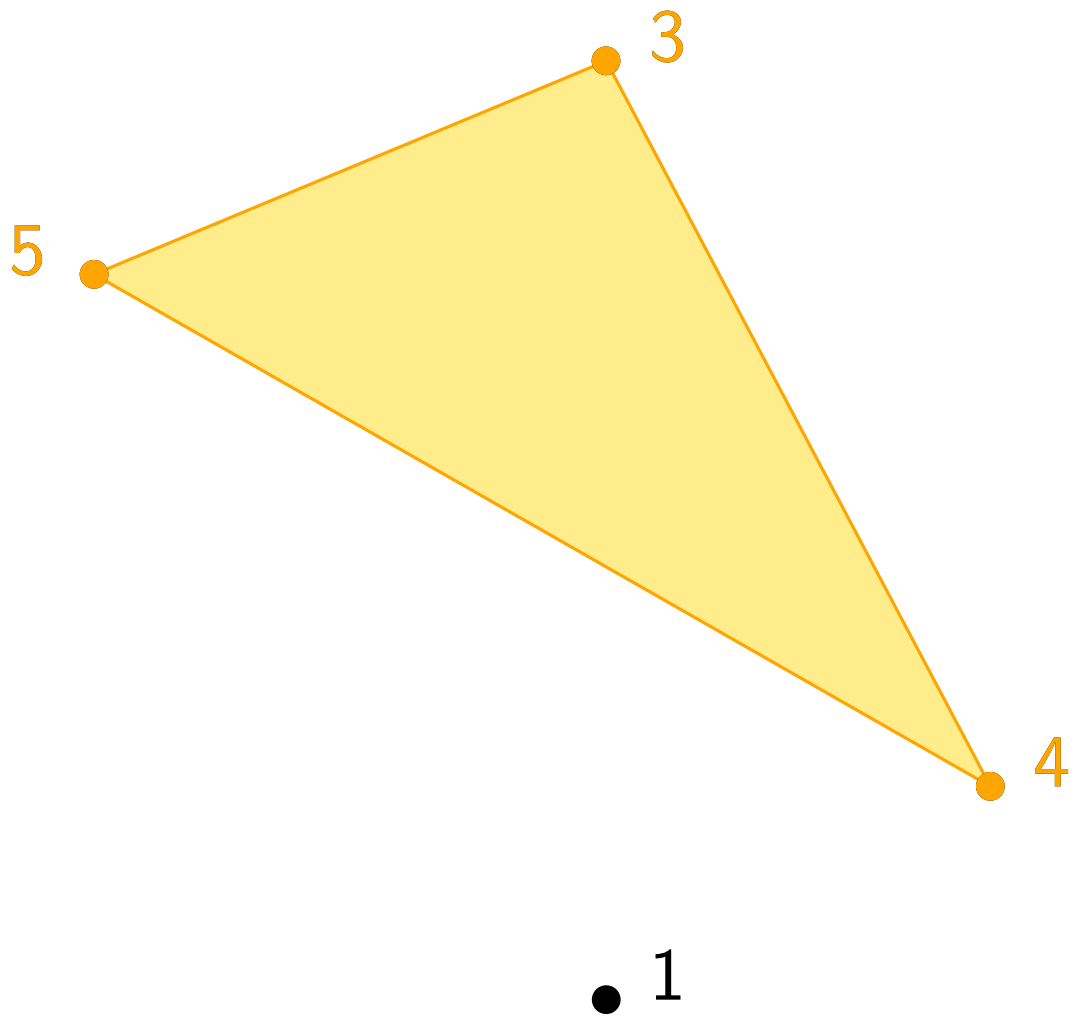


i	j
1	3
2	4
3	
4	
5	

Dynamic DS : Has property? Yes

Time-Windowed → Dynamic

CH area > 1?

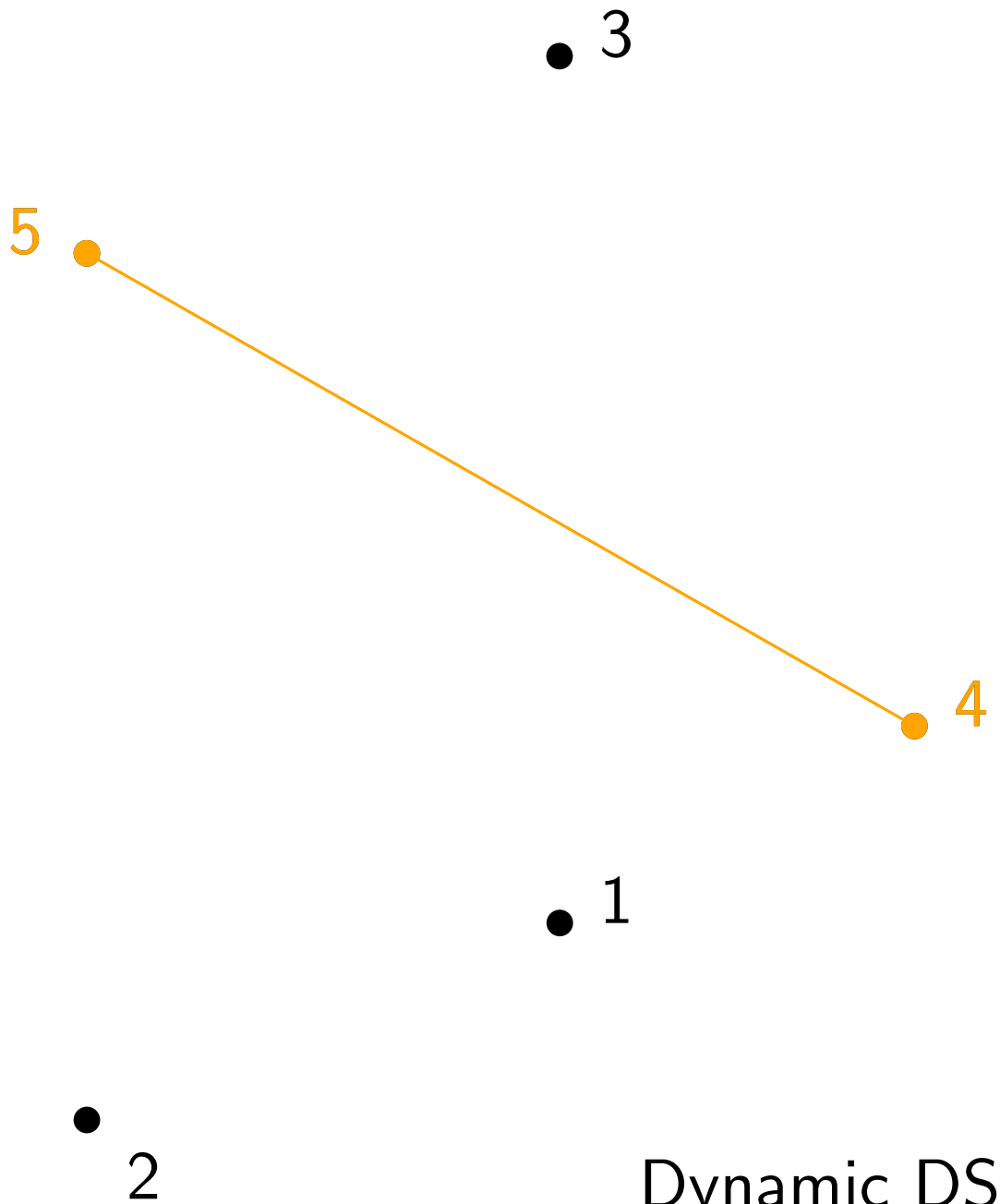


i	j
1	3
2	4
3	5
4	
5	

Dynamic DS : Has property? Yes

Time-Windowed $\rightarrow$ Dynamic

CH area > 1?

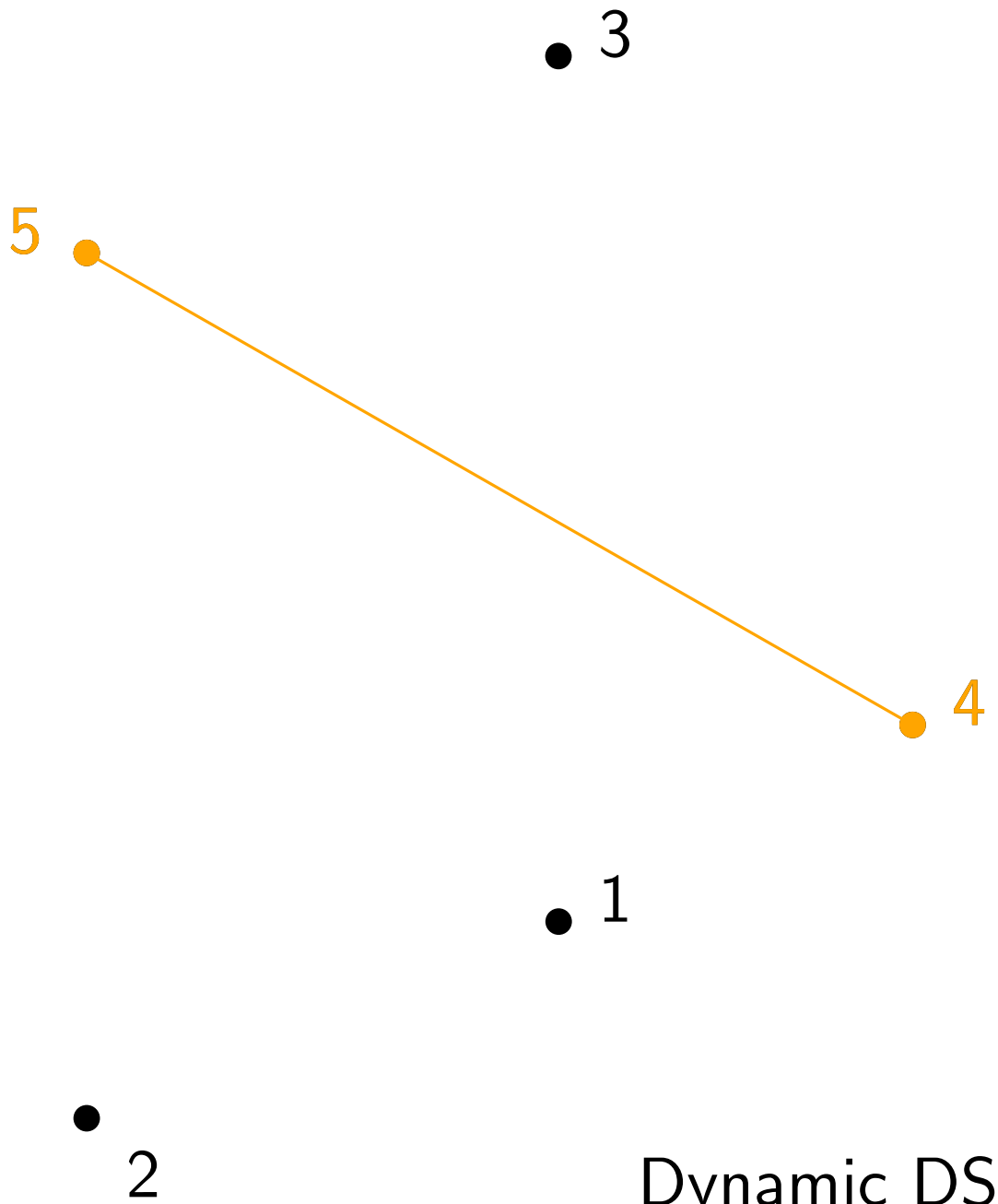


i	j
1	3
2	4
3	5
4	
5	

Dynamic DS : Has property? No

Time-Windowed $\rightarrow$ Dynamic

CH area > 1?

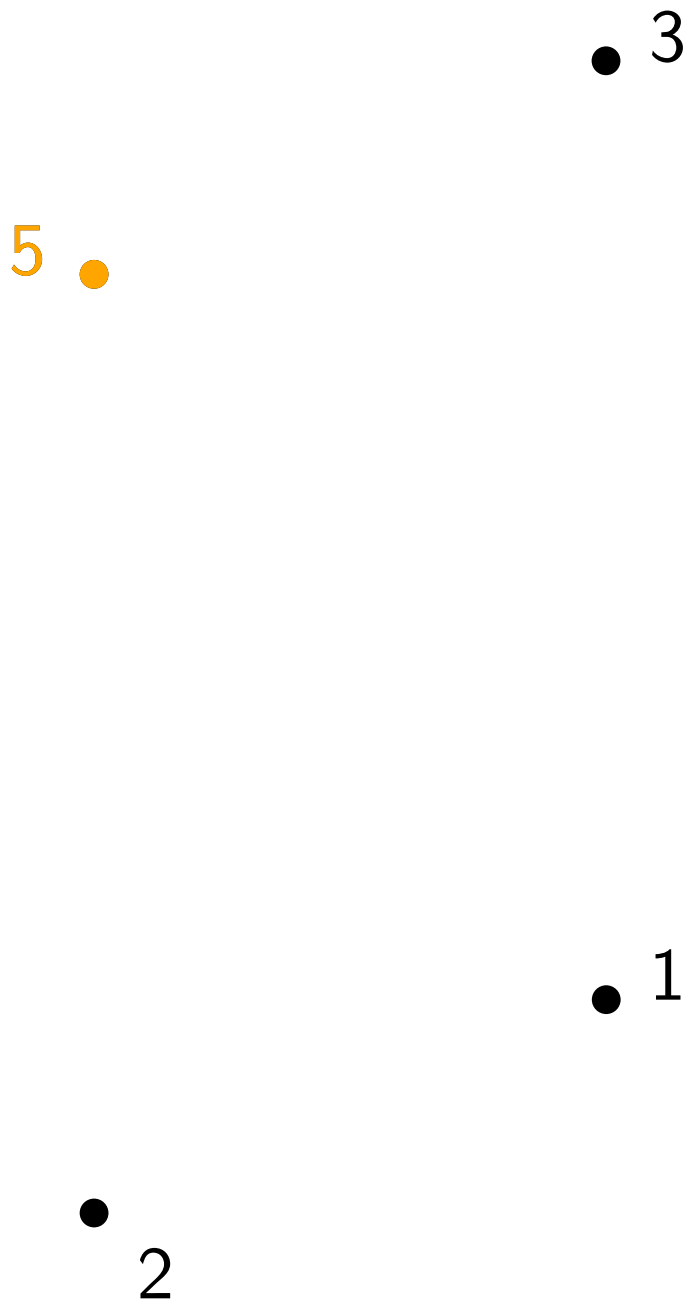


i	j
1	3
2	4
3	5
4	∞
5	

Dynamic DS : Has property? No

Time-Windowed $\rightarrow$ Dynamic

CH area > 1?

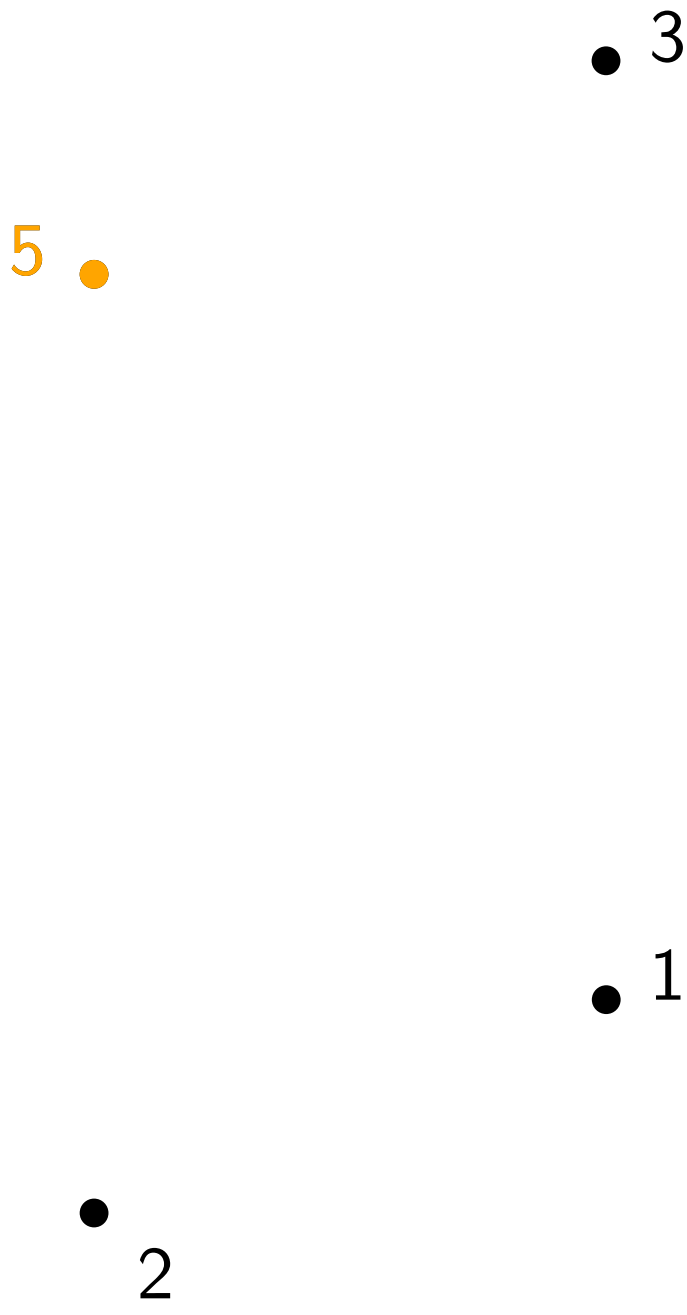


i	j
1	3
2	4
3	5
4	∞
5	

Dynamic DS : Has property? No

Time-Windowed $\rightarrow$ Dynamic

CH area > 1?

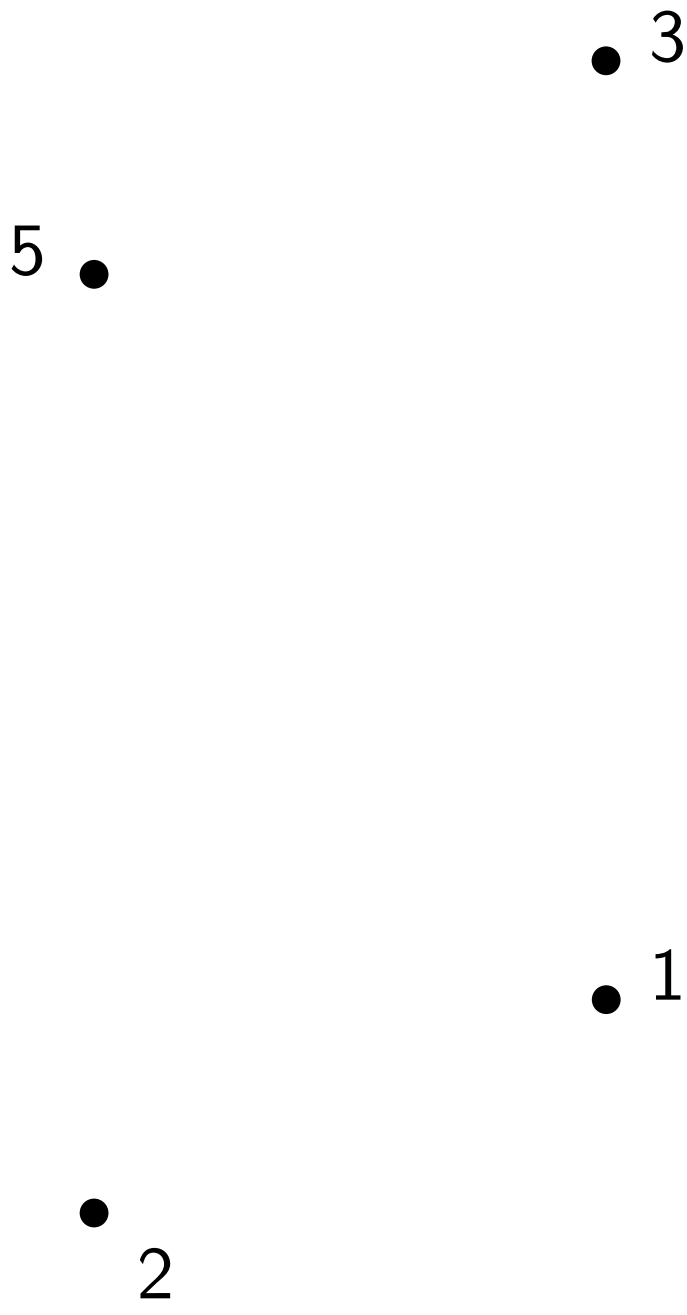


i	j
1	3
2	4
3	5
4	∞
5	∞

Dynamic DS : Has property? No

Time-Windowed $\rightarrow$ Dynamic

CH area > 1?

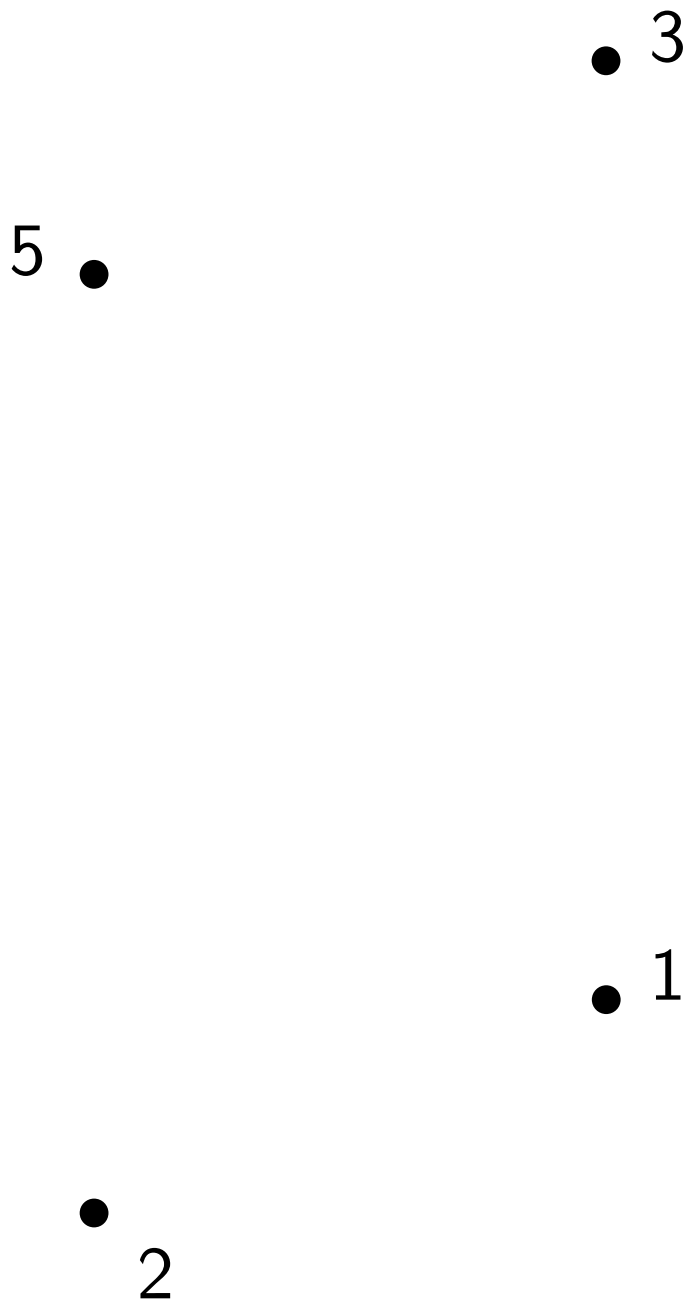


i	j
1	3
2	4
3	5
4	∞
5	∞

Dynamic DS : Has property? No

Time-Windowed $\rightarrow$ Dynamic

CH area > 1?



