Two Approaches to Building Time-Windowed Geometric Data Structures

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Problem Statement

Preprocess a set of n geometric objects each associated with a time value, s.t.:

given a query window $[t_1, t_2]$, determine whether the subset of objects whose time values are within that window have a given property.

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e.g. diameter > 1

Motivation

- *Geographic Information Systems* (GIS) data may consist not only of latitude, longitude, and altitude coordinates but also time
- Timestamped data (for example, social network data)

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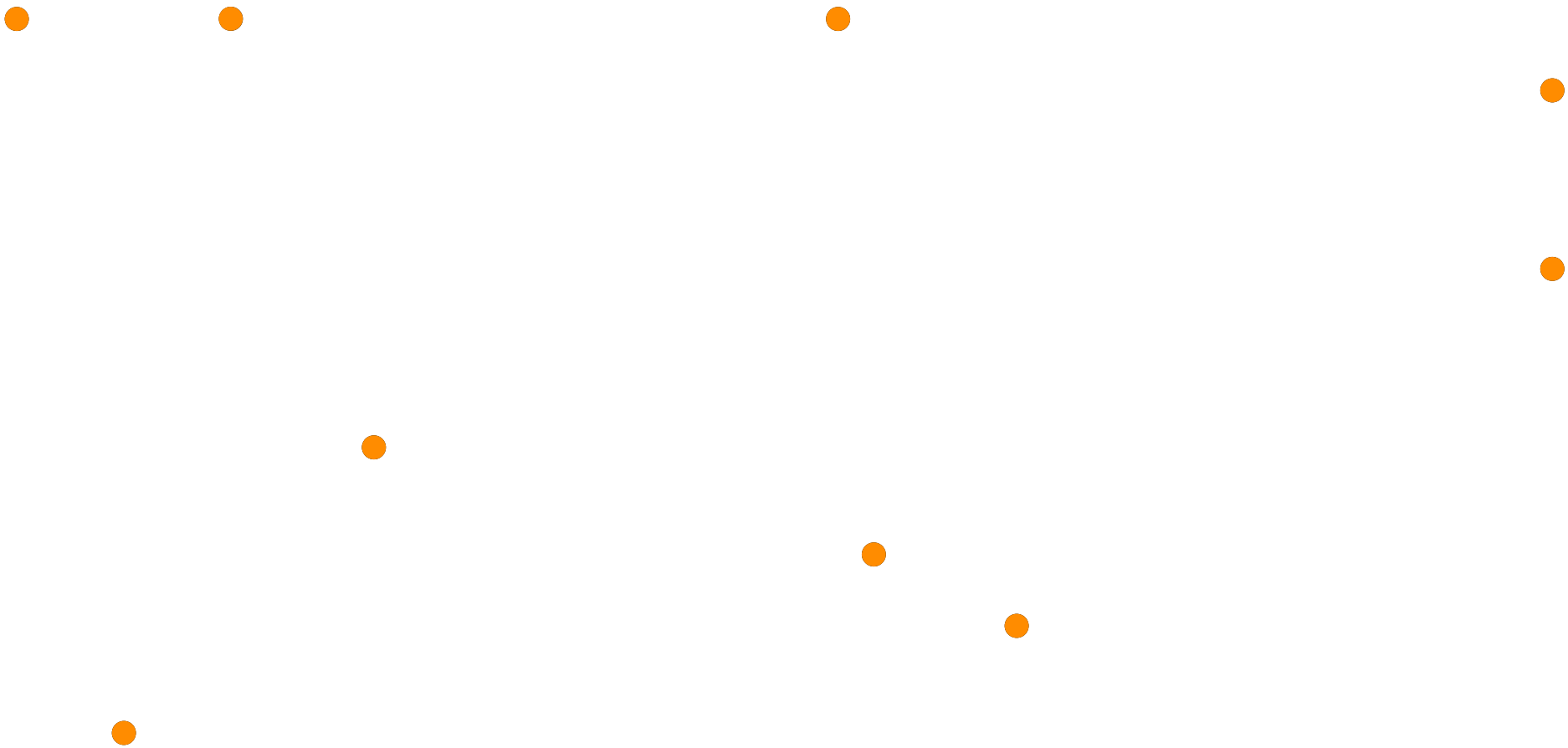