Notice the order of insertion/deletion

i	j
1	3
2	4
3	5
4	∞
5	∞

Dynamic DS : Has property? No

Time-Windowed $\rightarrow$ Dynamic

CH area > 1?

Notice the order of insertion/deletion

i	j
1	3
2	4
3	5
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5	∞

Dynamic DS : Has property? No

Time-Windowed $\rightarrow$ Dynamic

CH area > 1?

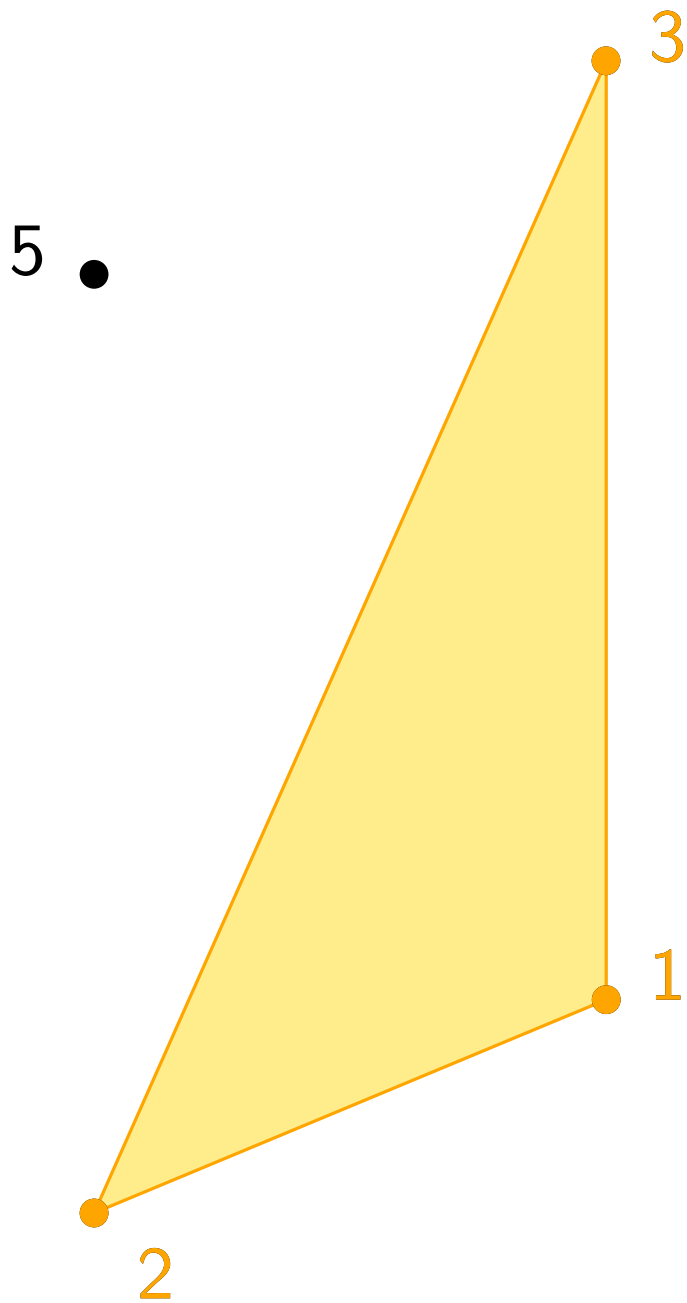
Notice the order of insertion/deletion

i	j
1	3
2	4
3	5
4	∞
5	∞

Dynamic DS : Has property? No

Time-Windowed $\rightarrow$ Dynamic

CH area > 1?



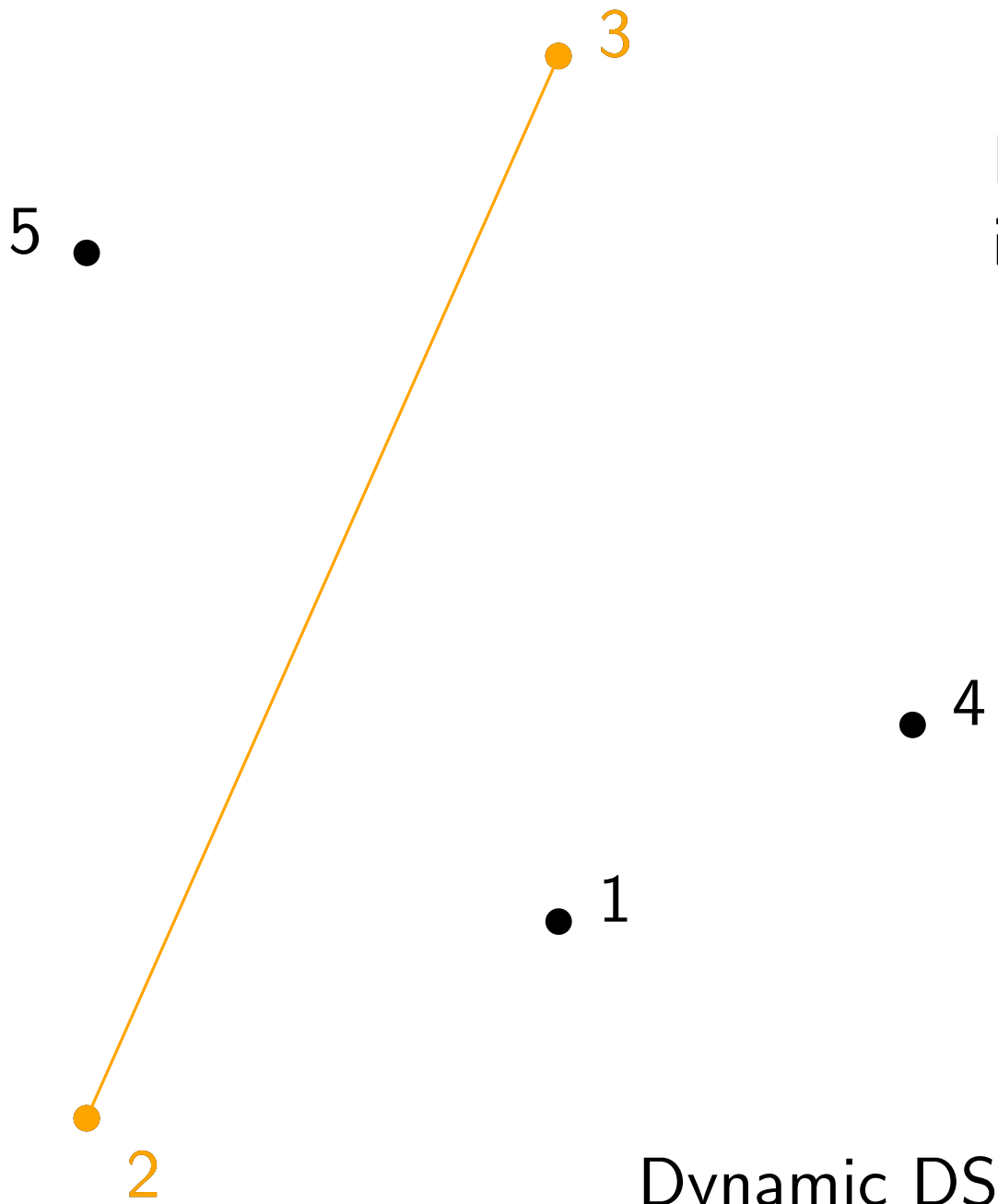
Notice the order of insertion/deletion

i	j
1	3
2	4
3	5
4	∞
5	∞

Dynamic DS : Has property? No

Time-Windowed $\rightarrow$ Dynamic

CH area > 1?



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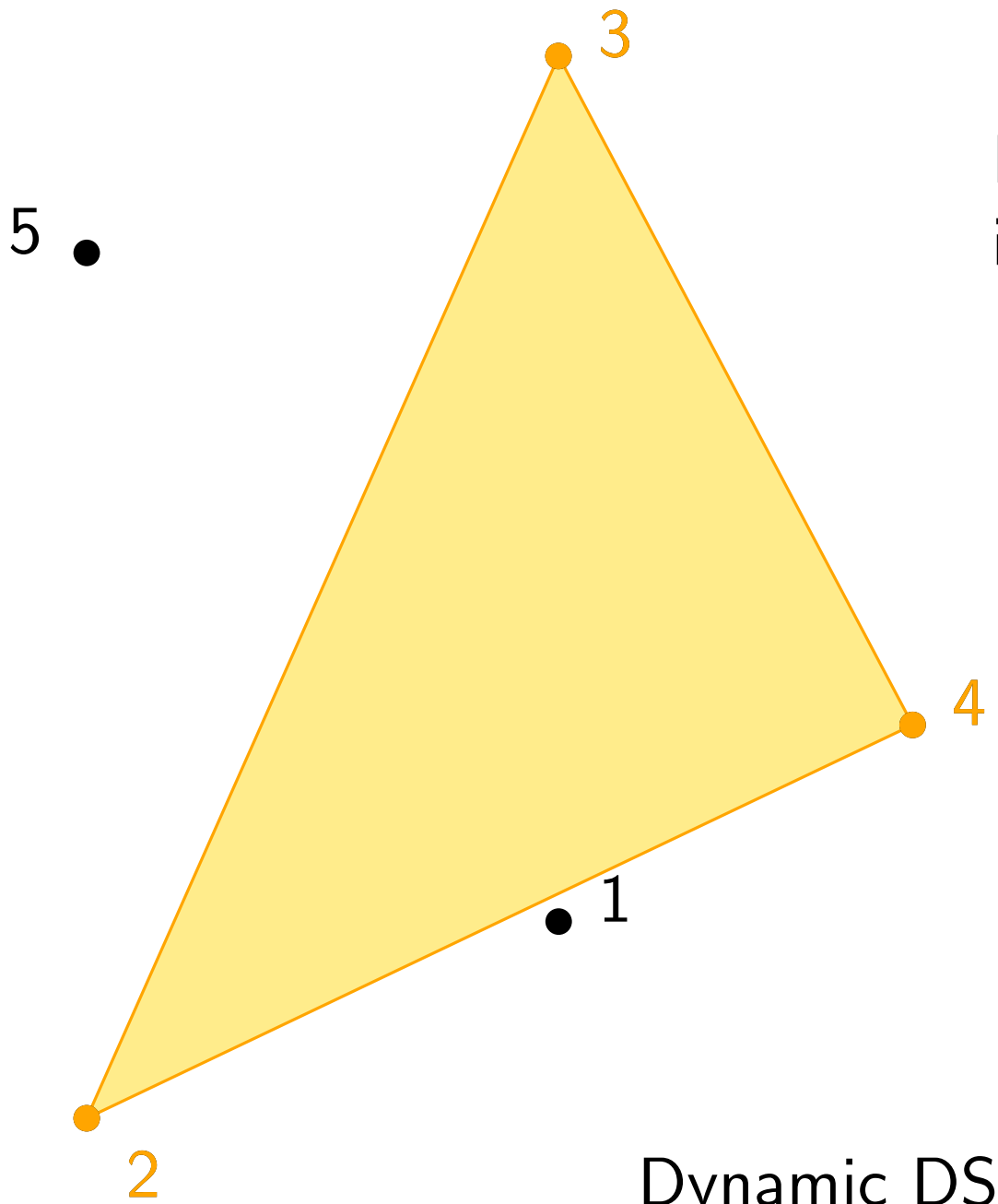
1	3
2	4
3	5
4	∞
5	∞

Dynamic DS : Has property? No

Time-Windowed $\rightarrow$ Dynamic

CH area > 1?



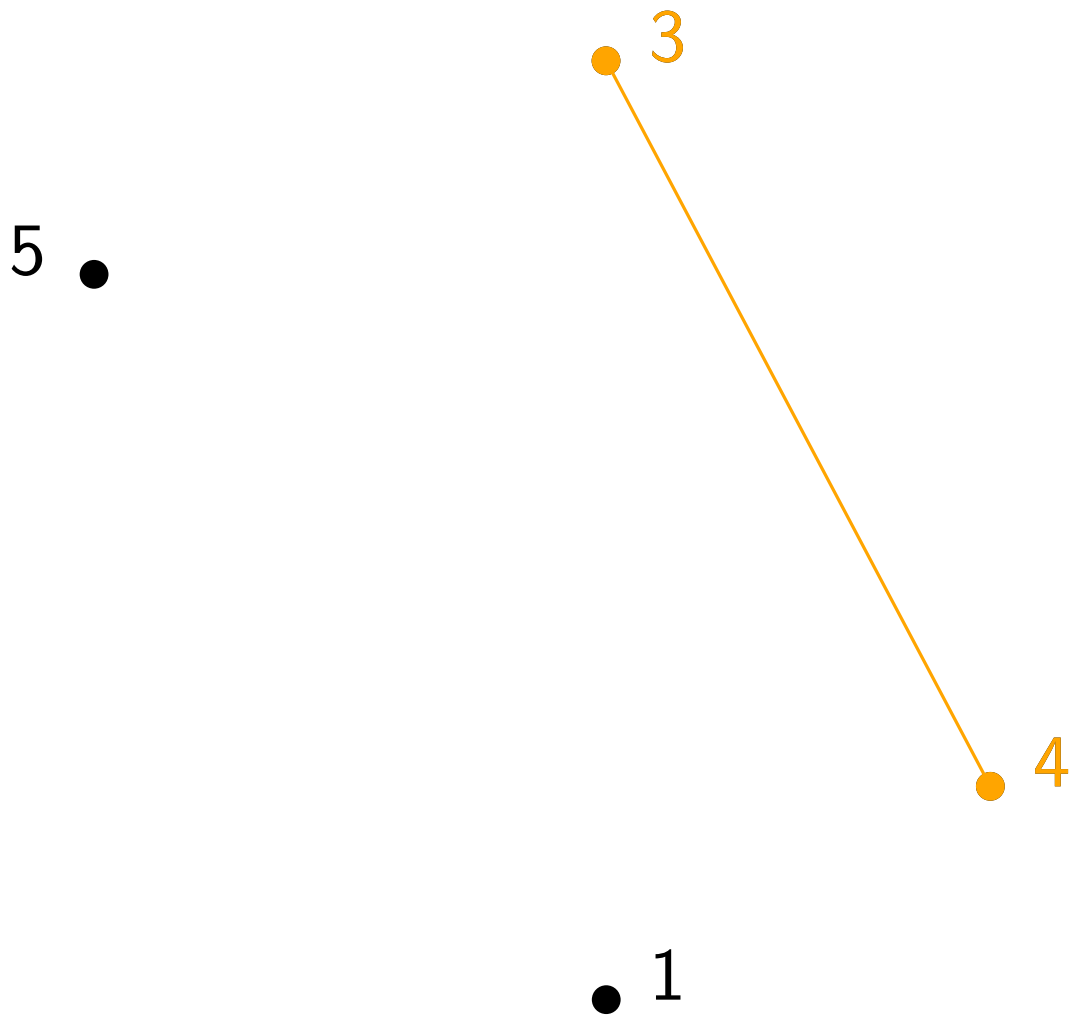
Notice the order of insertion/deletion

i	j
1	3
2	4
3	5
4	∞
5	∞

Dynamic DS : Has property? No

Time-Windowed $\rightarrow$ Dynamic

CH area > 1?



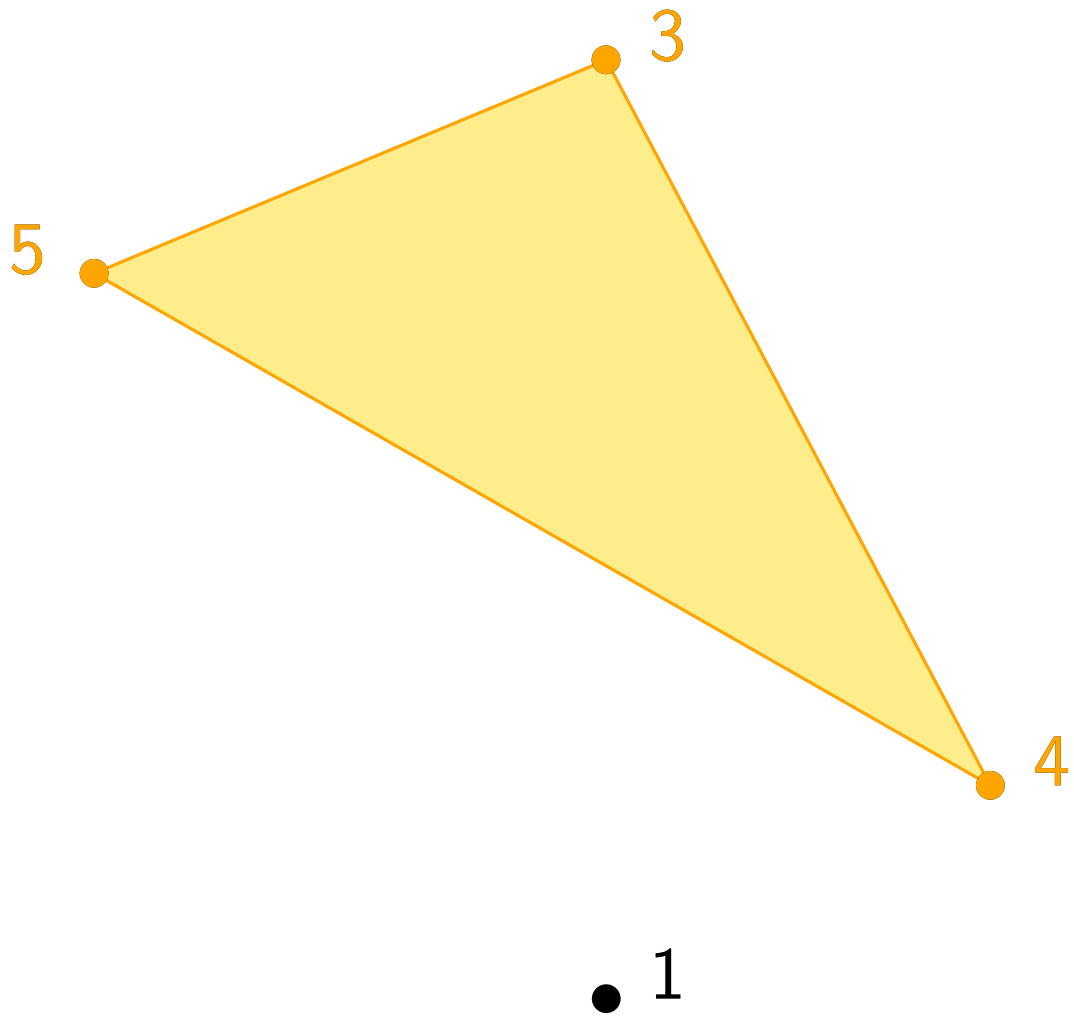
Notice the order of insertion/deletion

i	j
1	3
2	4
3	5
4	∞
5	∞

Dynamic DS : Has property? No

Time-Windowed $\rightarrow$ Dynamic

CH area > 1?



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i	j
1	3
2	4
3	5
4	∞
5	∞

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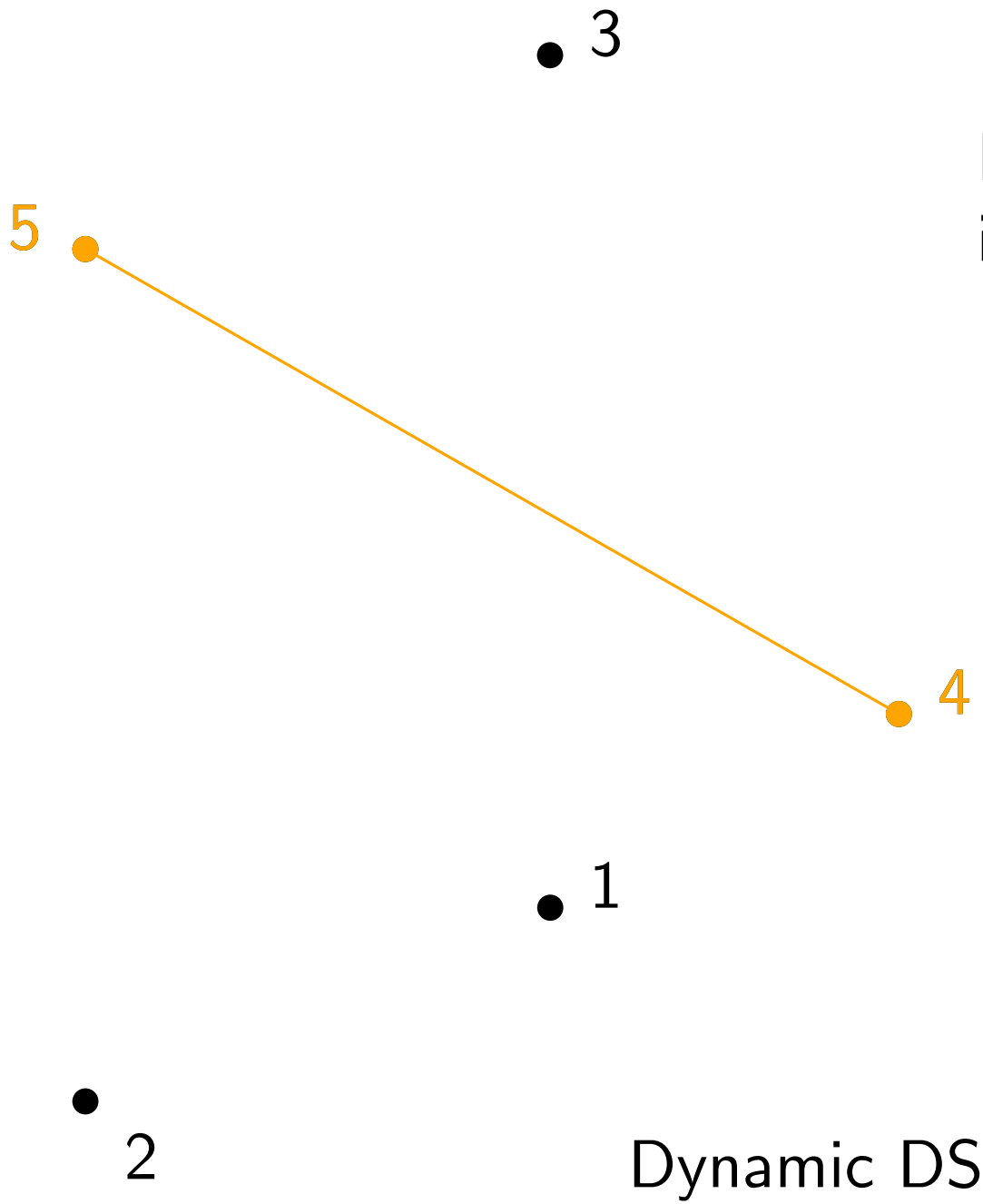
CH area > 1?



Notice the order of insertion/deletion

i	j
1	3
2	4
3	5
4	∞
5	∞

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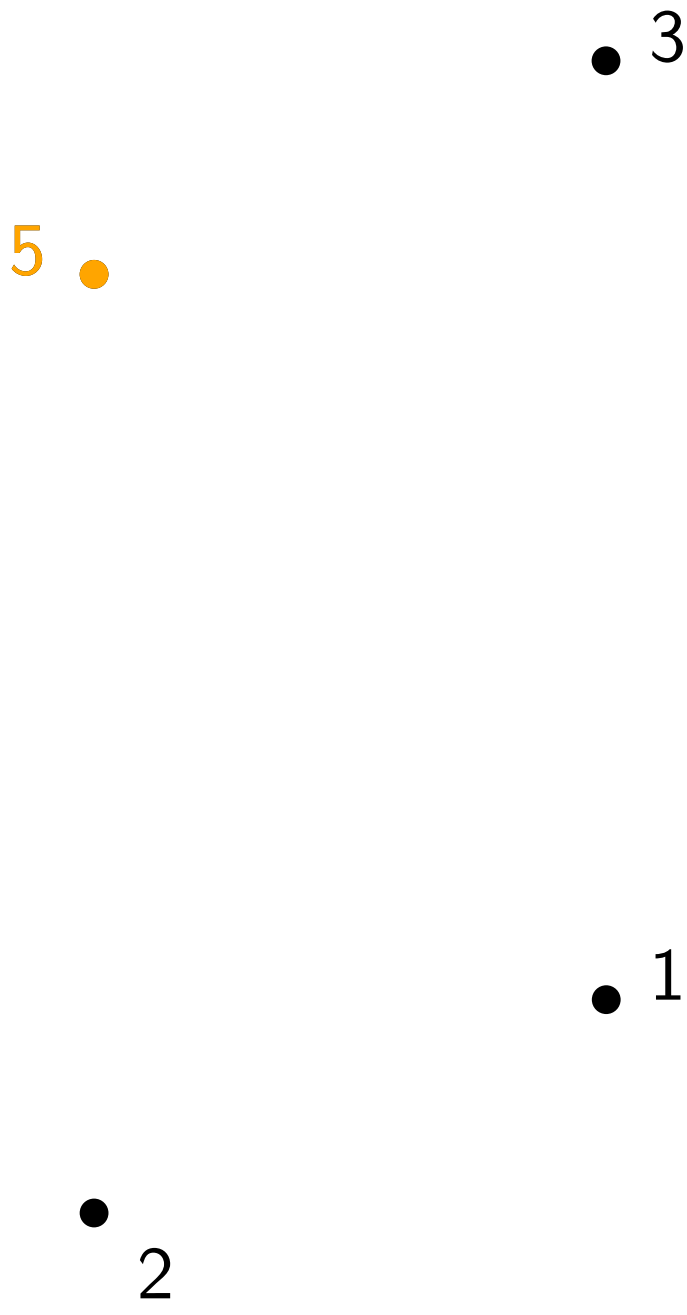
Time-Windowed $\rightarrow$ Dynamic

CH area > 1?



Notice the order of insertion/deletion

i	j
1	3
2	4
3	5
4	∞
5	∞

Dynamic DS : Has property? No

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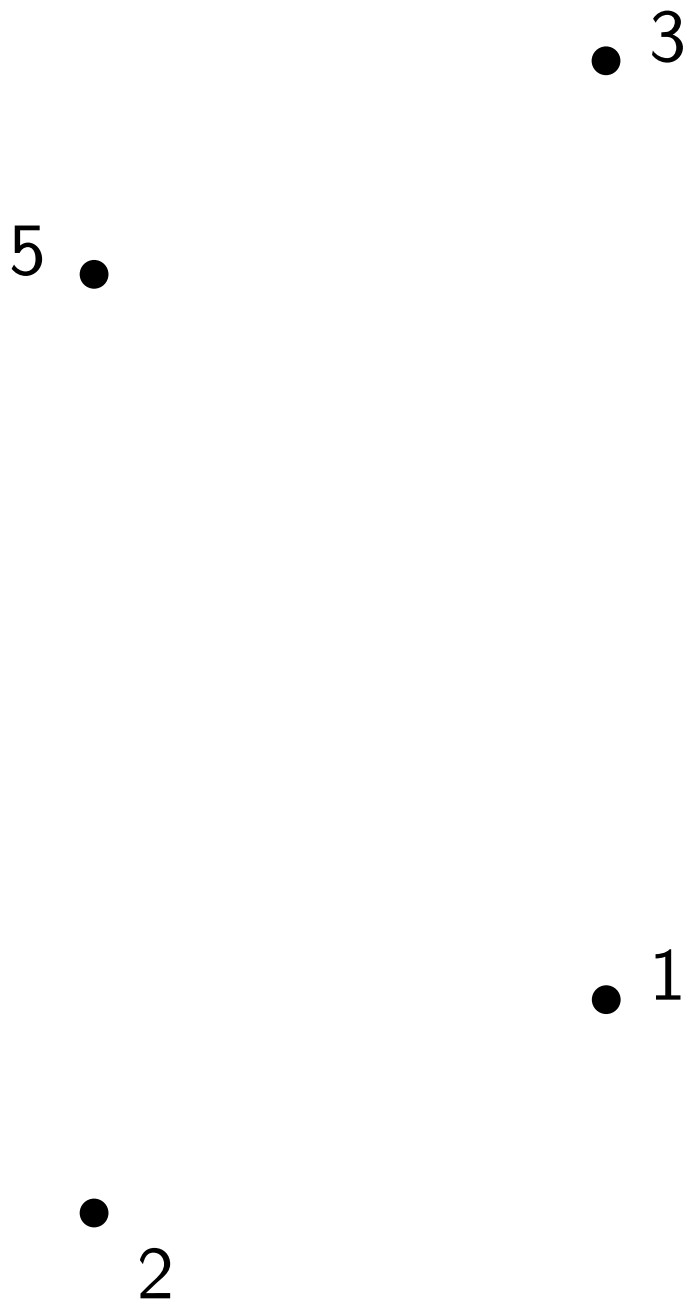


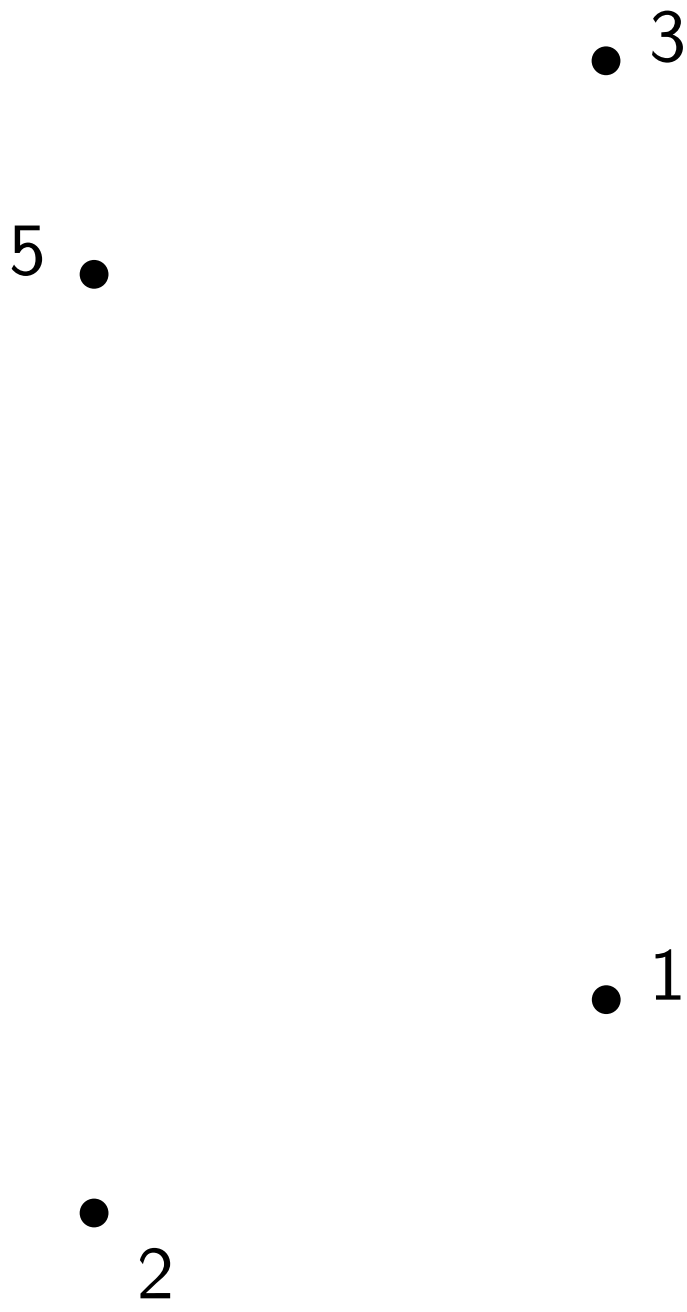
Notice the order of insertion/deletion

i	j
1	3
2	4
3	5
4	∞
5	∞

Dynamic DS : Has property? No

Time-Windowed $\rightarrow$ Dynamic

CH area > 1?



Notice the order of insertion/deletion

First-in, first-out: like a queue

i	j
1	3
2	4
3	5
4	∞
5	∞

Dynamic DS : Has property? No

Time-Windowed → Dynamic

CH area > 1?

Notice the order of insertion/deletion

First-in, first-out: like a queue

FIFO update sequence

i	j
1	3
2	4
3	5
4	∞
5	∞

Dynamic DS : Has property? No

Dynamic Convex Hull Complexity

Generally, $O(n^2)$ changes over $O(n)$ updates

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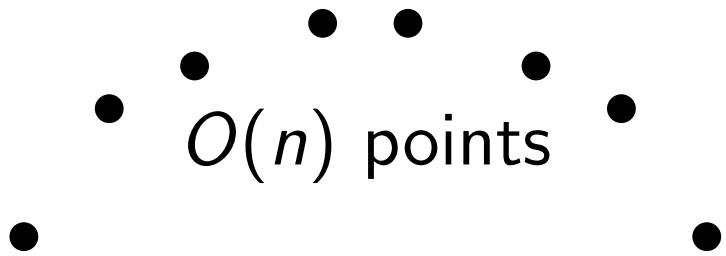
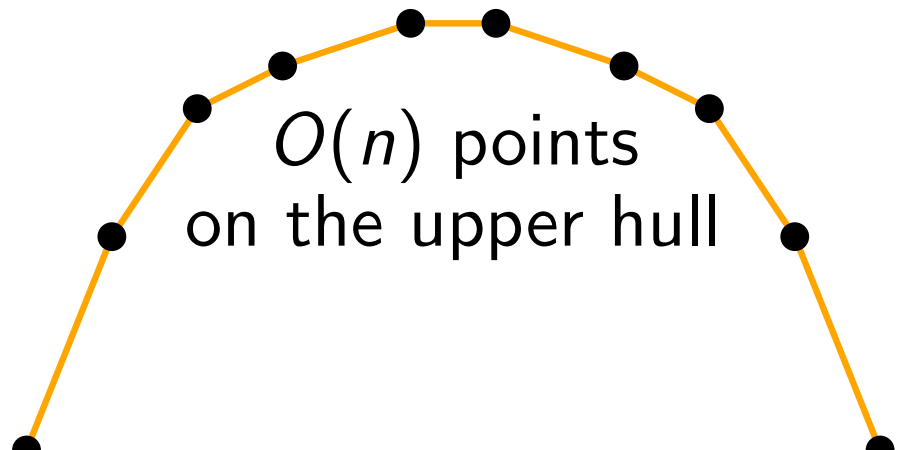


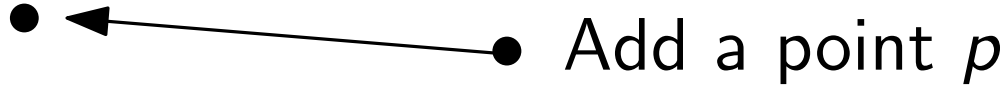
Dynamic Convex Hull Complexity

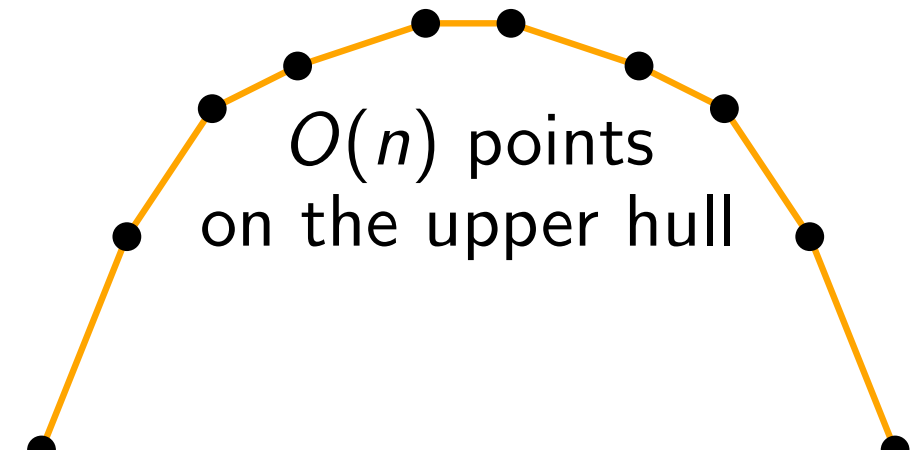
Generally, $O(n^2)$ changes over $O(n)$ updates



Dynamic Convex Hull Complexity

Generally, $O(n^2)$ changes over $O(n)$ updates

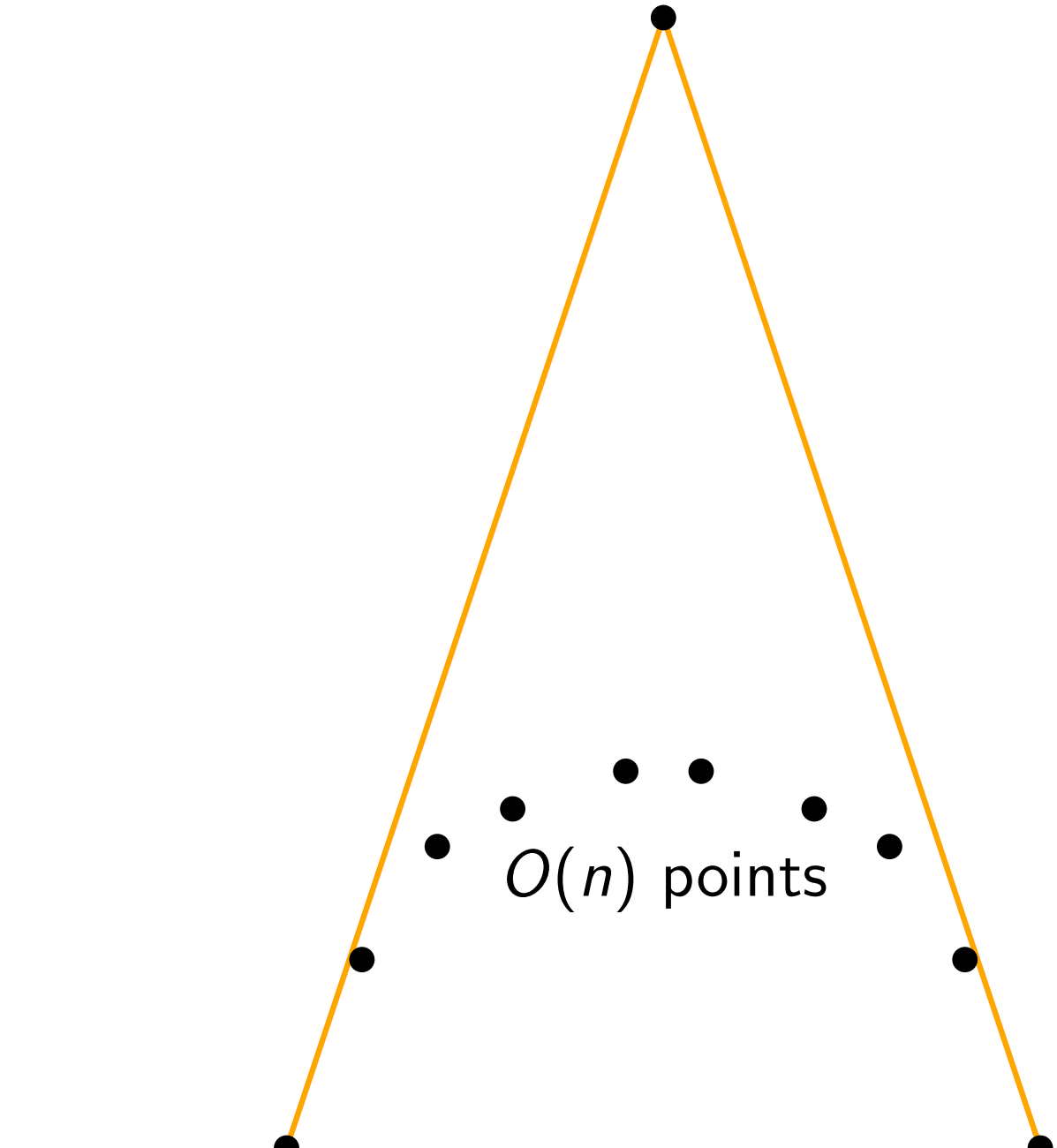




Dynamic Convex Hull Complexity

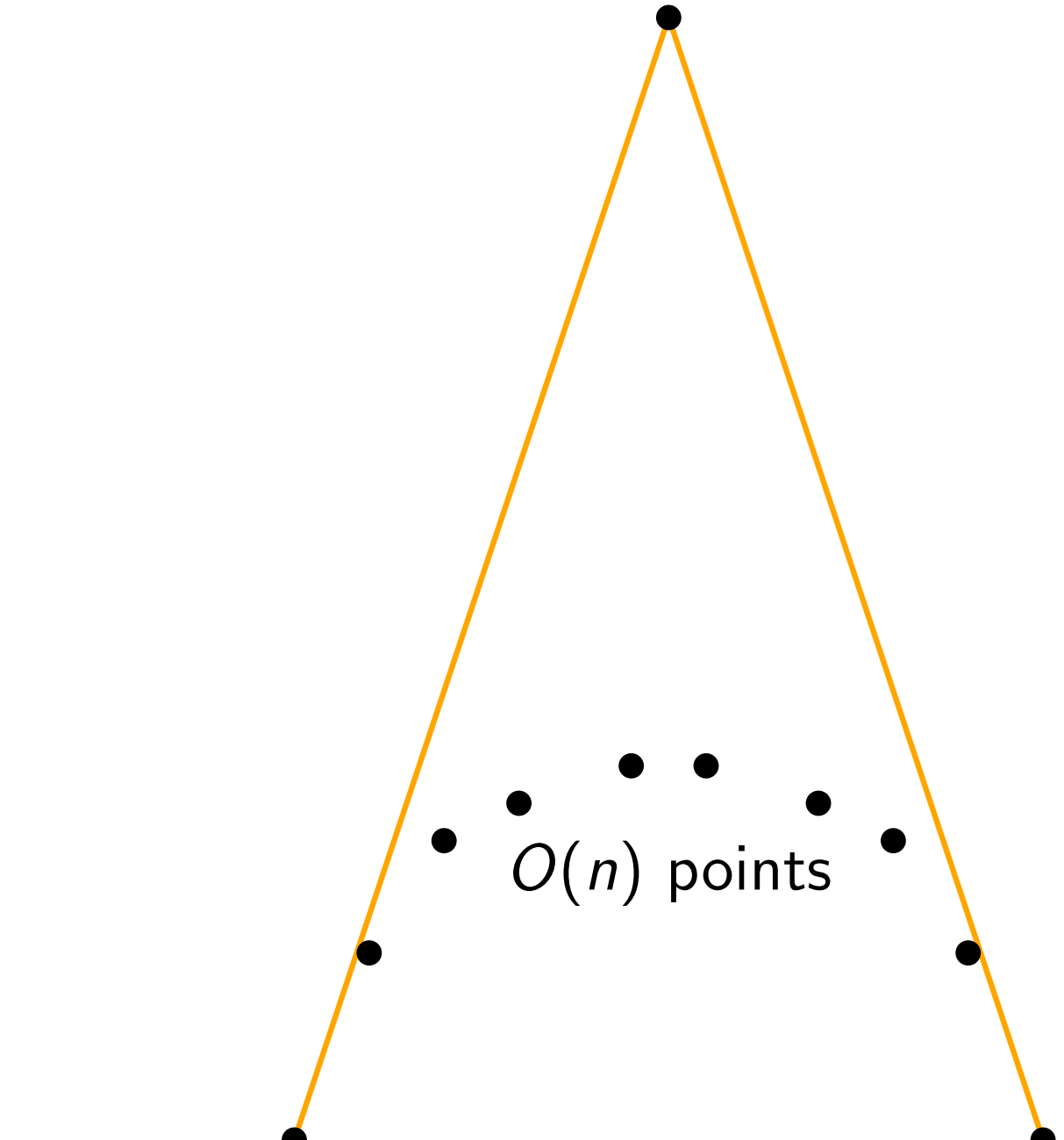
Generally, $O(n^2)$ changes over $O(n)$ updates