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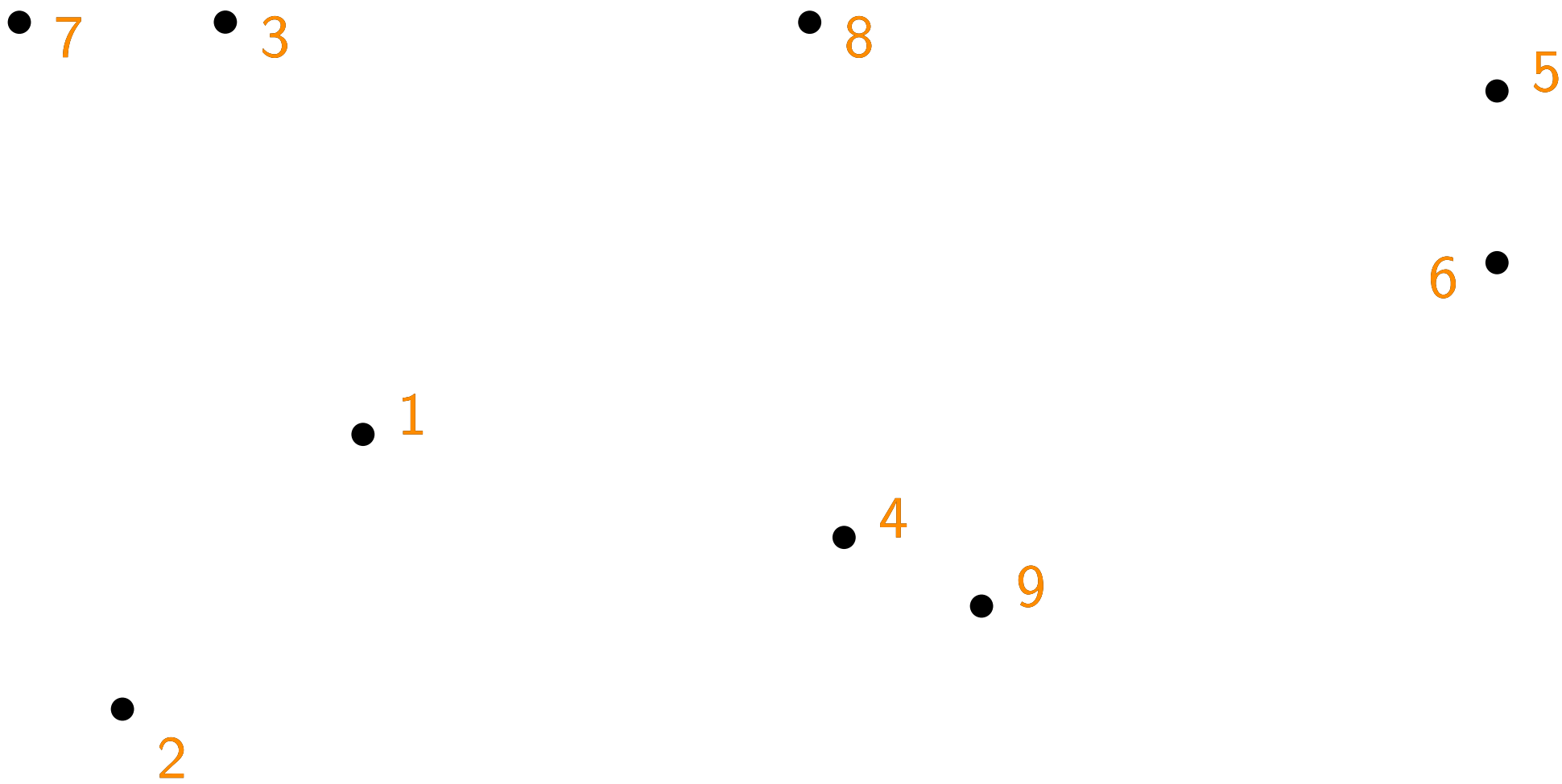

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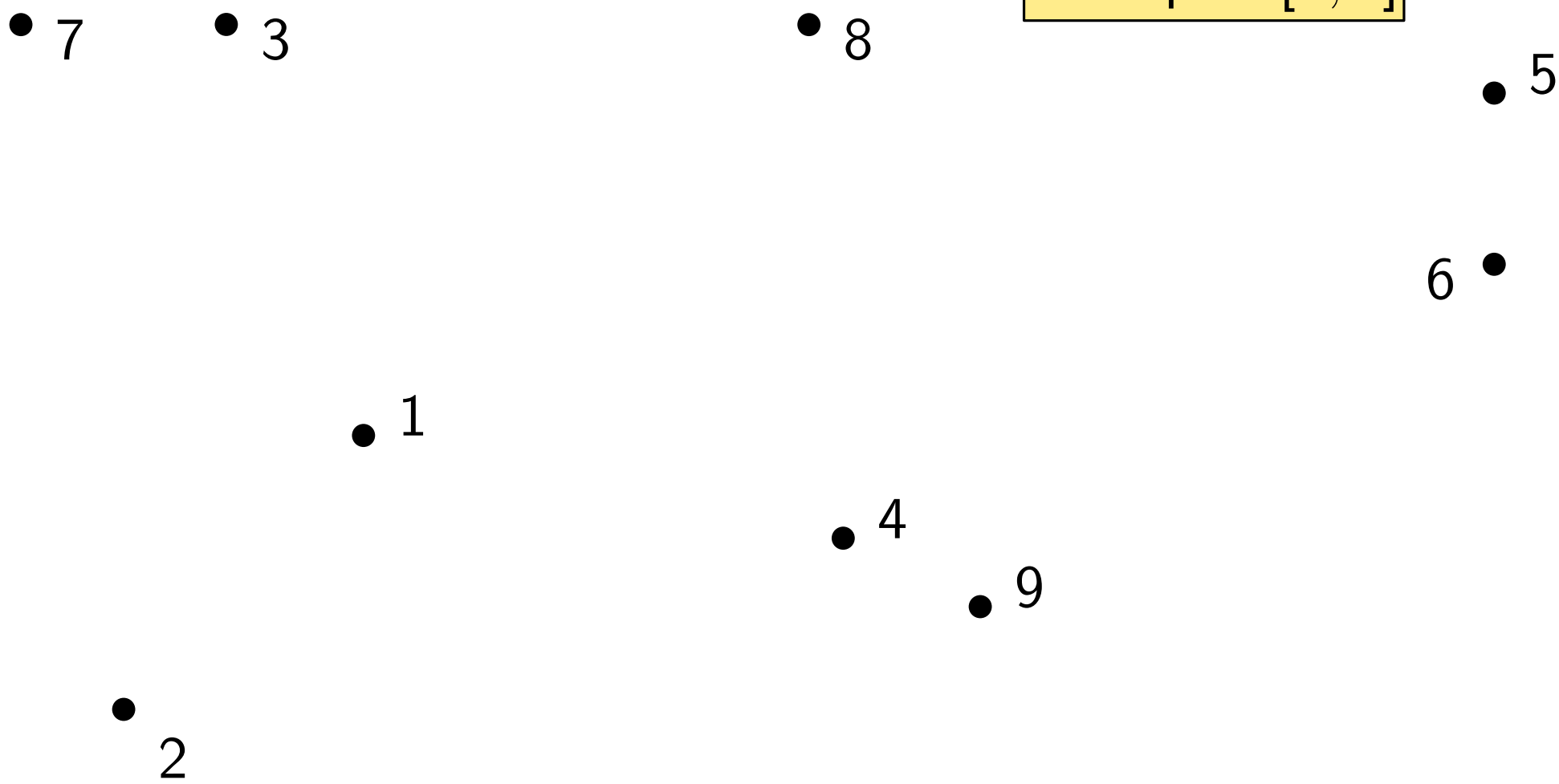