- Add a point p
- $O(n)$ changes



Dynamic Convex Hull Complexity

Generally, $O(n^2)$ changes over $O(n)$ updates



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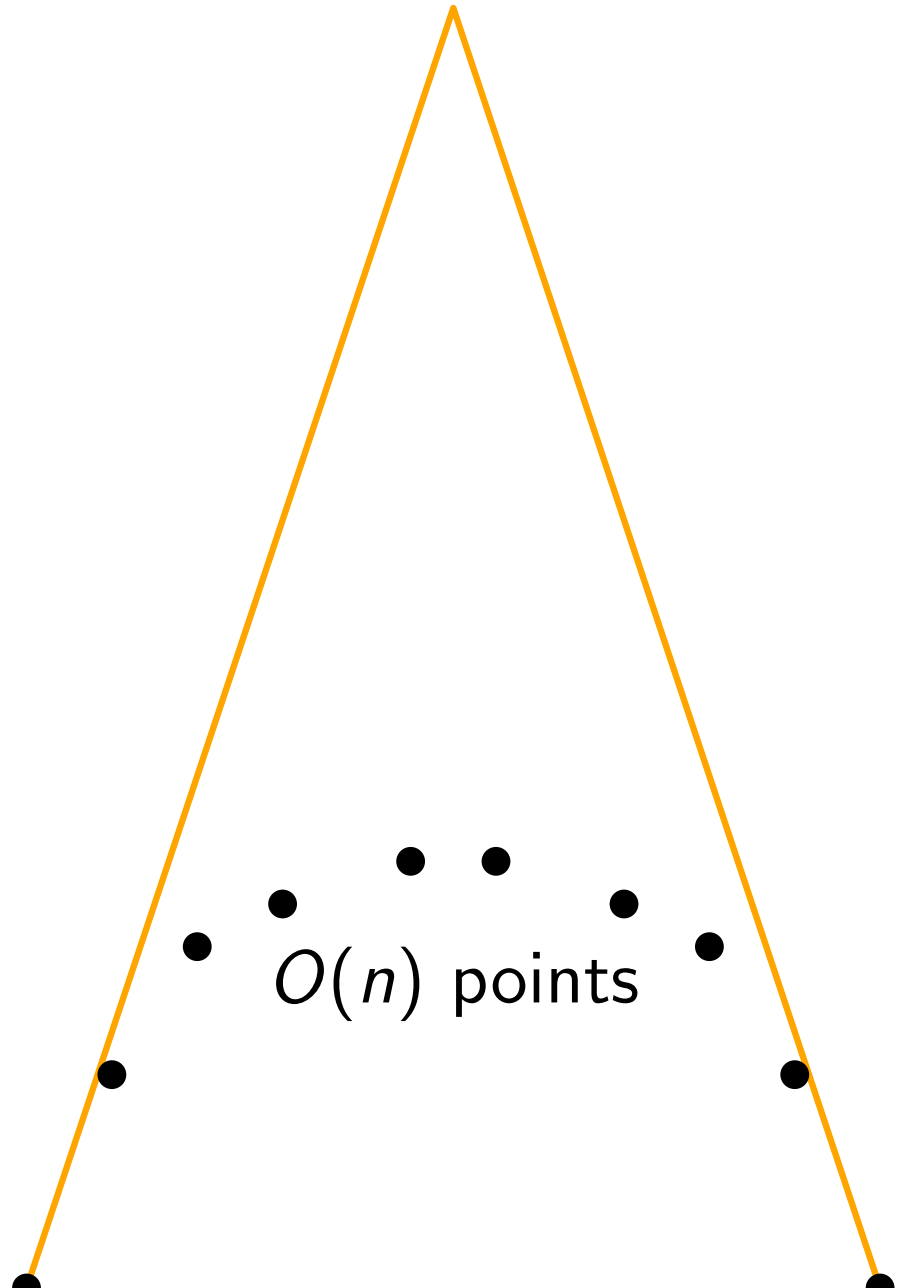

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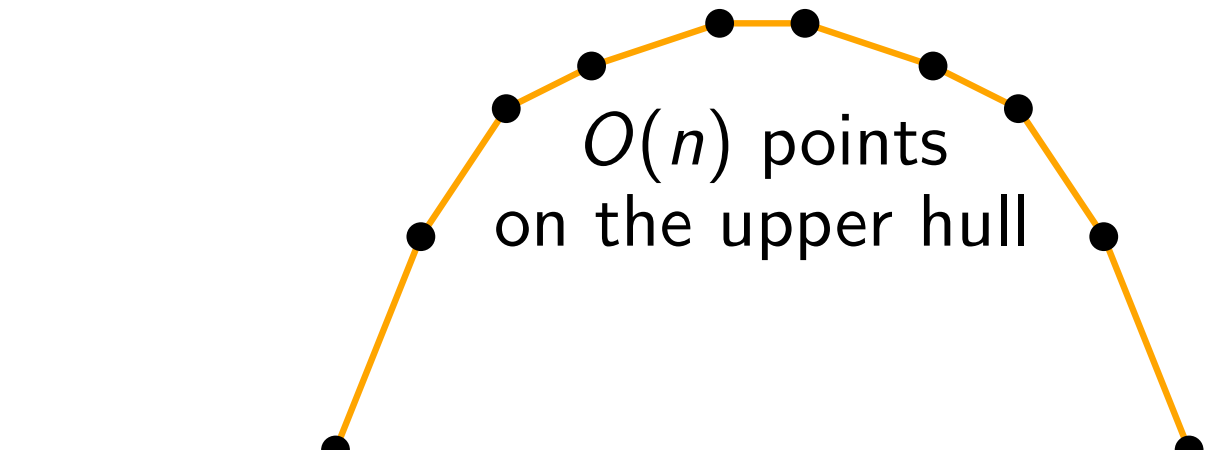

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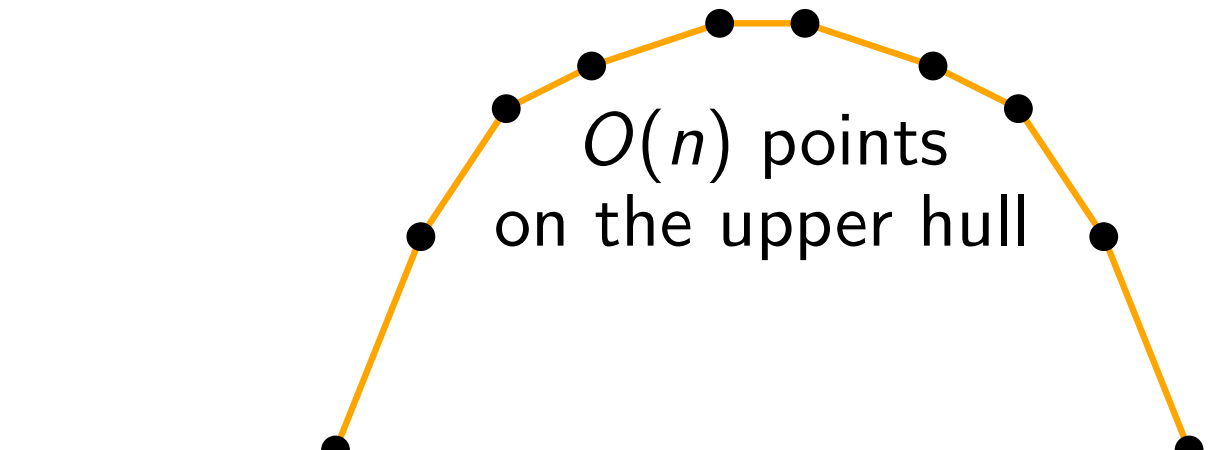

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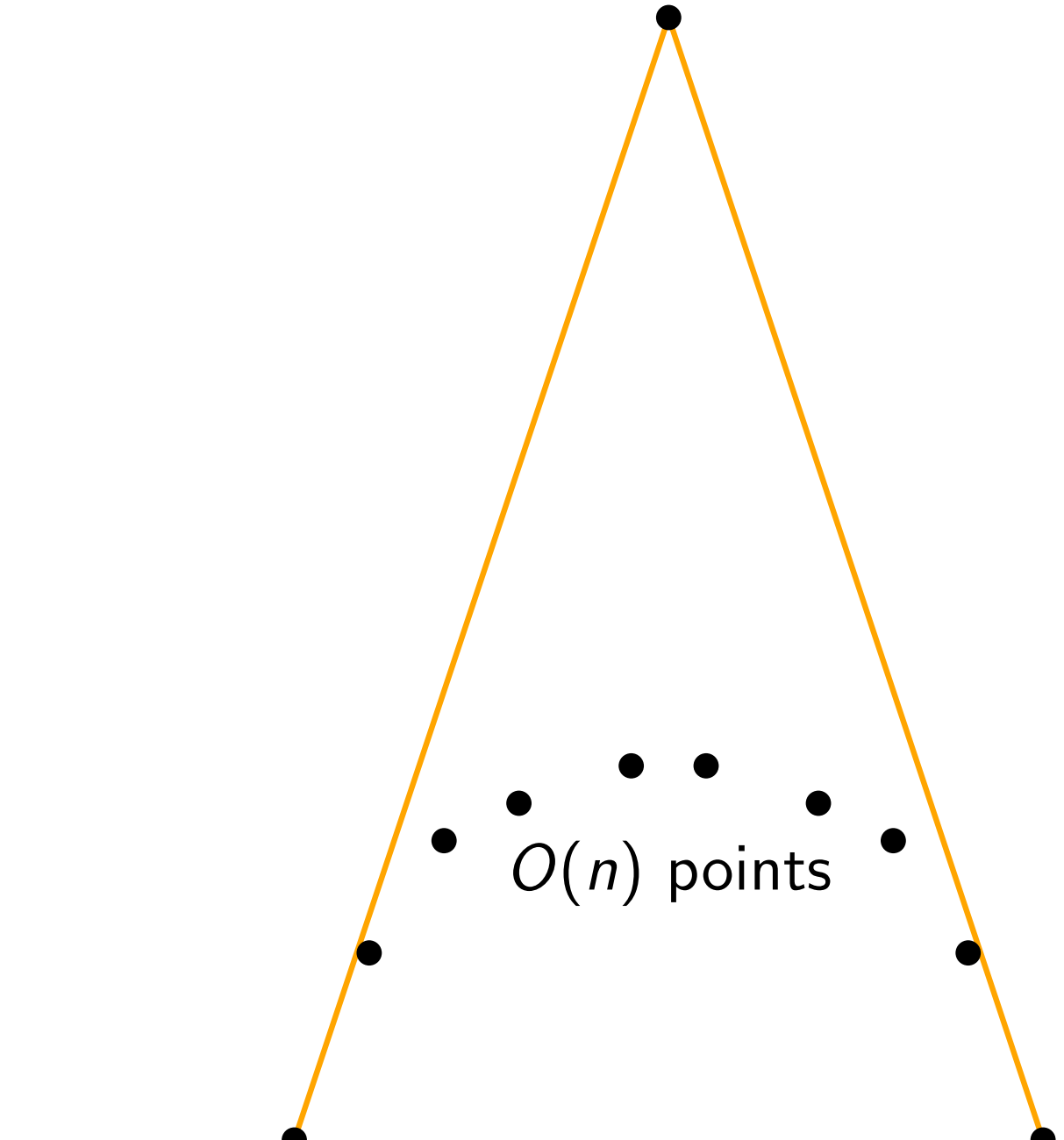

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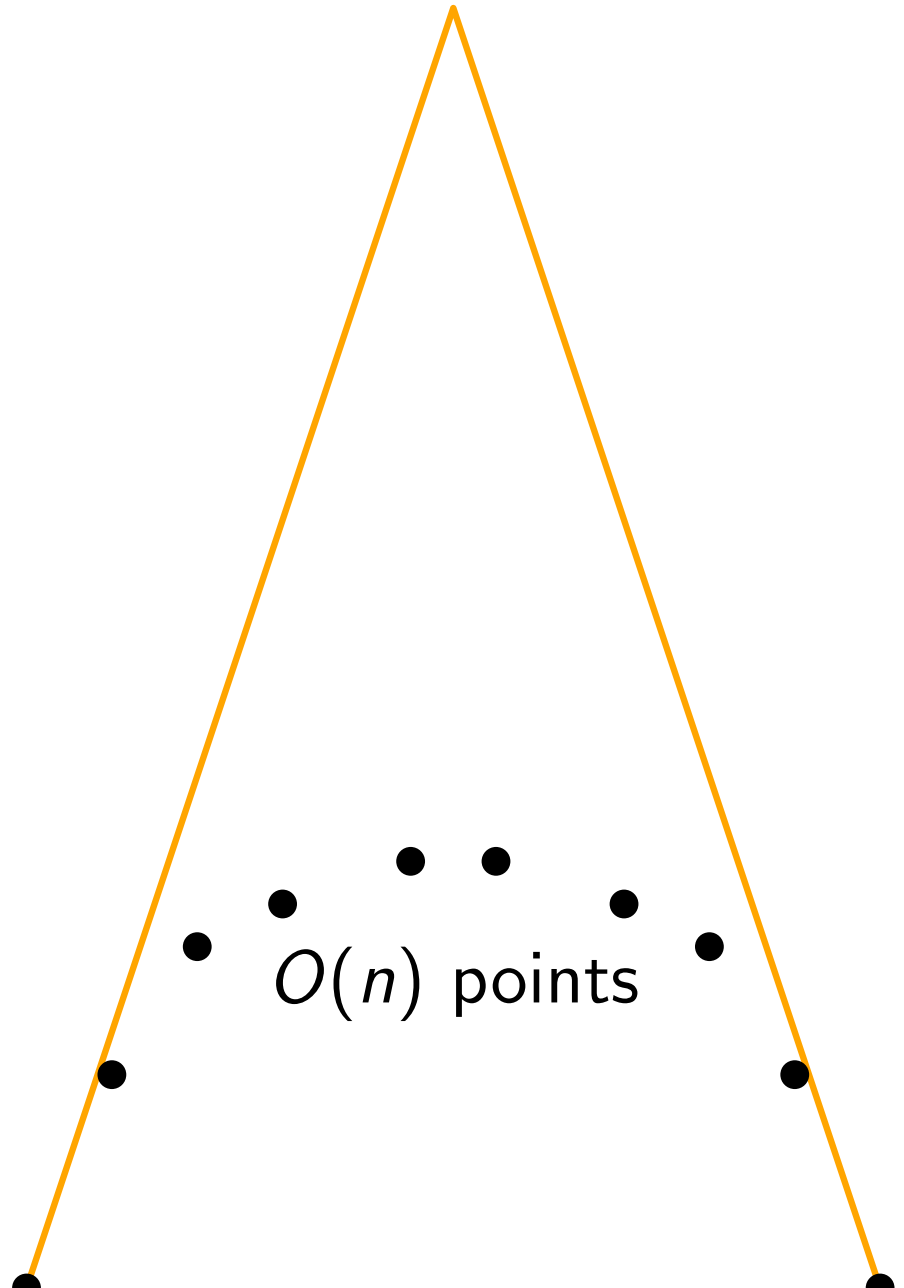

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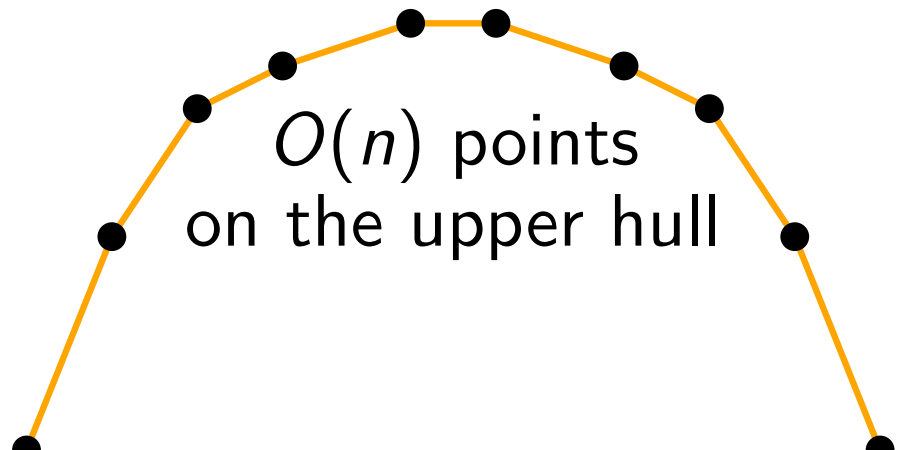

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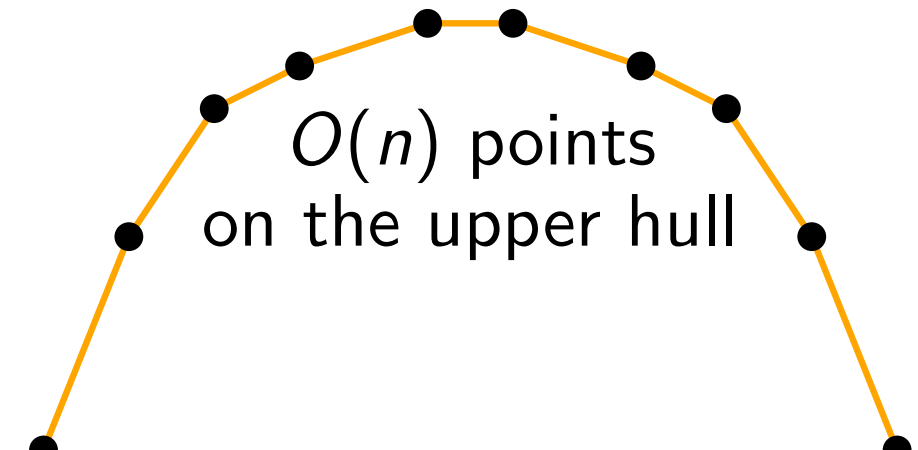

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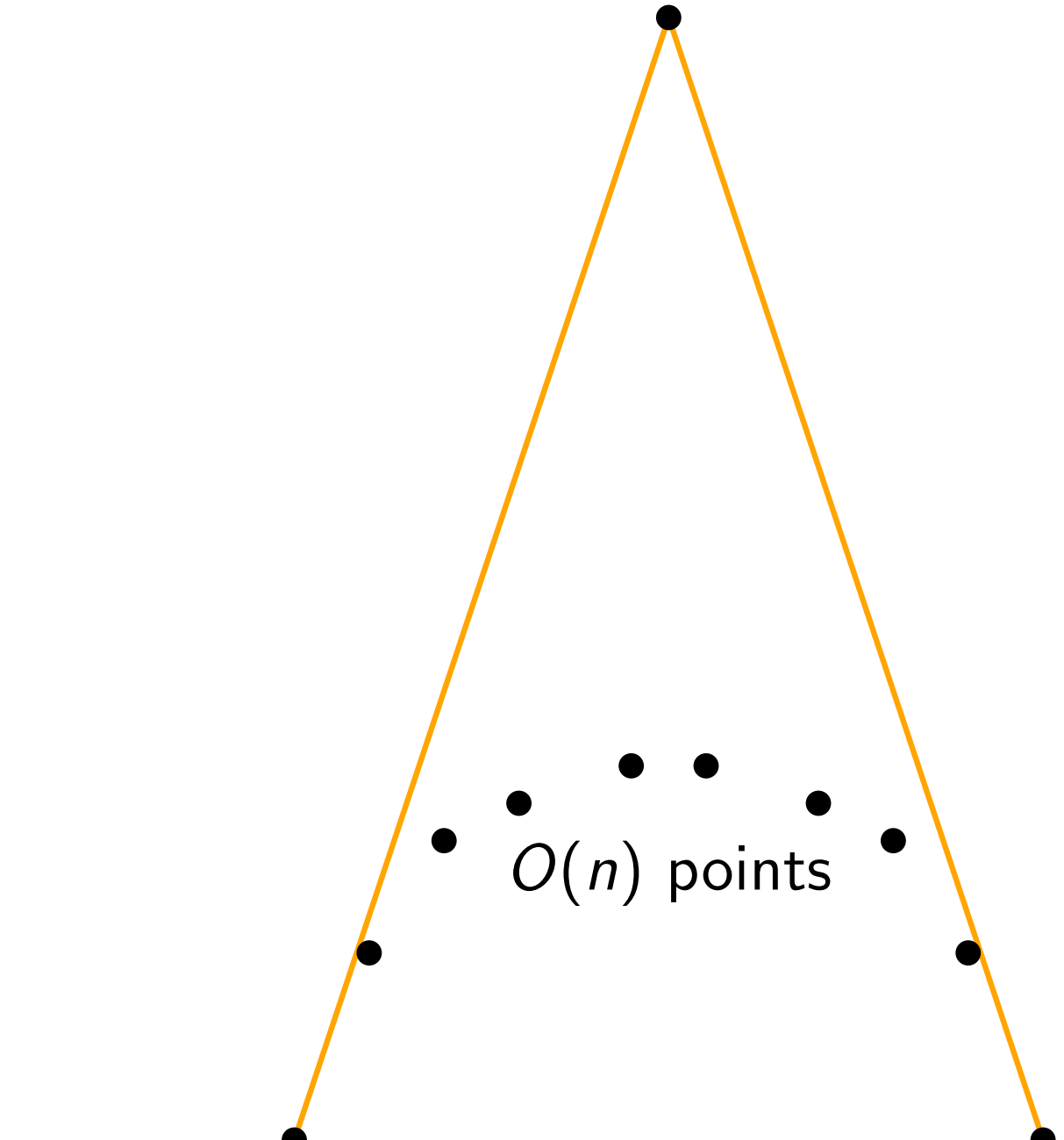

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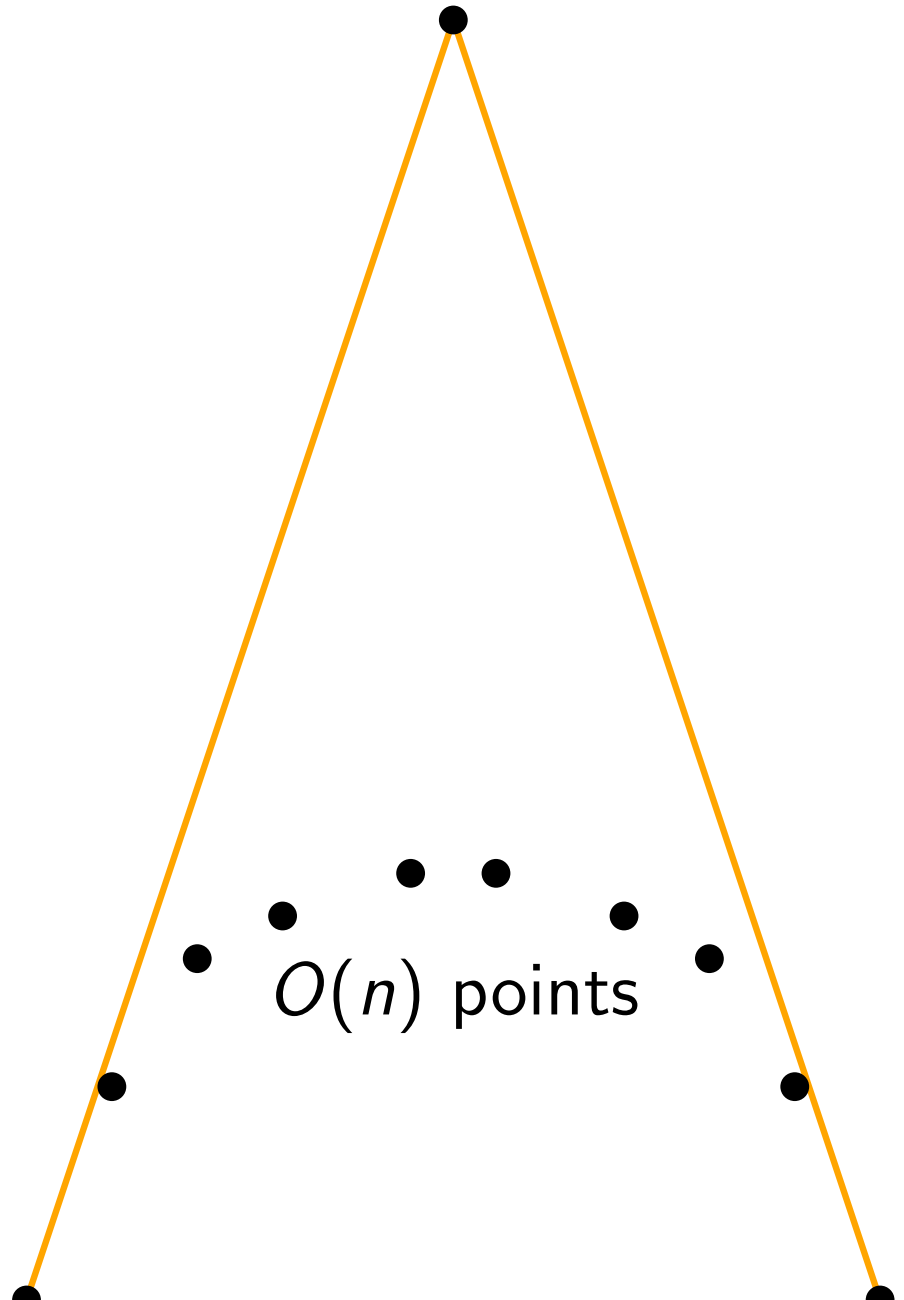

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FIFO Convex Hull Complexity

Lemma: # of structural changes to upper hull after n FIFO updates is $O(n \log n)$.

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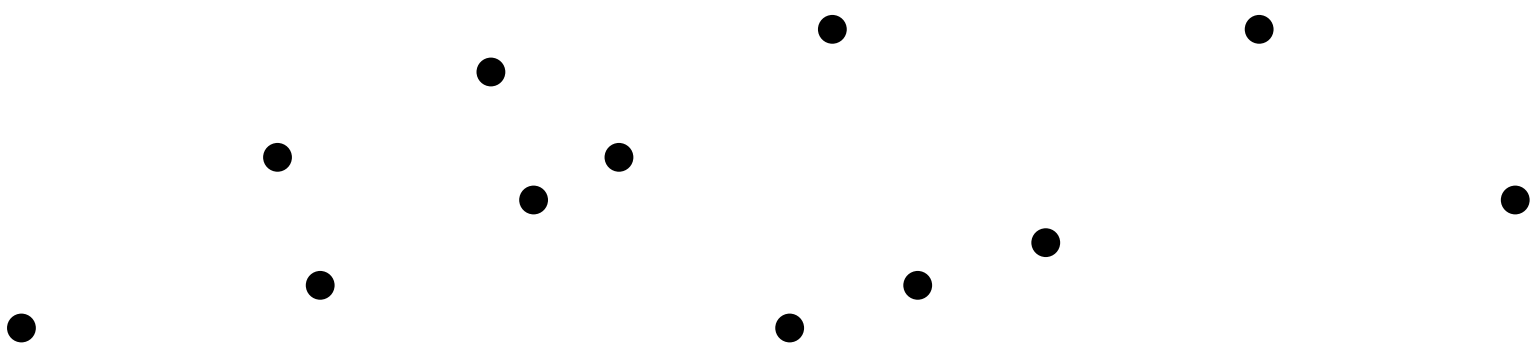

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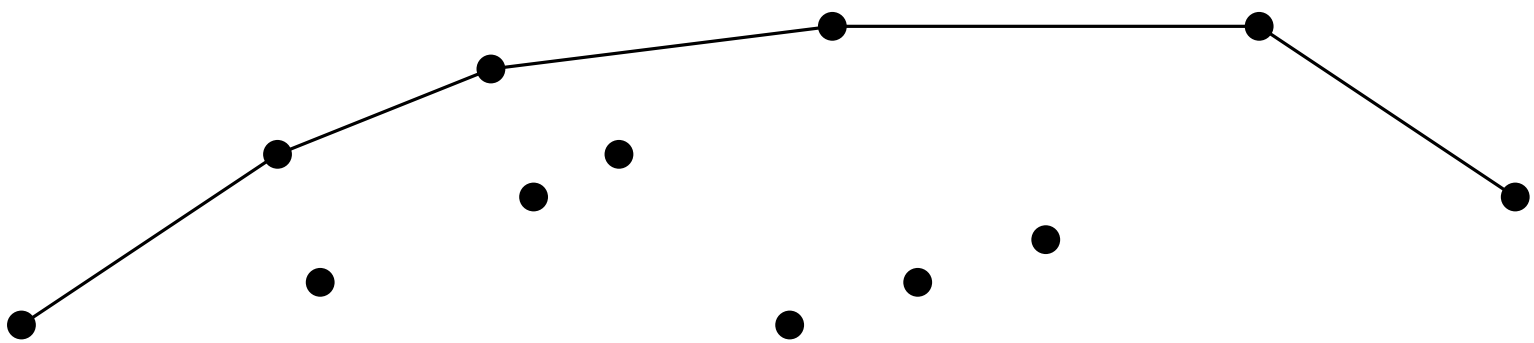

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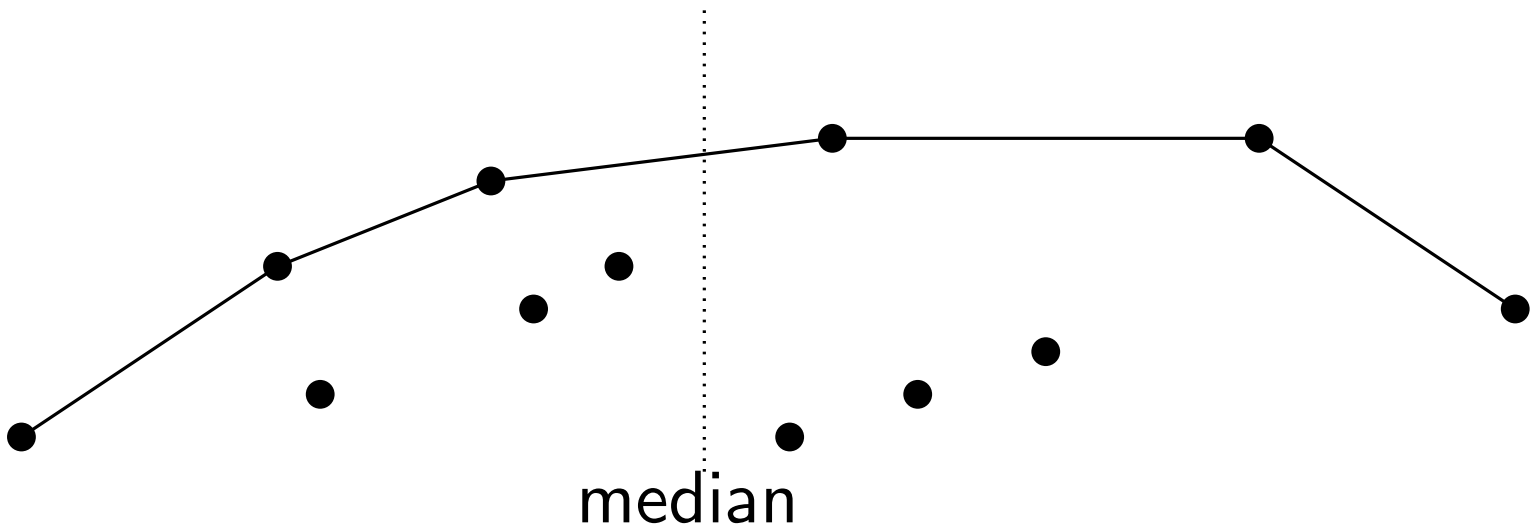

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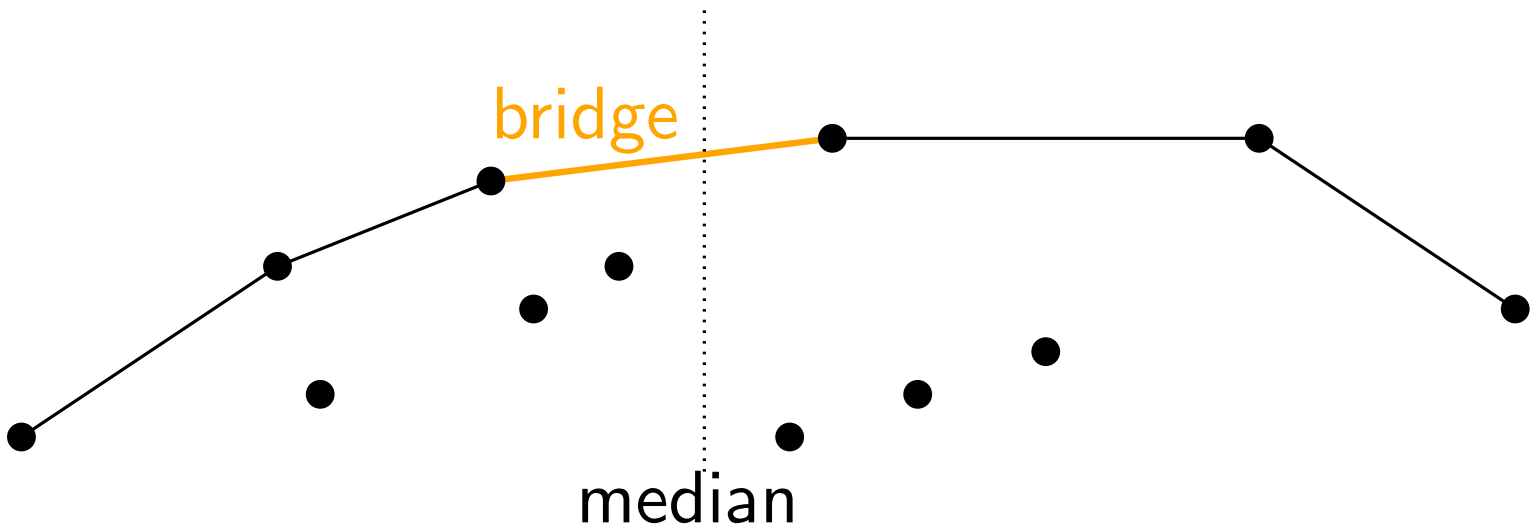

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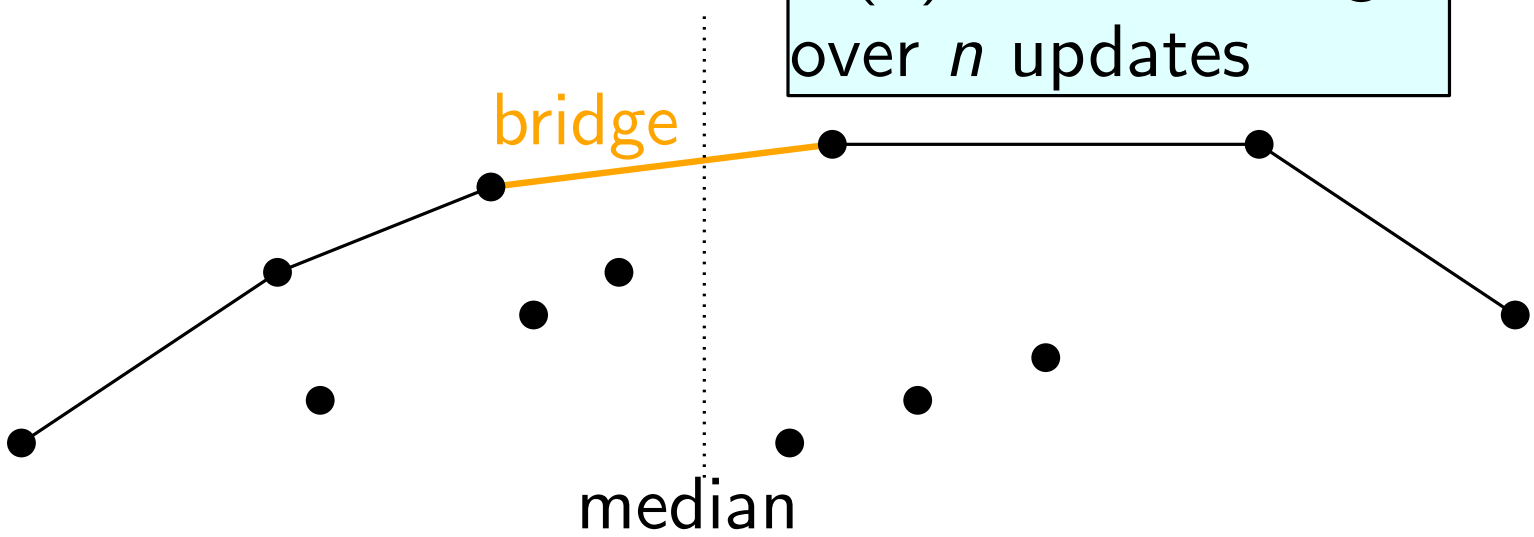

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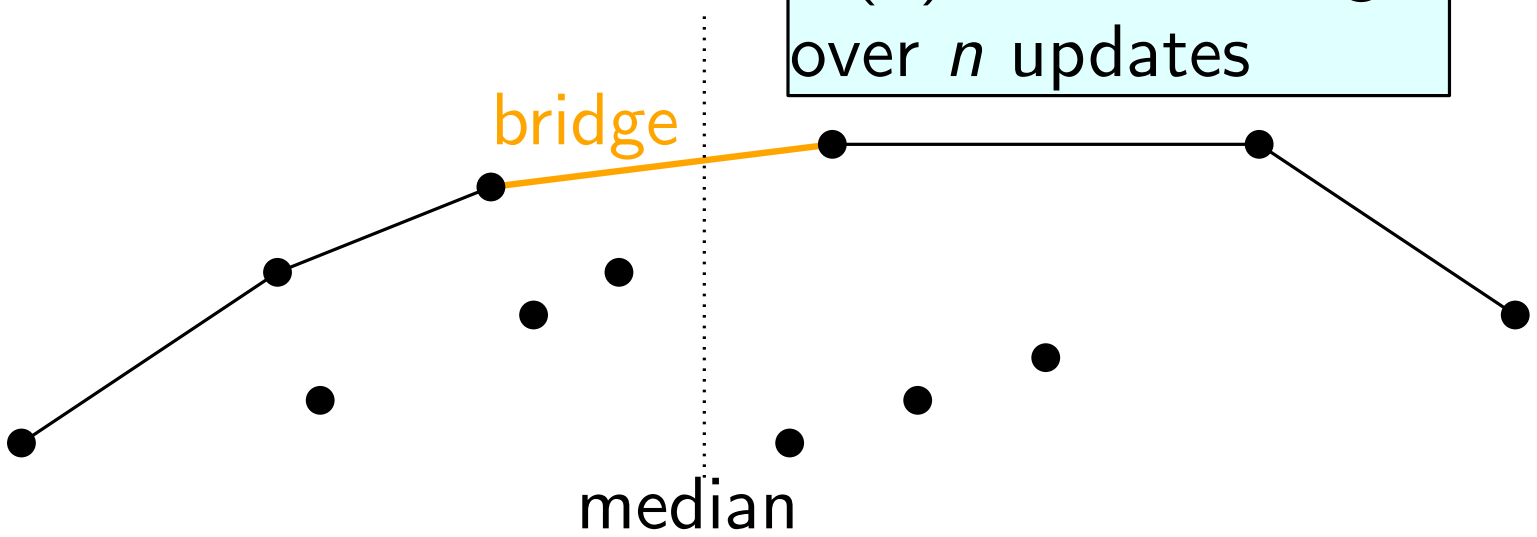

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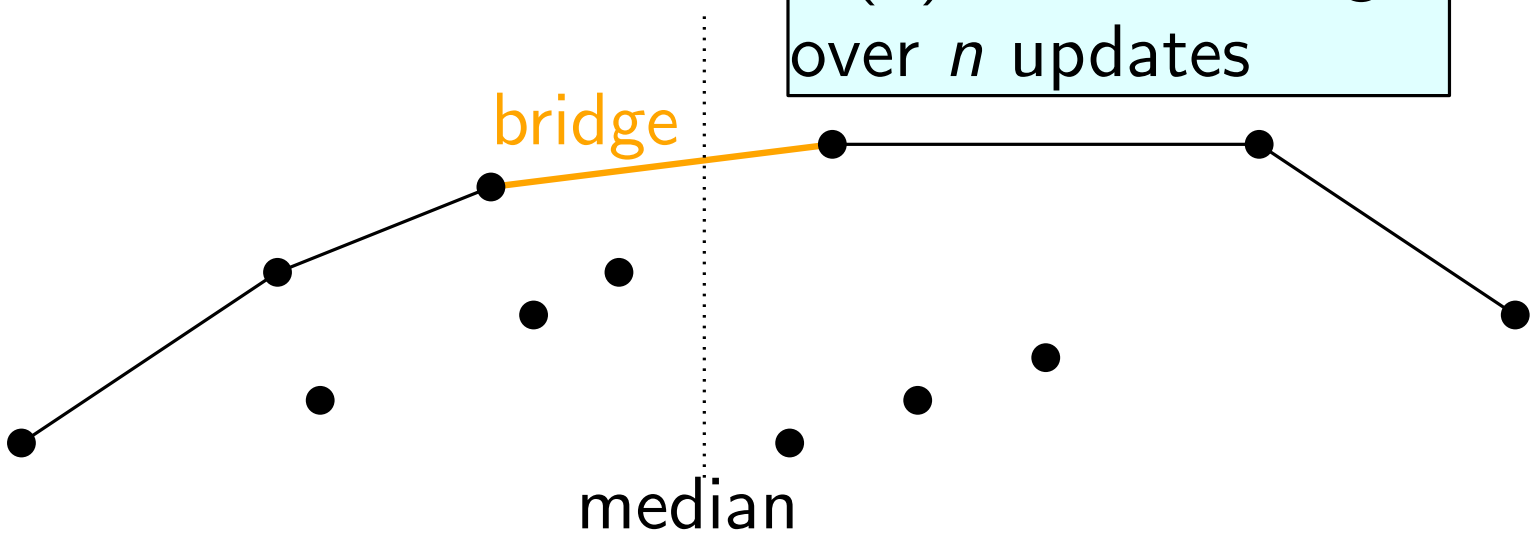

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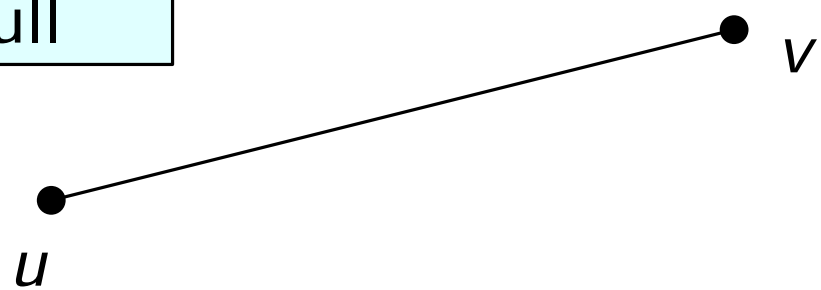