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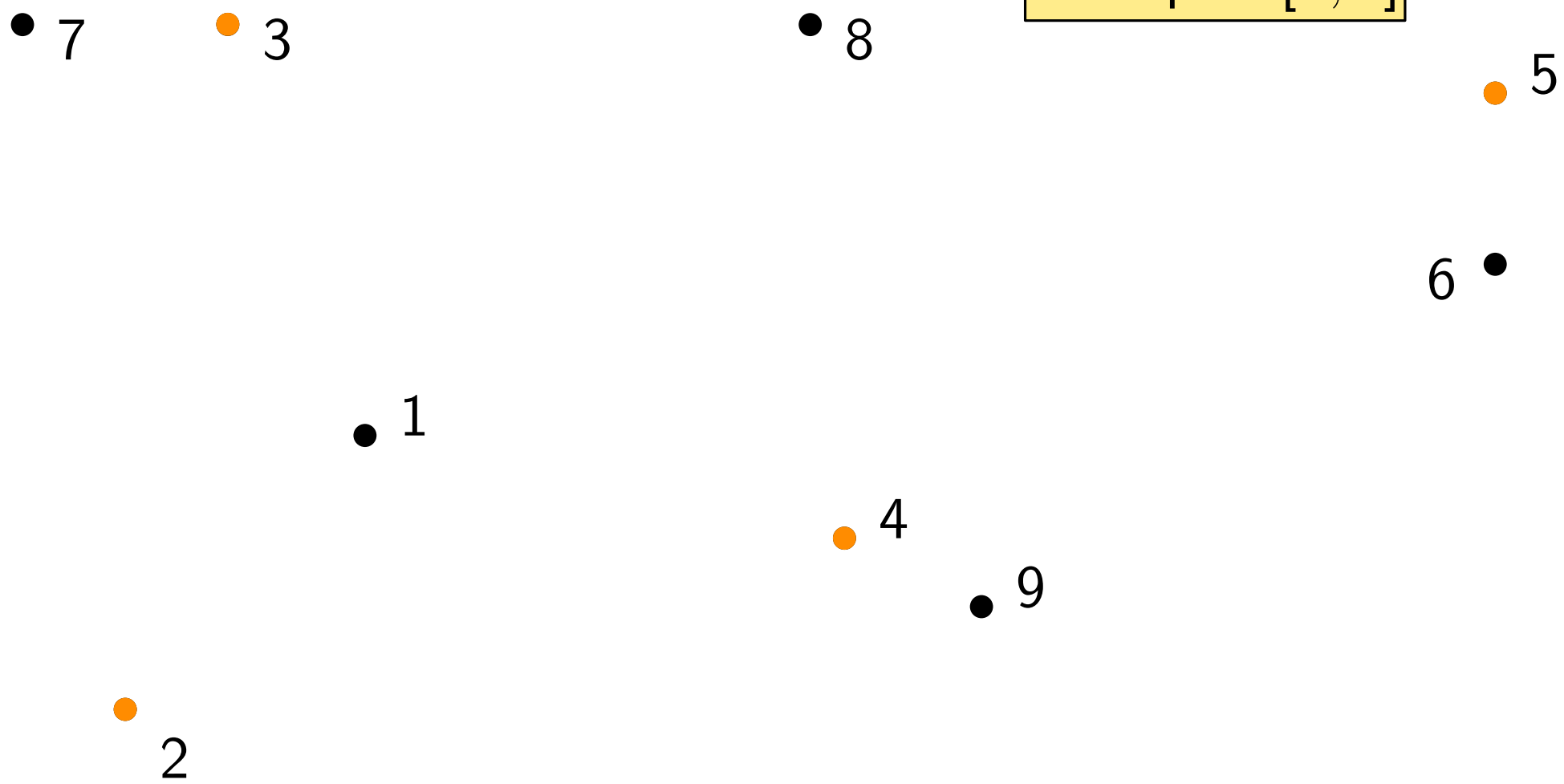