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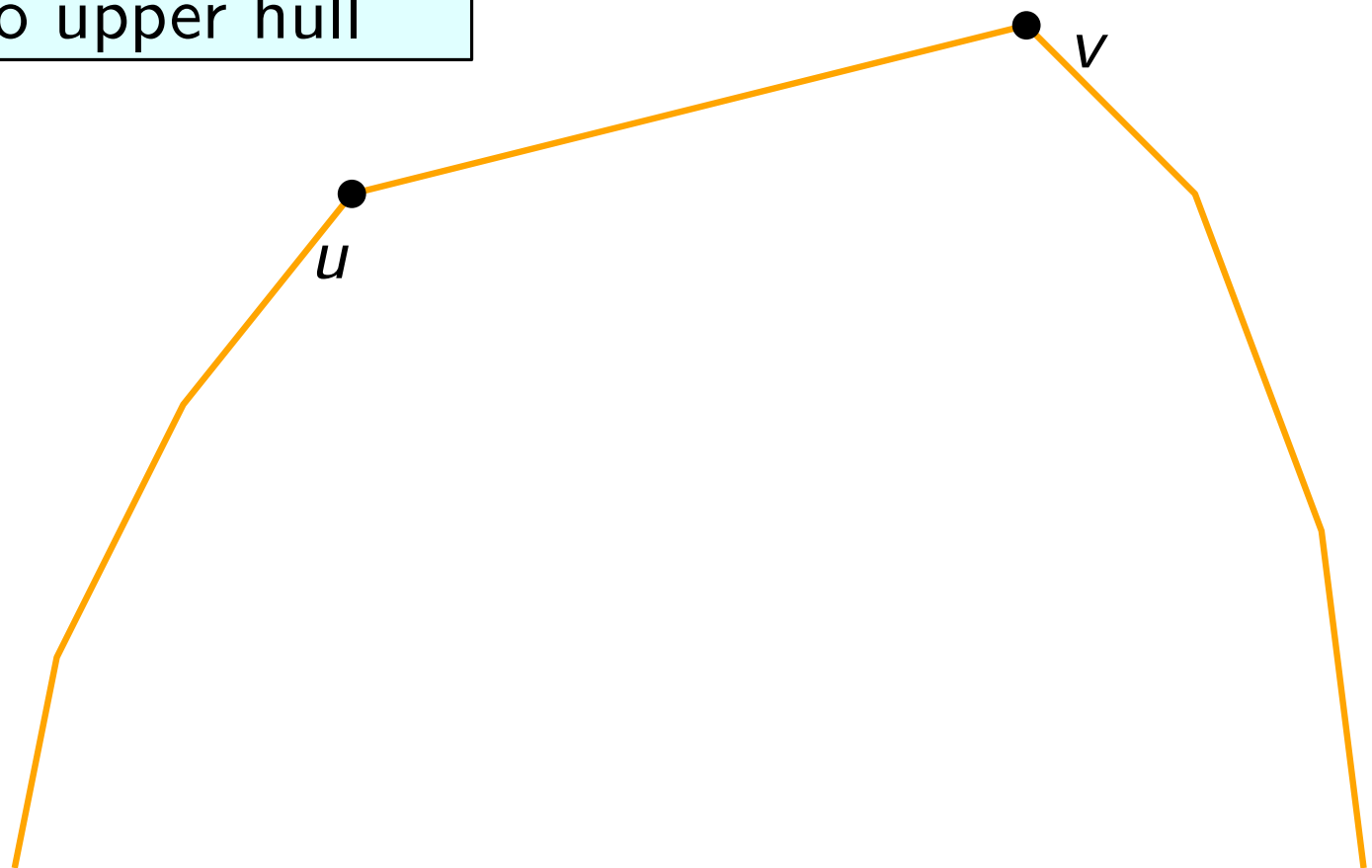

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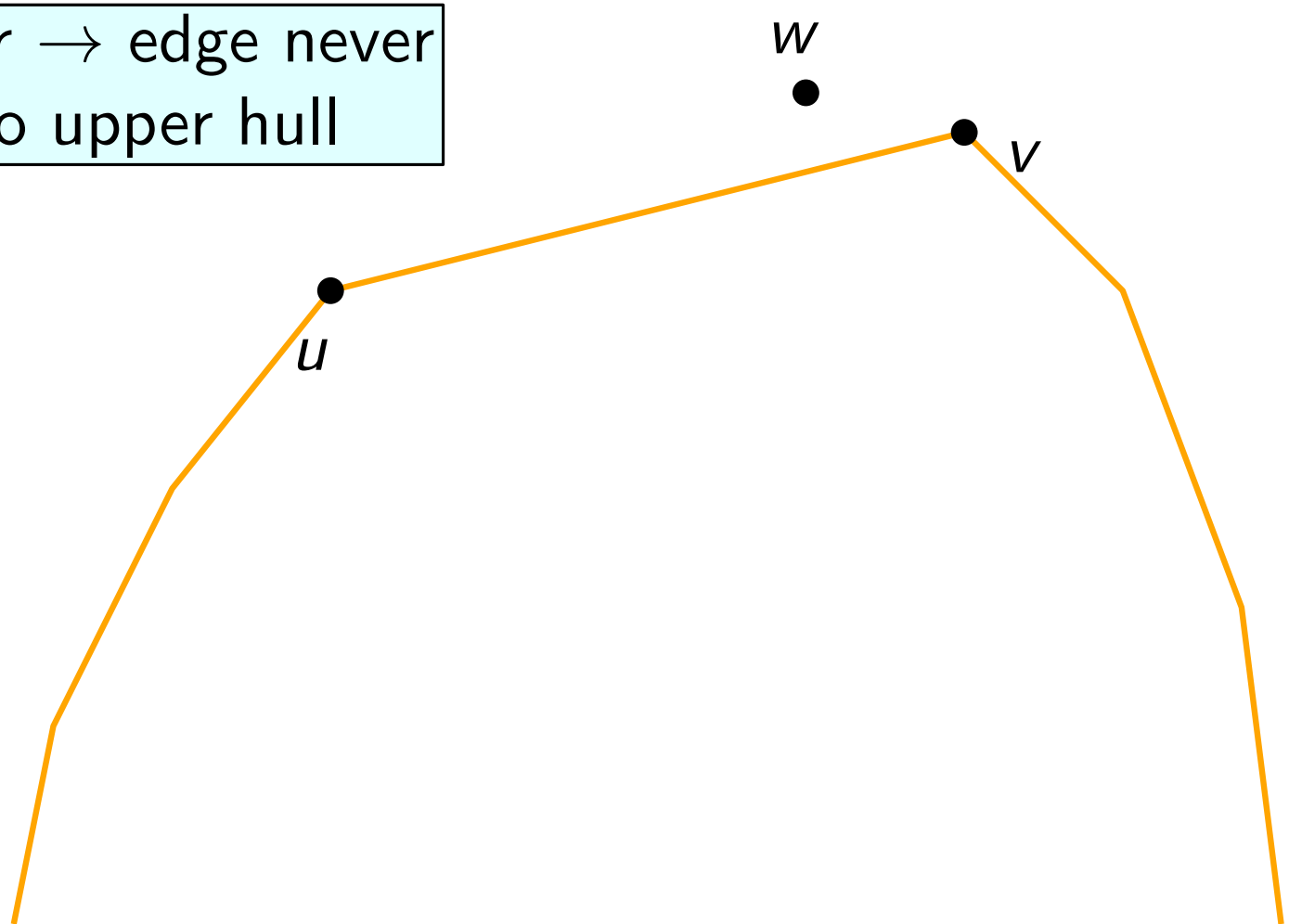

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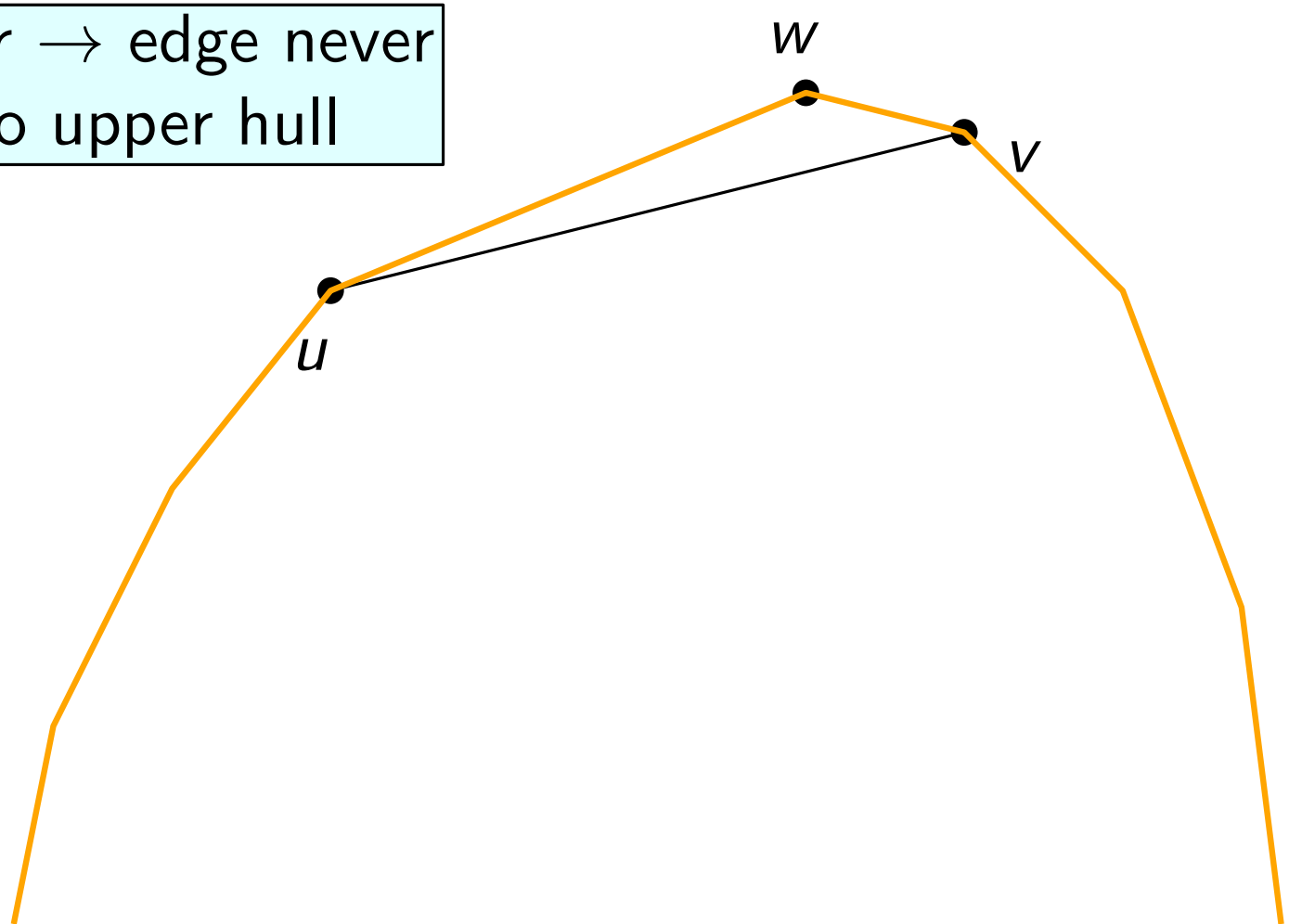

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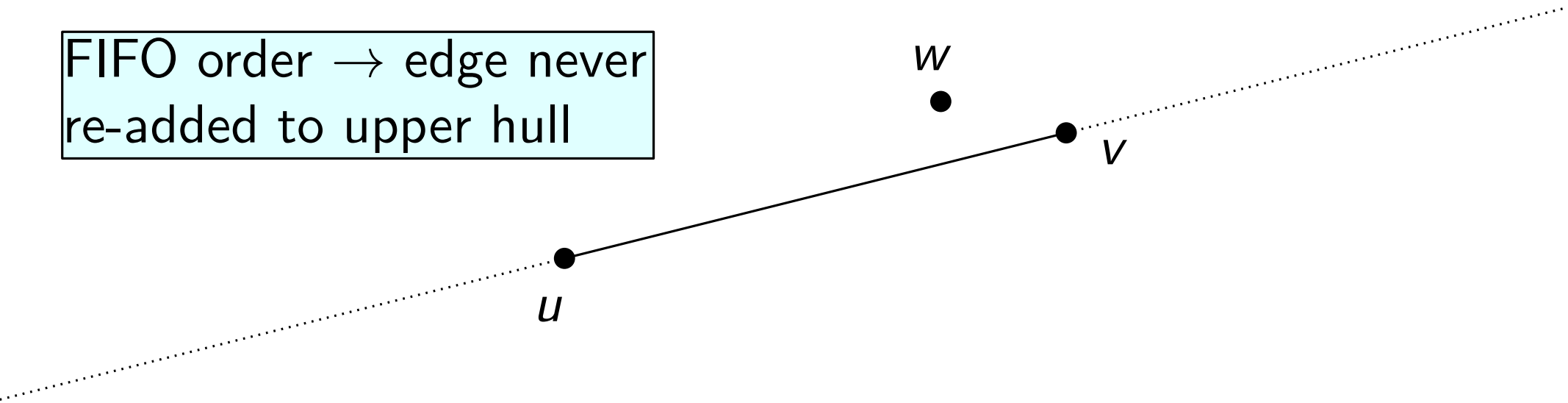

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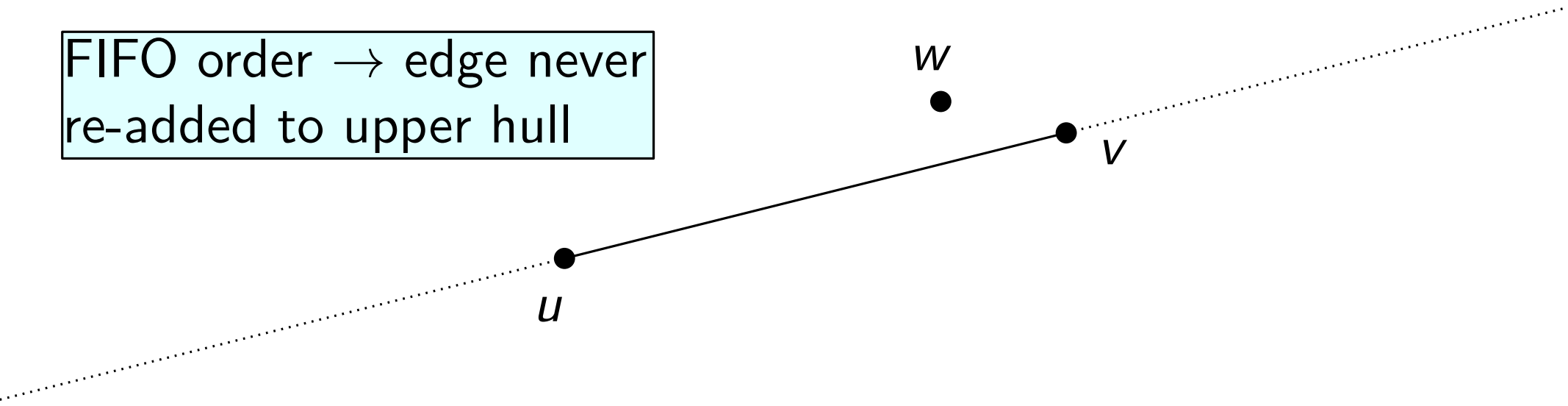

FIFO: Conclusion

Combinatorial Result

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Can we do better?

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Can we do better? Hershberger says yes!

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New Results

Approach #1: Grids & Quadtrees

Problem	Preprocessing	Space	Query
CP Decision	$O(n)$ exp.	$O(n)$ bits	$O(1)$
Closest Pair	$O(n \lg n \lg \lg n)$	$O(n \lg n)$	$O(\lg \lg n)$

Approach #2: Generalized Range Successor

Problem	Preprocessing Time
2D diameter decision	$O(n \lg^2 n)$
$\rightarrow$ improved to	$O(n \lg n)$
3D diameter decision	$O(n \lg^2 n)$
2D orth. seg. intersection detection	$O(n \lg n \lg \lg n)$

Approach #3: FIFO Update Sequence

Problem	Preprocessing Time
2D convex hull area decision	$O(n \lg n \lg \lg n)$
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Hershberger's Combinatorial Result

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- Plot each point p_i as segment $[(i(p_i), i), (d(p_i), i)]$

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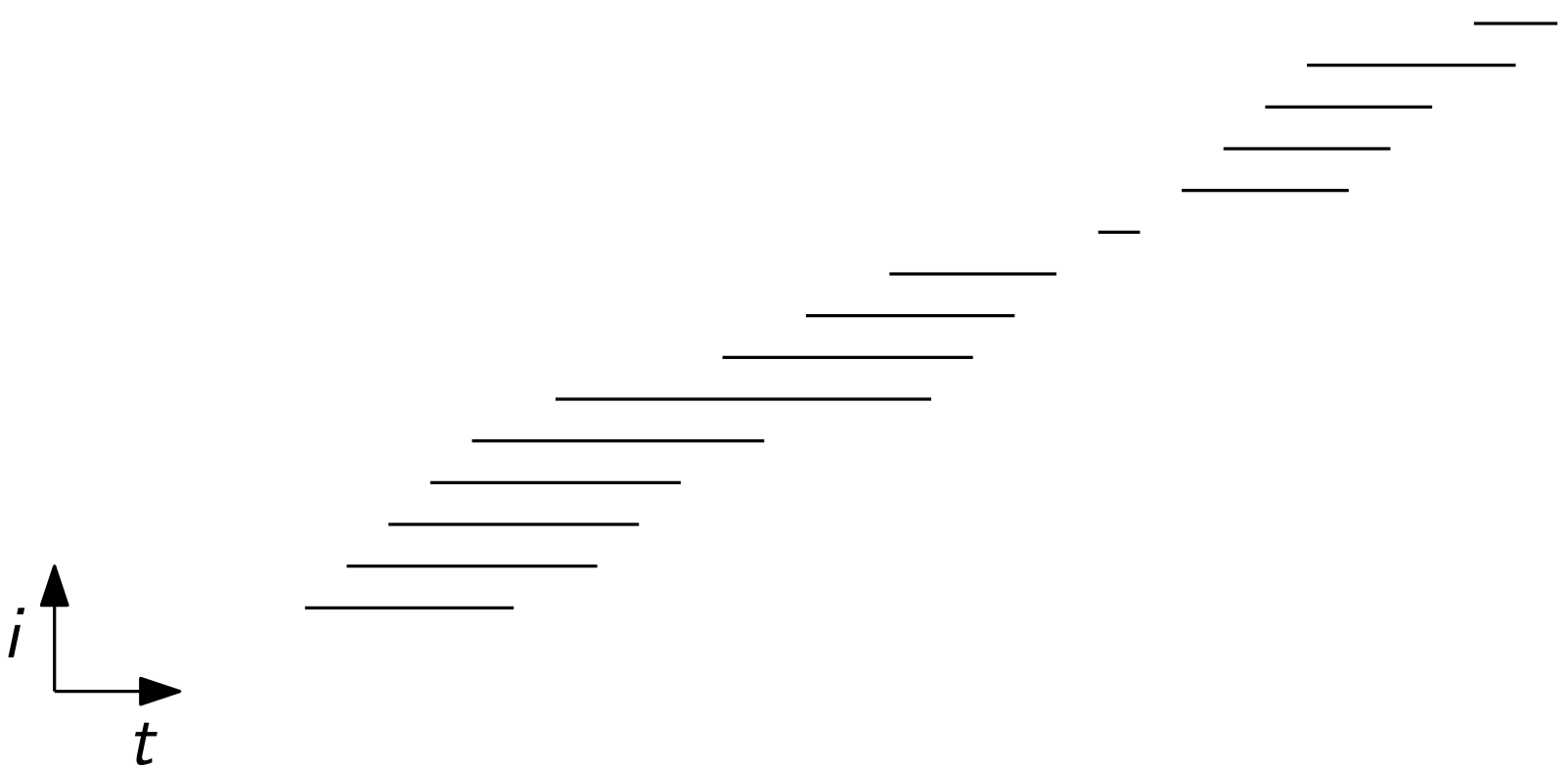

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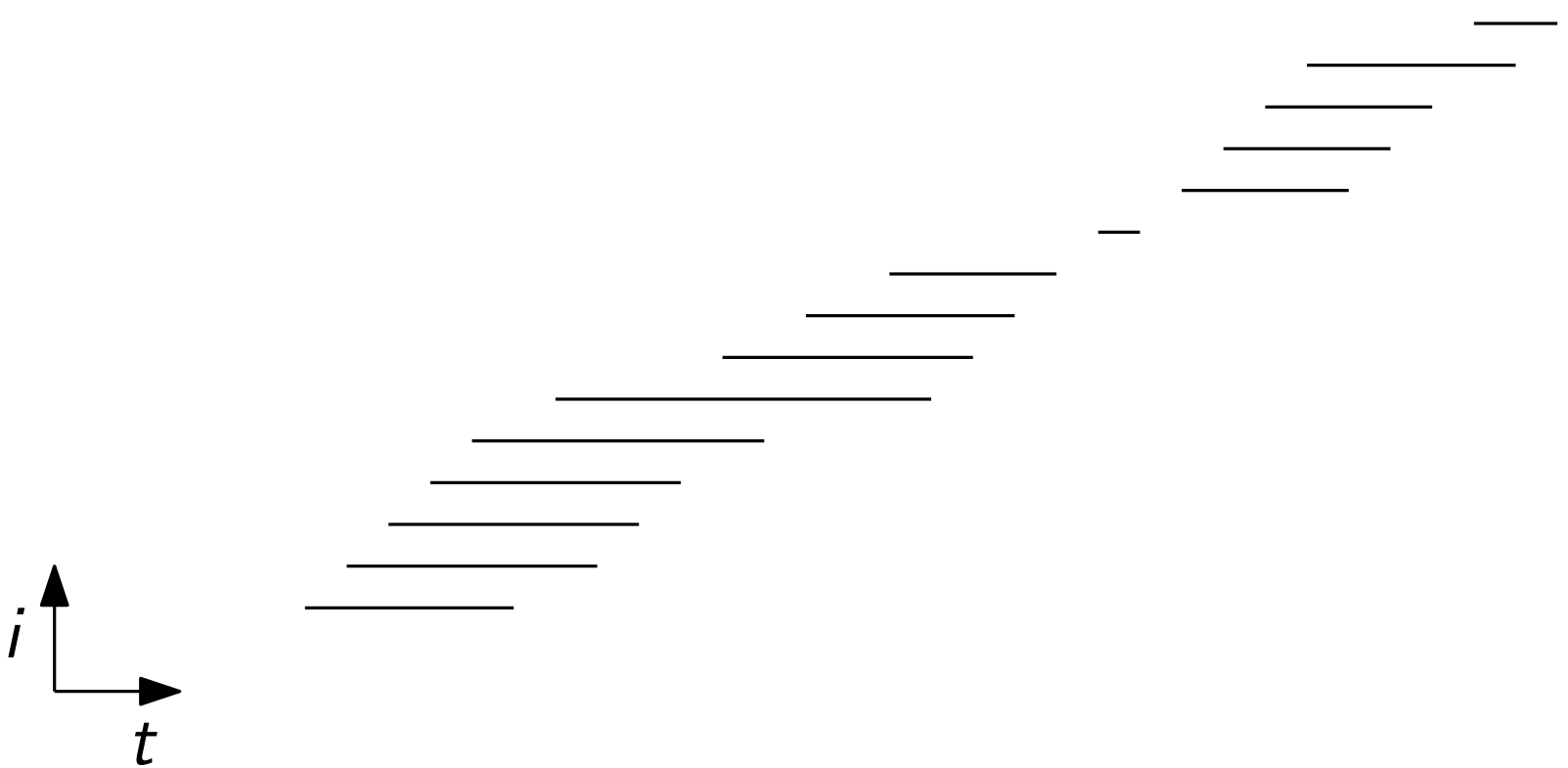

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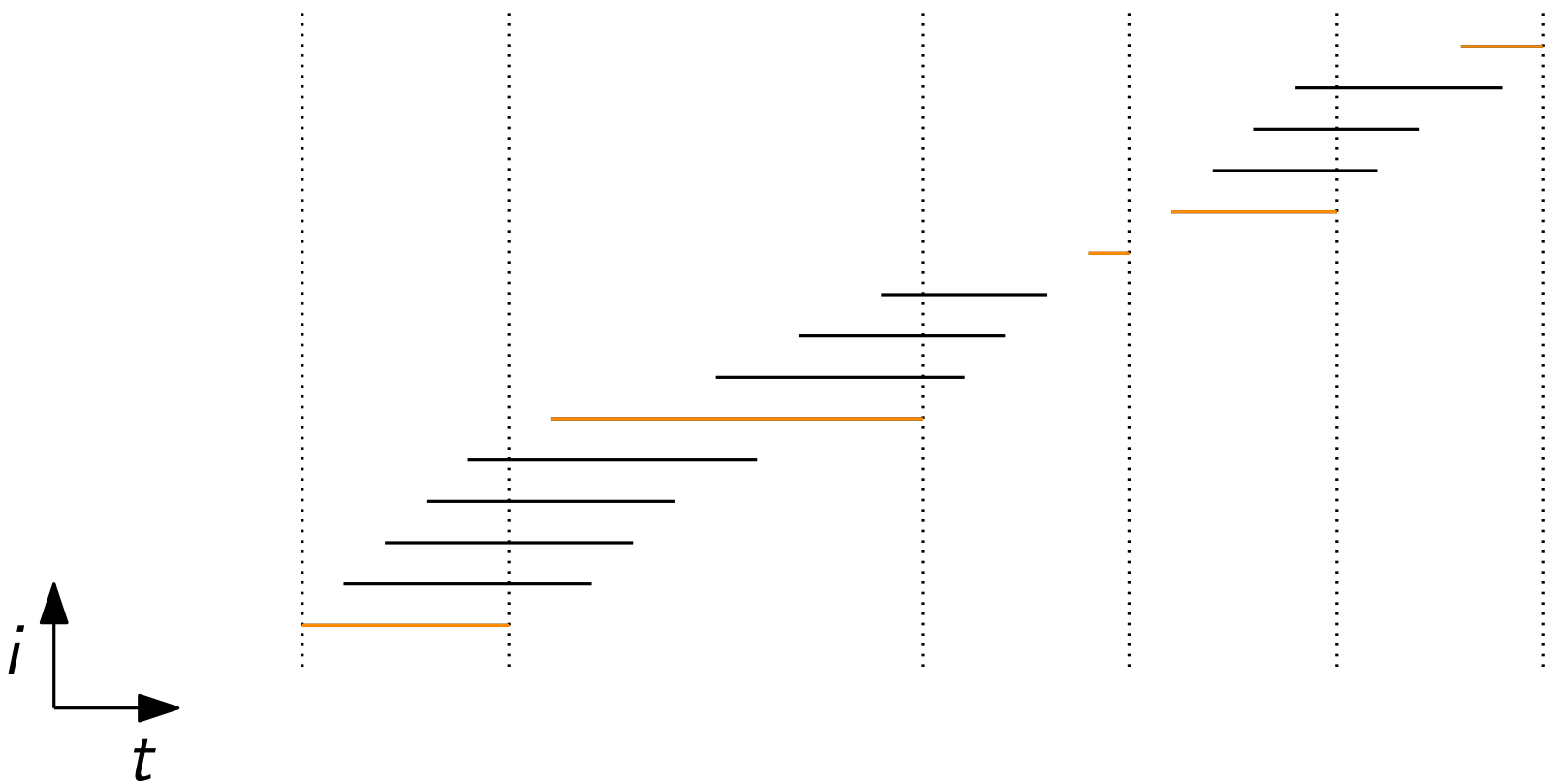

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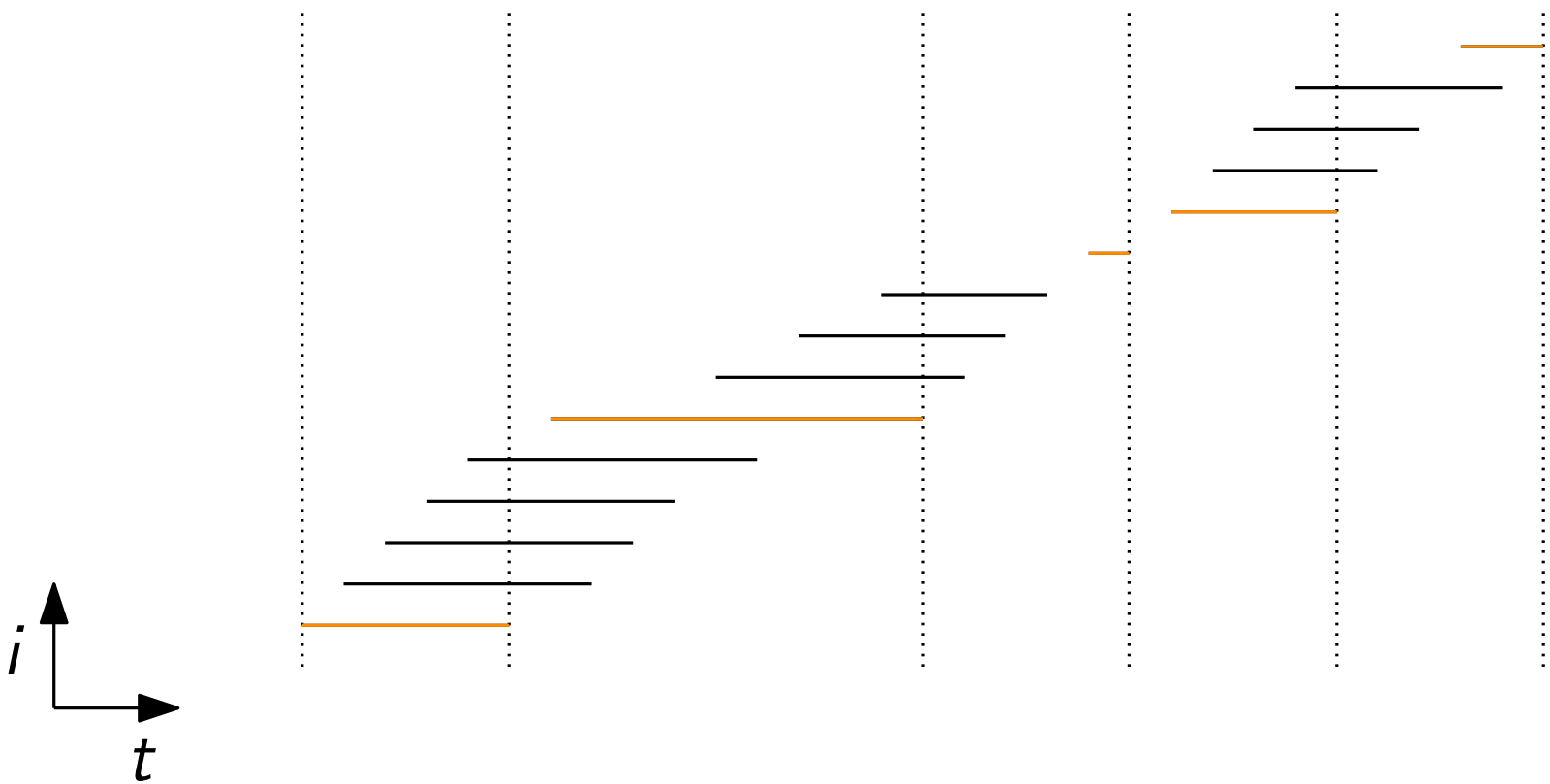

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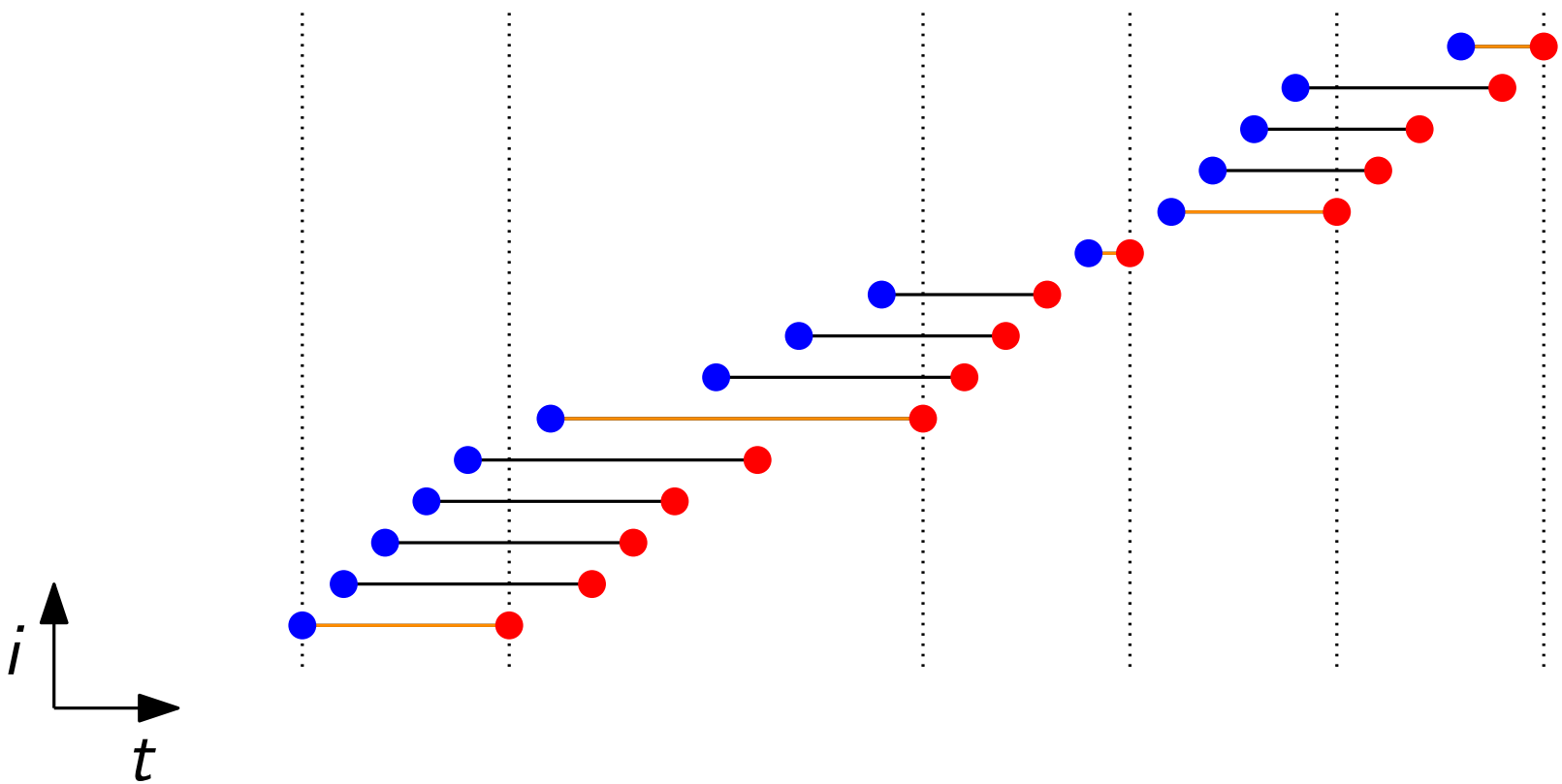

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- Plot each point p_i as segment $[(i(p_i), i), (d(p_i), i)]$
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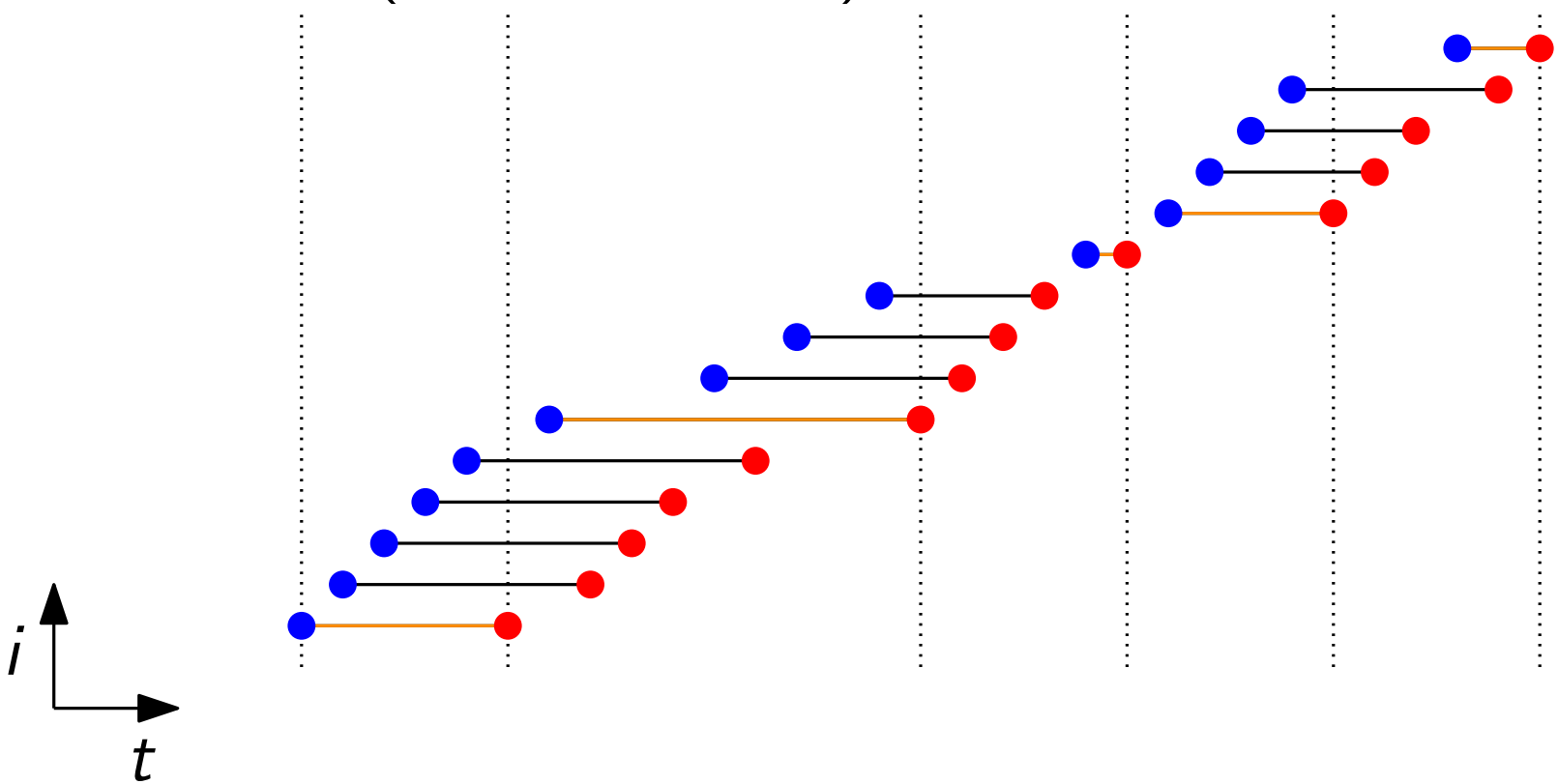
- 3 types of edges:
  - ◇ Insert-only
  - ◇ Delete-only
  - ◇ Mixed (1 ins. & 1 del.)



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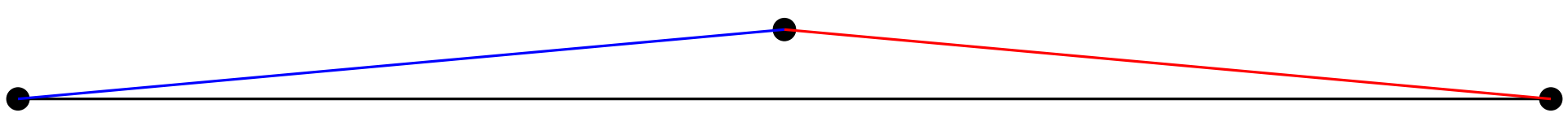

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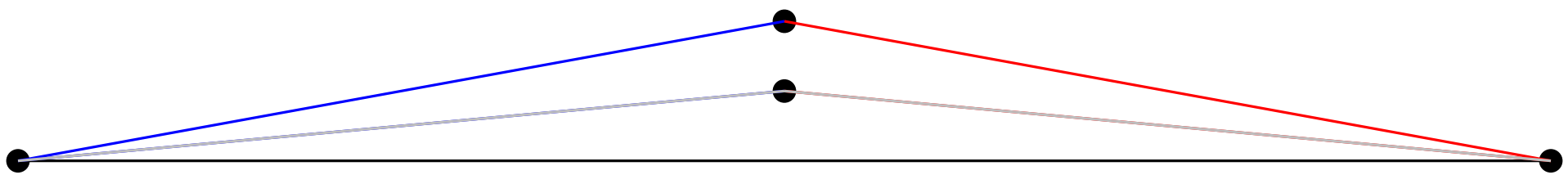

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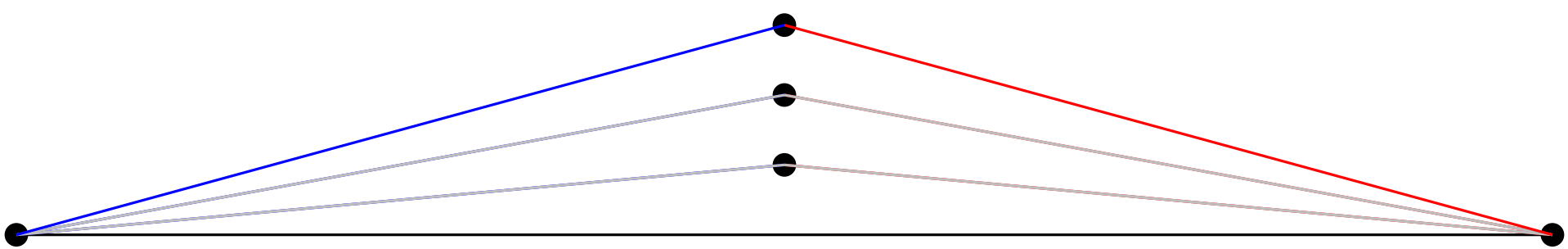

Hershberger: bounding insert-only

- Insertion-only: each point creates at most 2 edges
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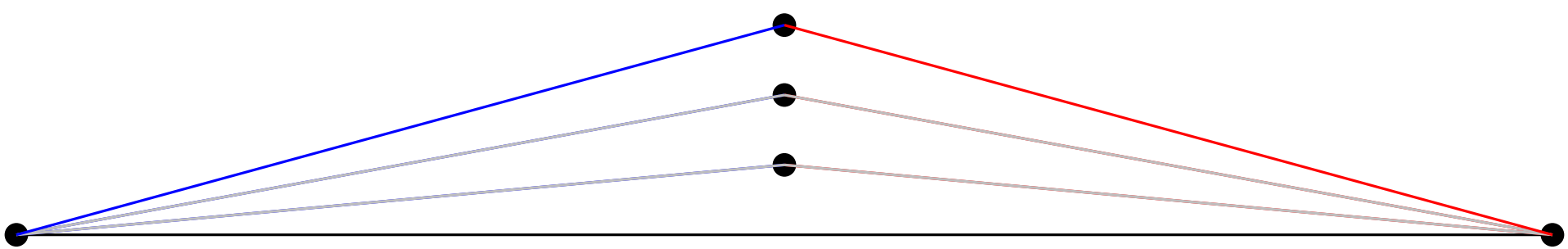
- Deletion-only: same in reverse



Hershberger: bounding insert-only

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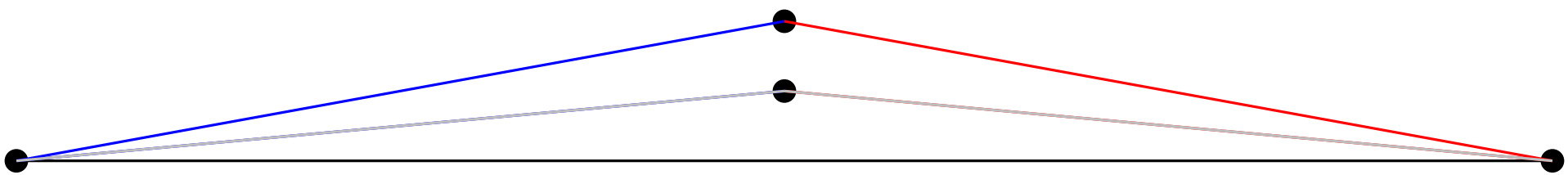

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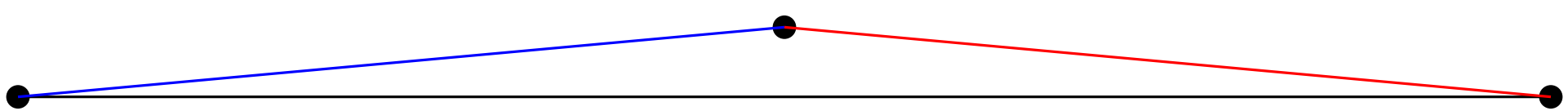

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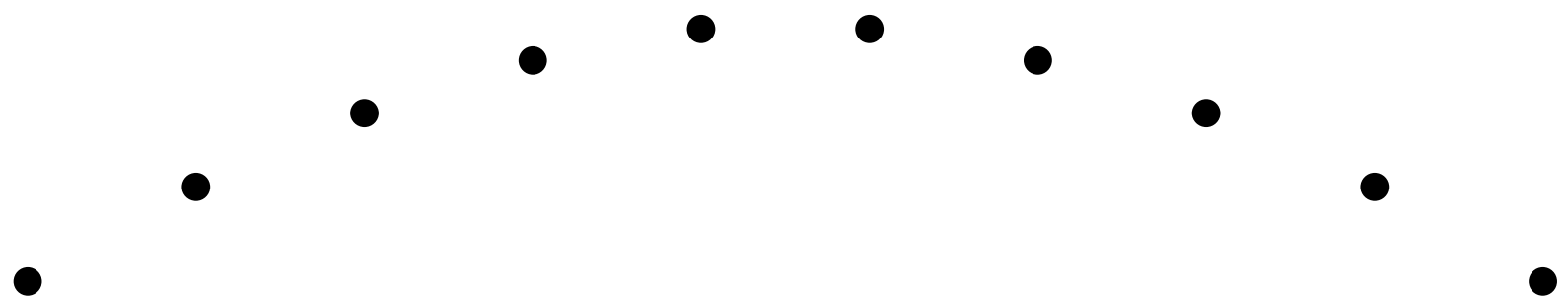