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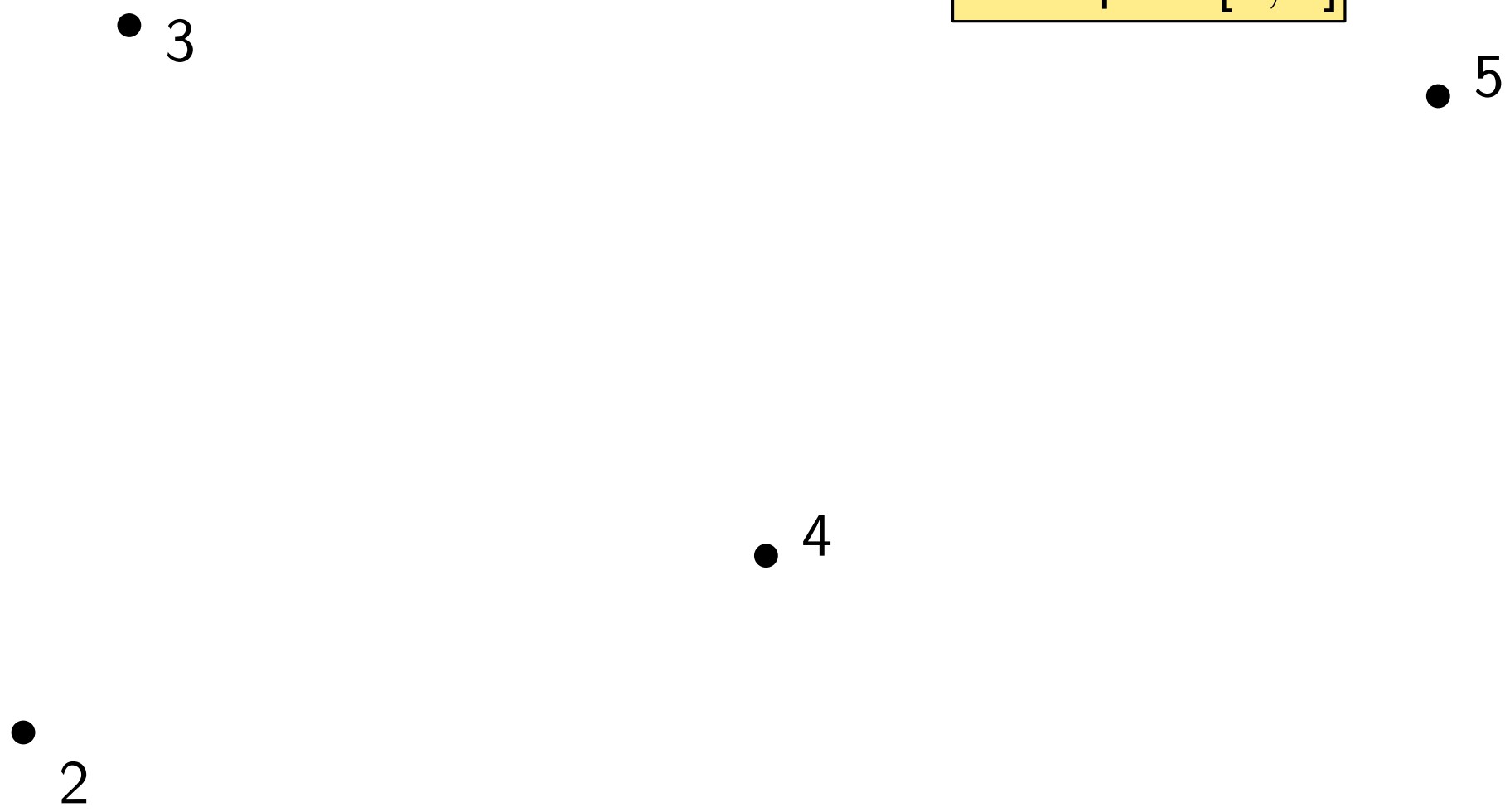

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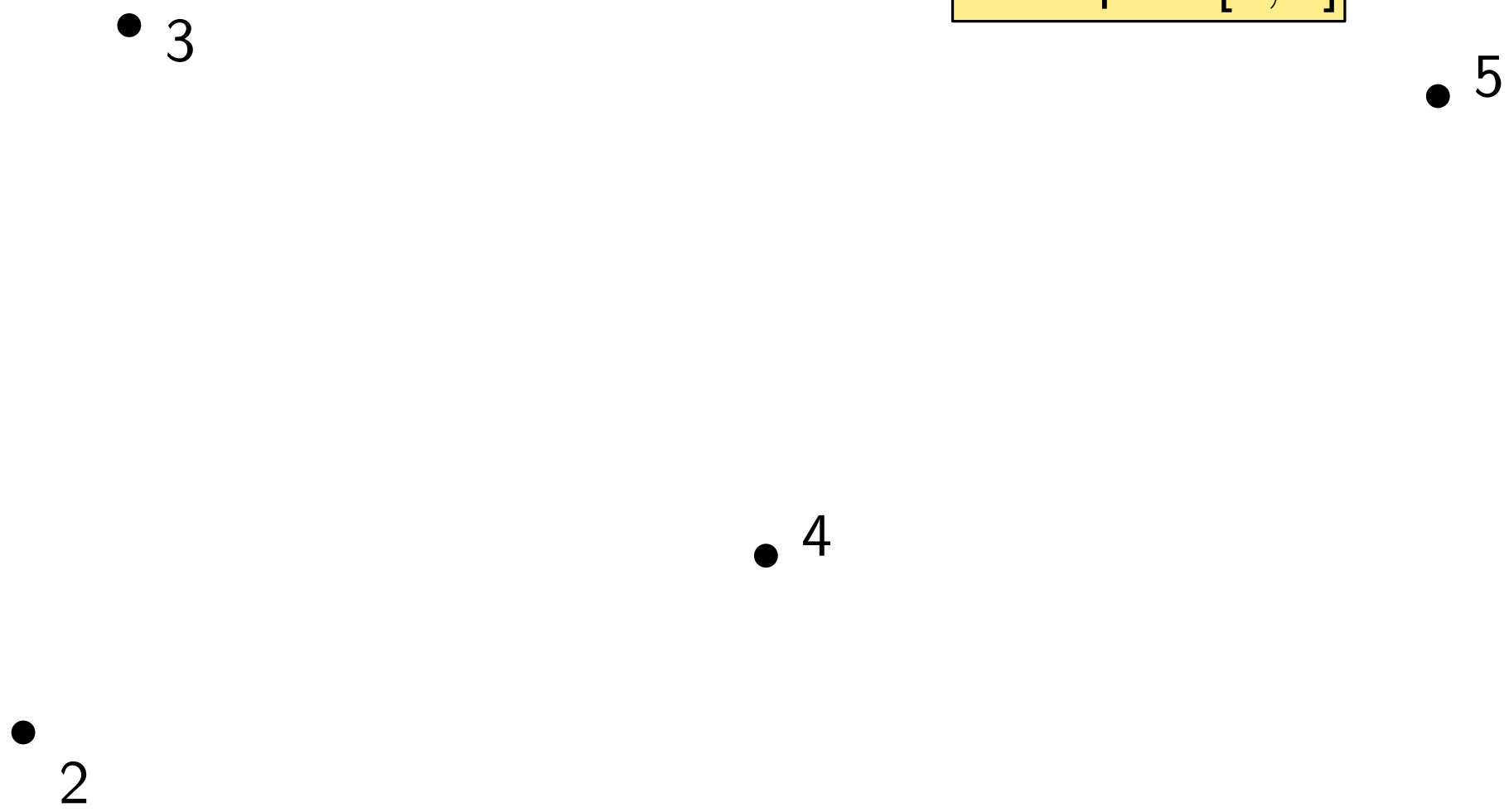

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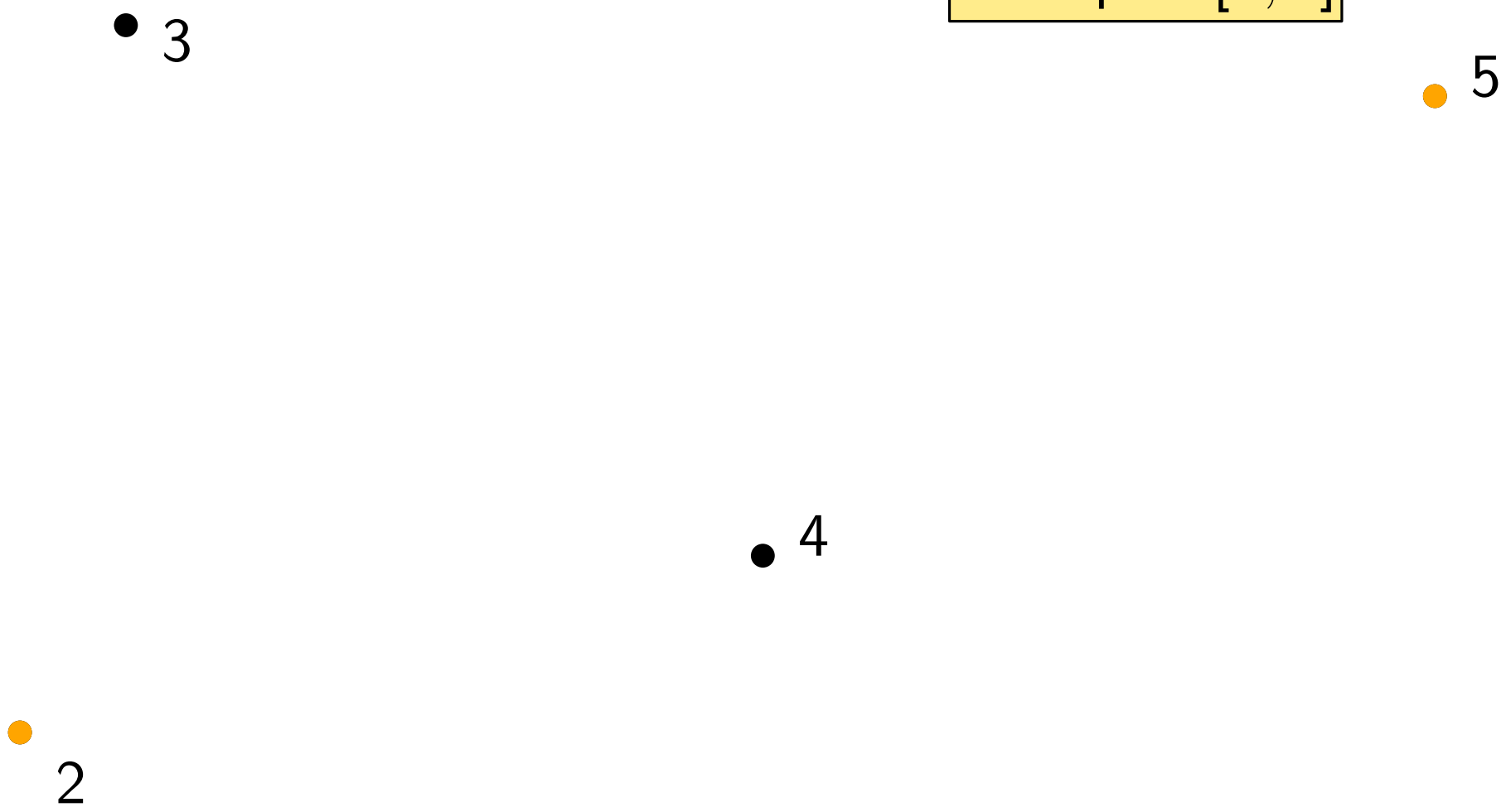