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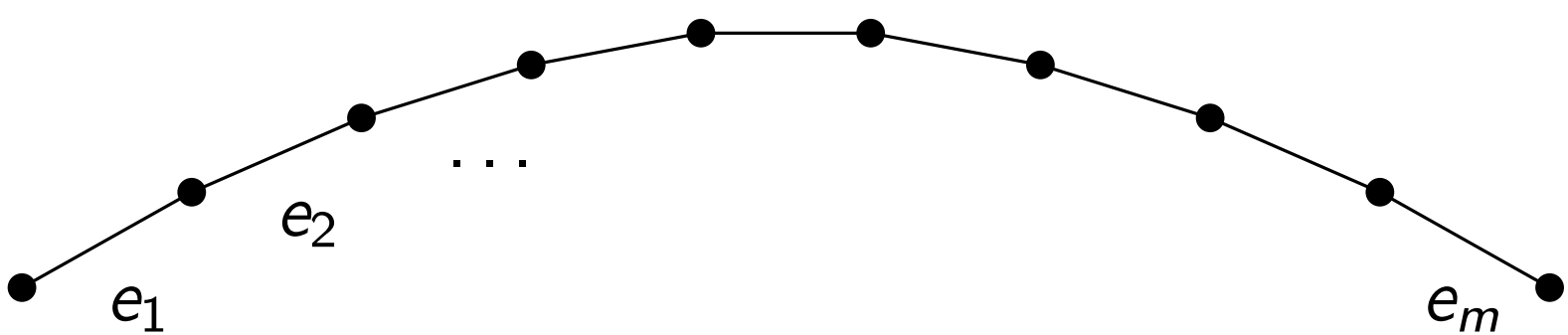

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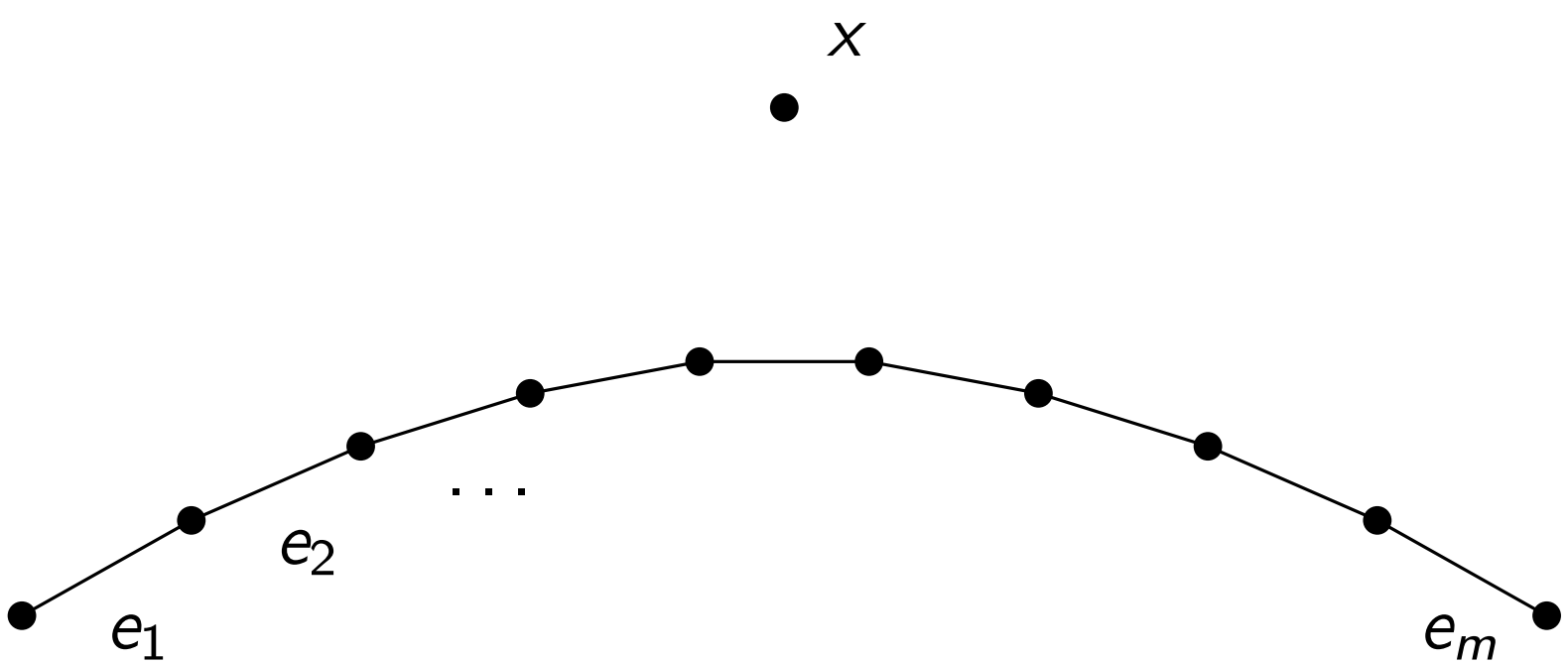

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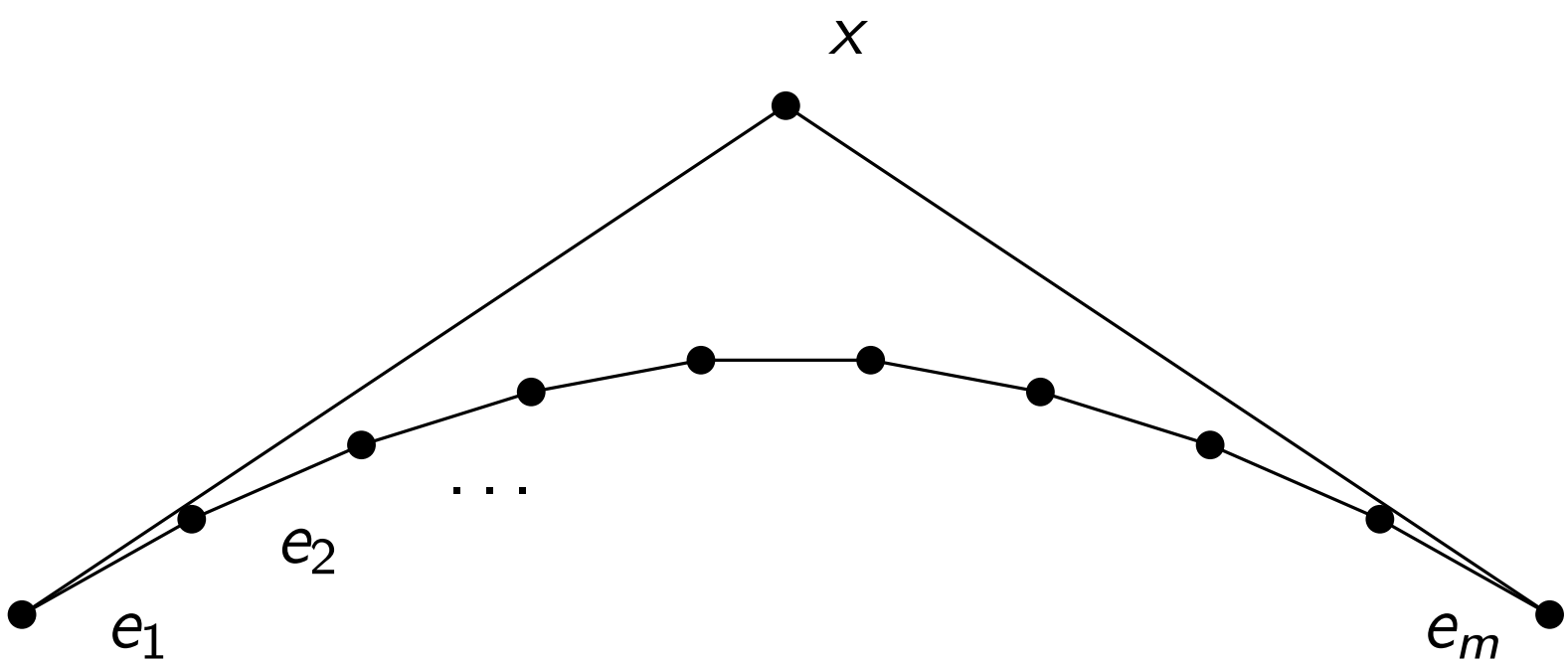

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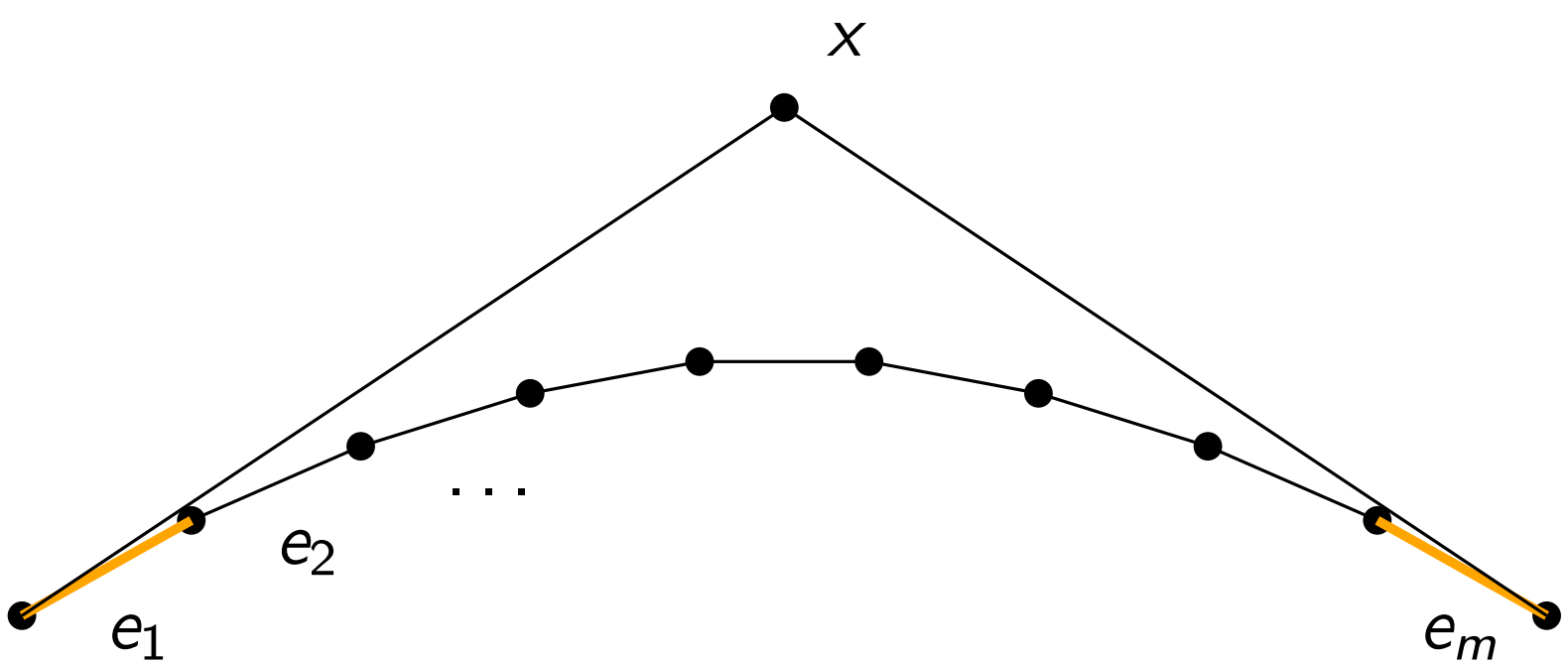

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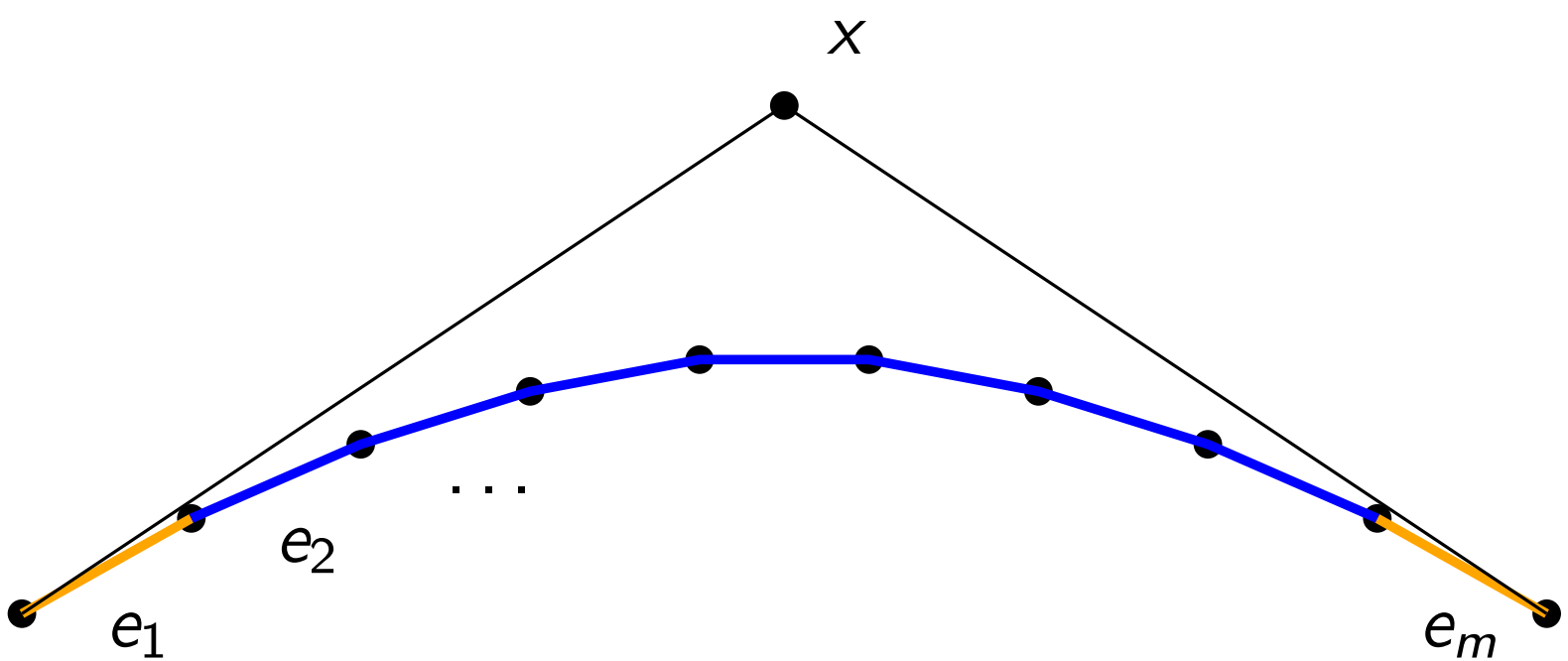

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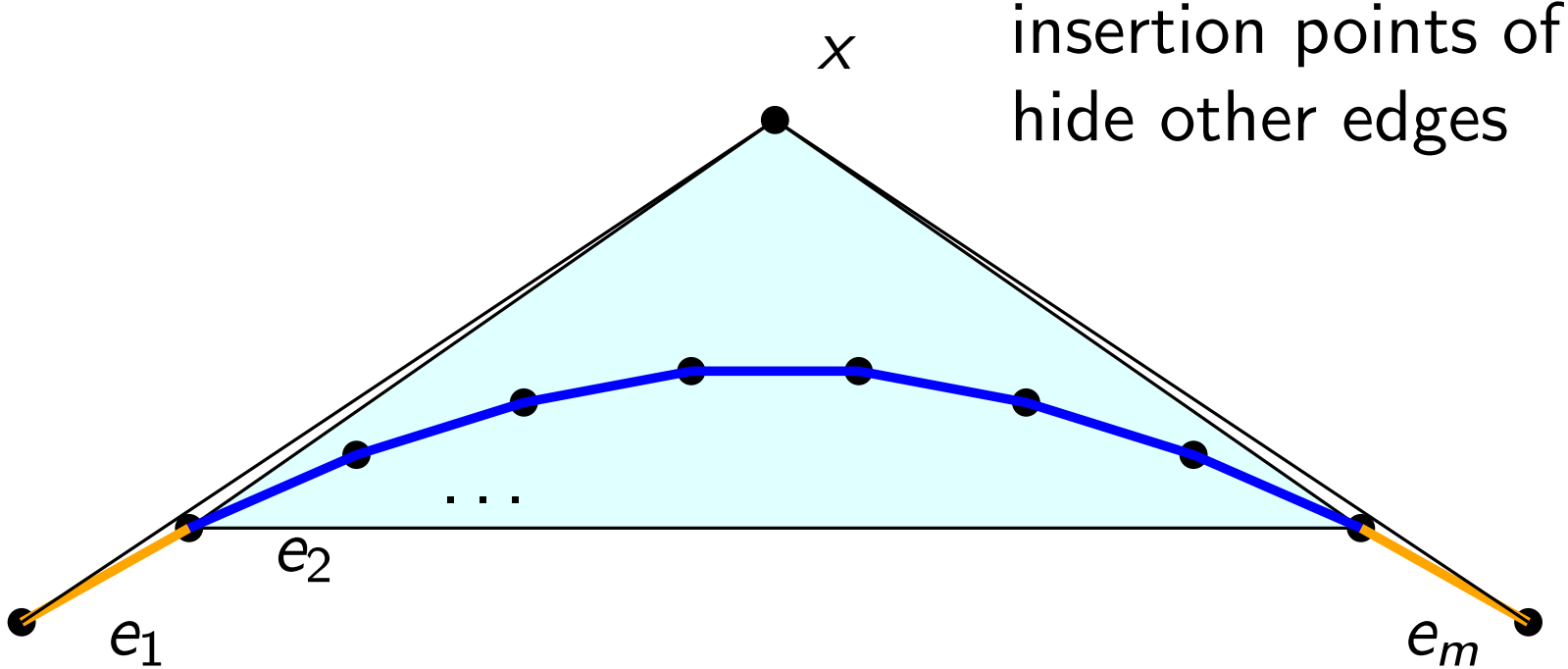

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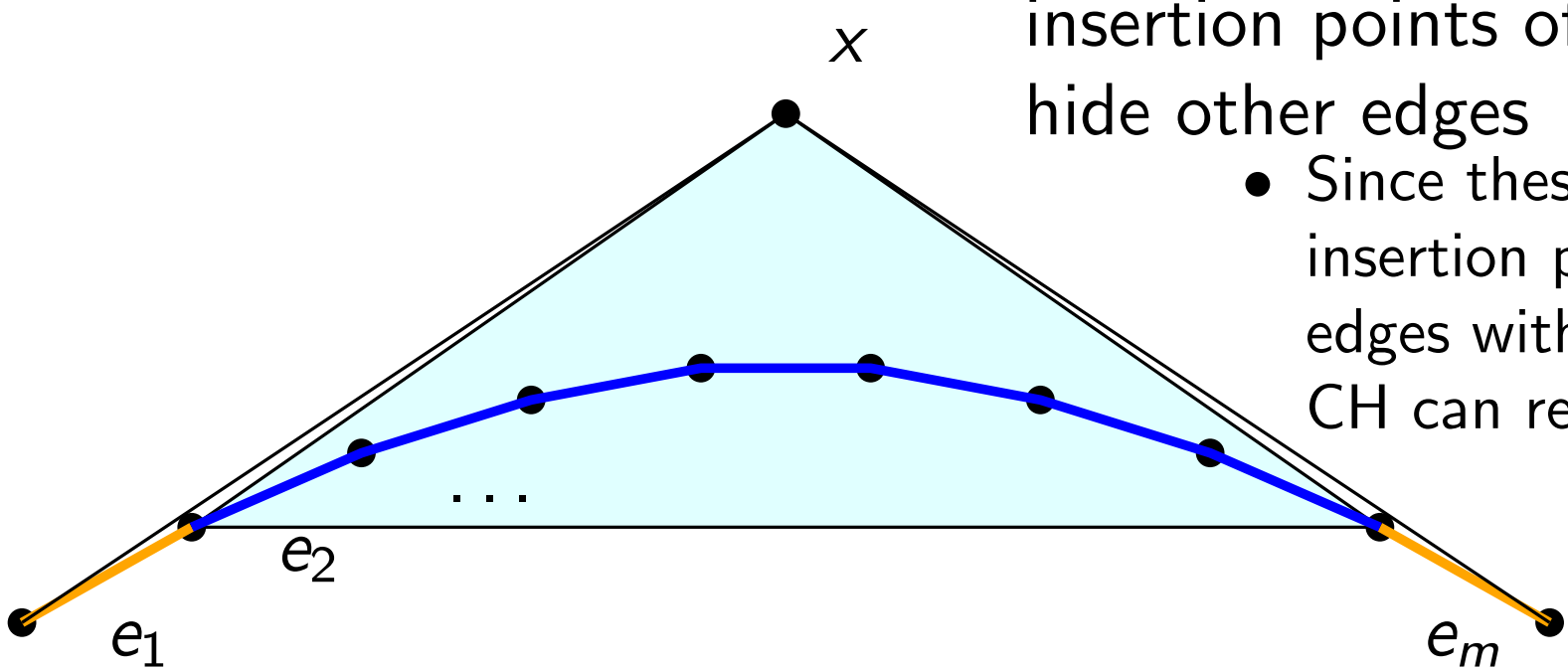
  - Since these are insertion points, no edges within their CH can re-appear.



New Results

Approach #1: Grids & Quadtrees

Problem	Preprocessing	Space	Query
CP Decision	$O(n)$ exp.	$O(n)$ bits	$O(1)$
Closest Pair	$O(n \lg n \lg \lg n)$	$O(n \lg n)$	$O(\lg \lg n)$

Approach #2: Generalized Range Successor

Problem	Preprocessing Time
2D diameter decision	$O(n \lg^2 n)$
$\rightarrow$ improved to	$O(n \lg n)$
3D diameter decision	$O(n \lg^2 n)$
2D orth. seg. intersection detection	$O(n \lg n \lg \lg n)$

Approach #3: FIFO Update Sequence

Problem	Preprocessing Time
2D convex hull area decision	$O(n \lg n \lg \lg n)$
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2D width decision	$O(n \lg^7 n)$

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