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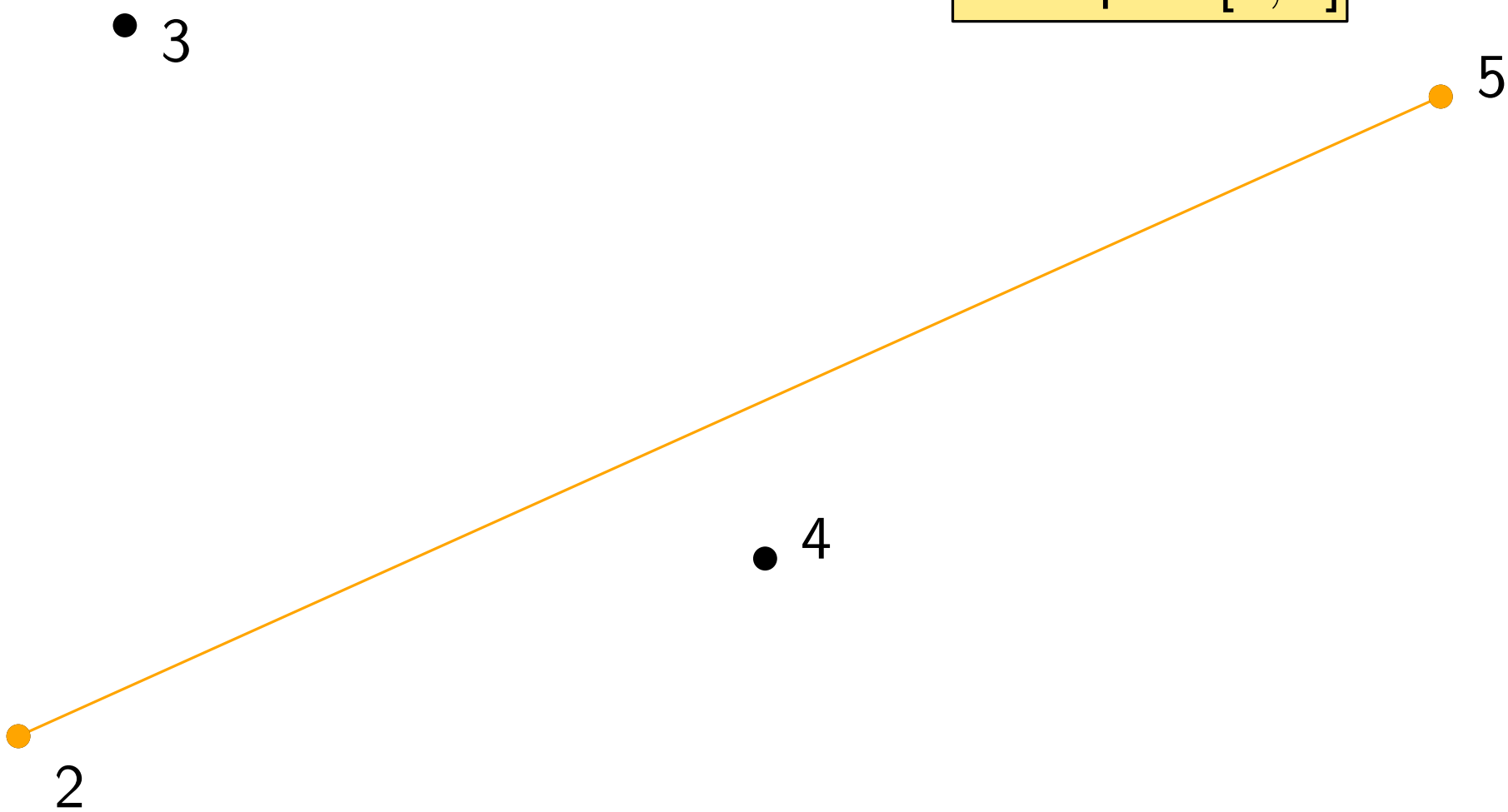

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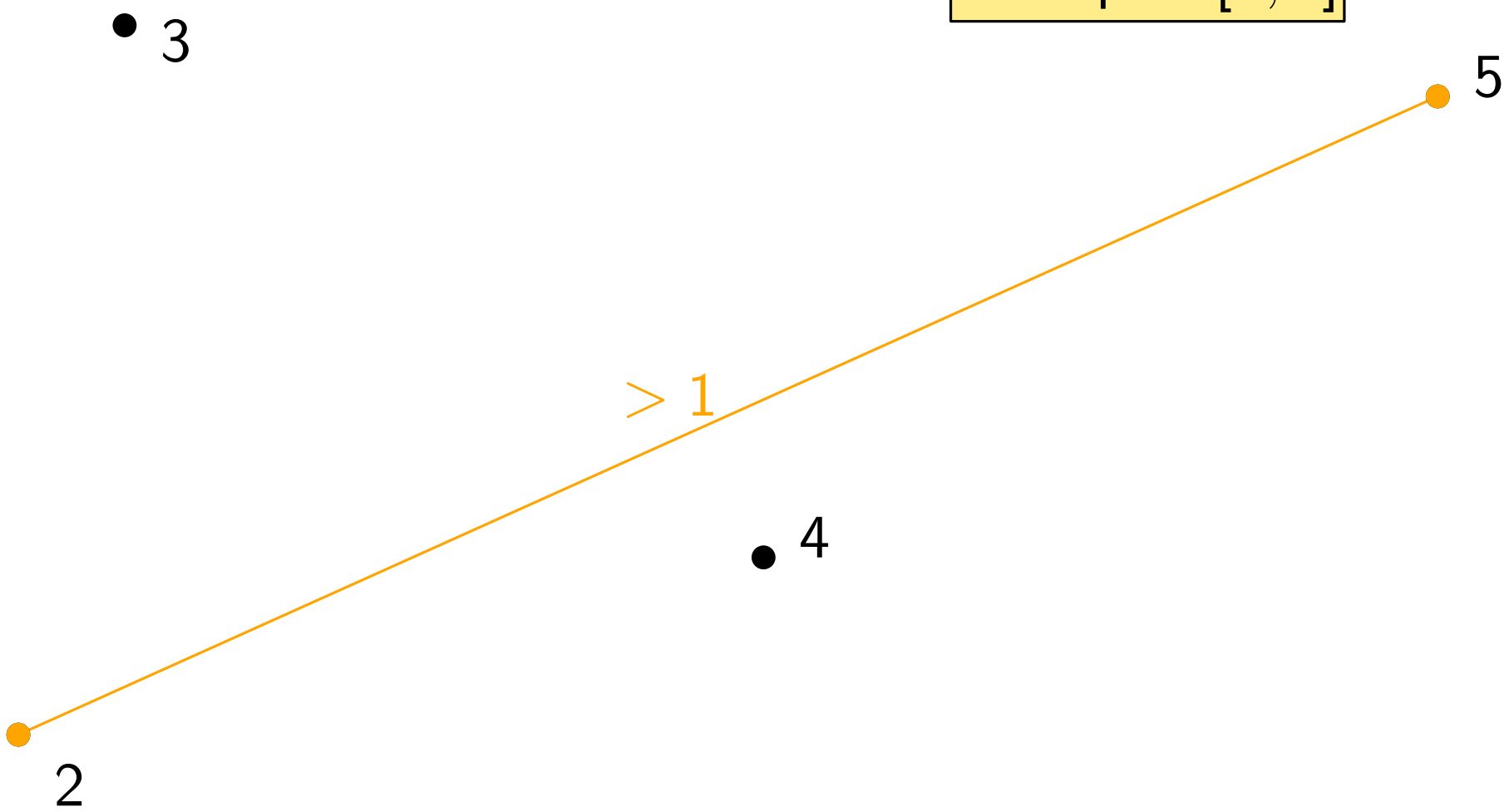

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If a set S has a *hereditary* property $\mathcal{P}$, then any superset $S' \supseteq S$ has $\mathcal{P}$.

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Examples:

Problem	Property
diameter decision	diameter > 1
convex hull area decision	area of convex hull > 1
orthogonal segment inters. detect.	$\exists$ intersecting segments
width decision	width > 1

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For each i:

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For each i: store min j s.t. $[i, j]$ has $\mathcal{P}$

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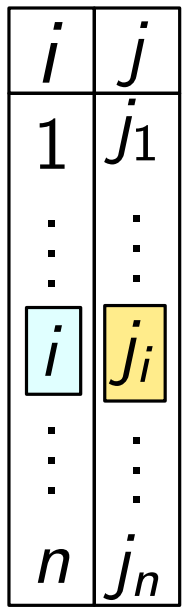

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Henceforth, only concerned with preprocessing.

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Bannister, Devanny, Goodrich, Simons, Trott; *CCCG 2014*:

- Time-windowed convex hull
  - Gift wrapping & tangent $O(\log^2 w)$
  - Linear programming $O(\log w)$
  - Line decision $O(\log w)$
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$w = t_2 - t_1$, the size of the query time window

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Chan and Pratt; *CCCG 2015*:

- Time-windowed closest pair decision problem
  - Preprocessing: $O(n)$
  - Query: $O(1)$
  - Space: $O(n)$ *bits* of space
- Time-windowed closest pair
  - Preprocessing: $O(n \log n \log \log n)$
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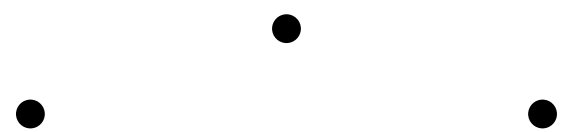

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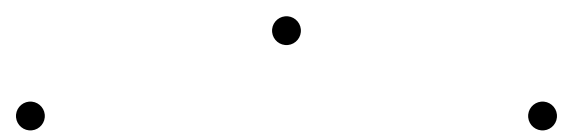

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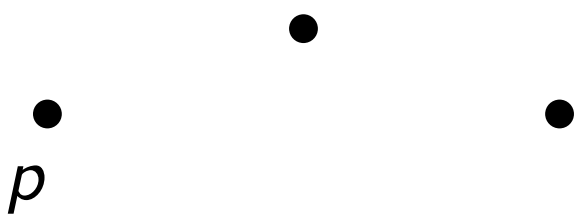

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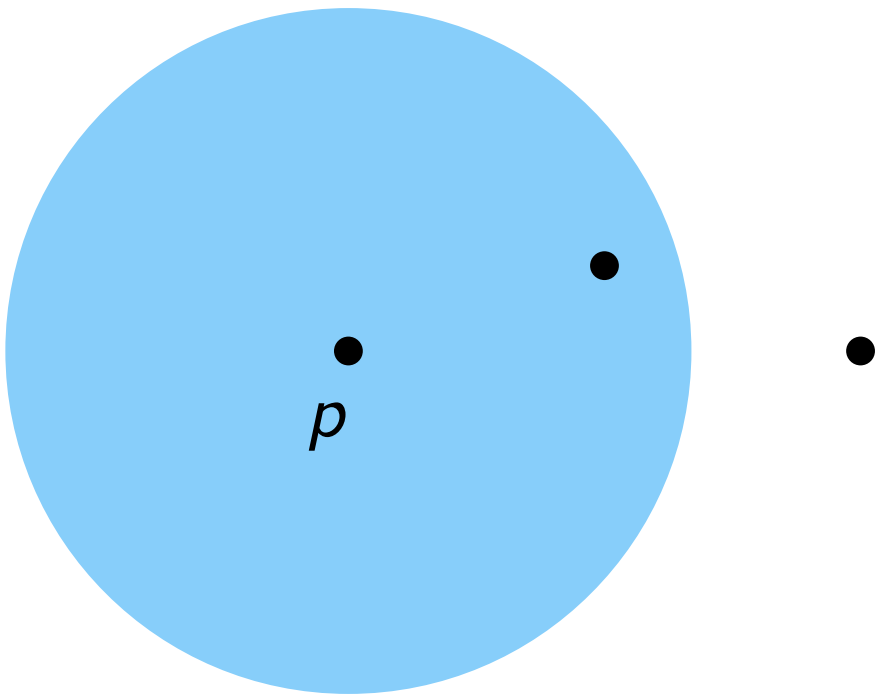

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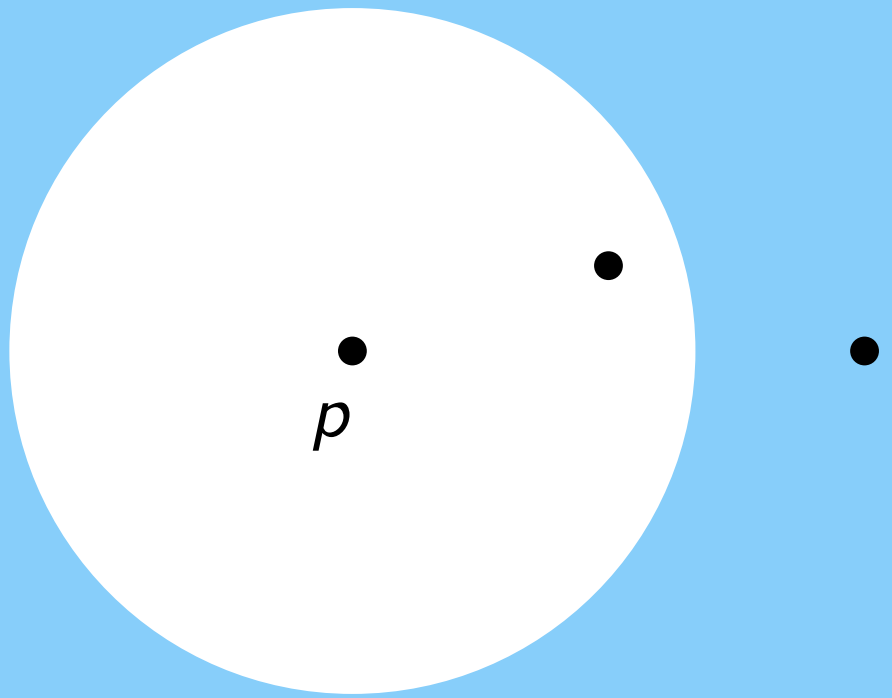

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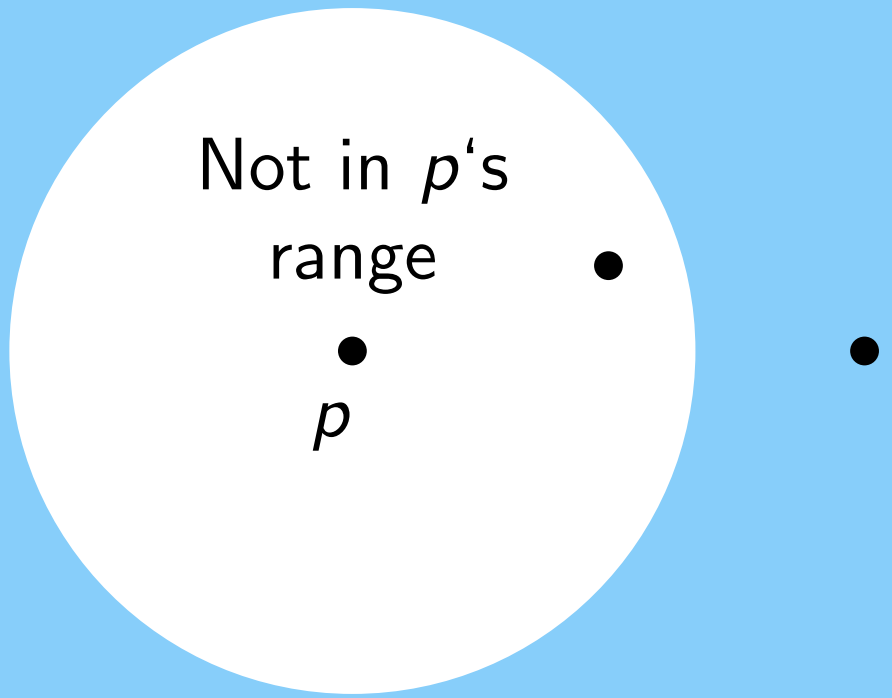

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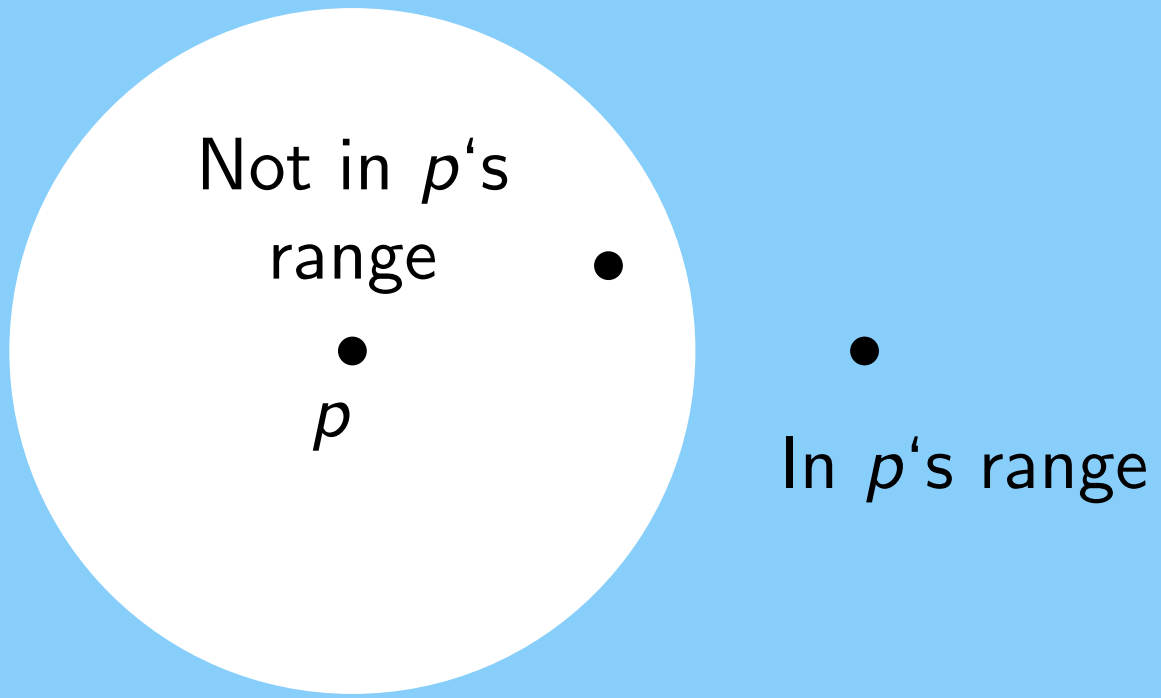

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Recall this table

i	j
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Given time

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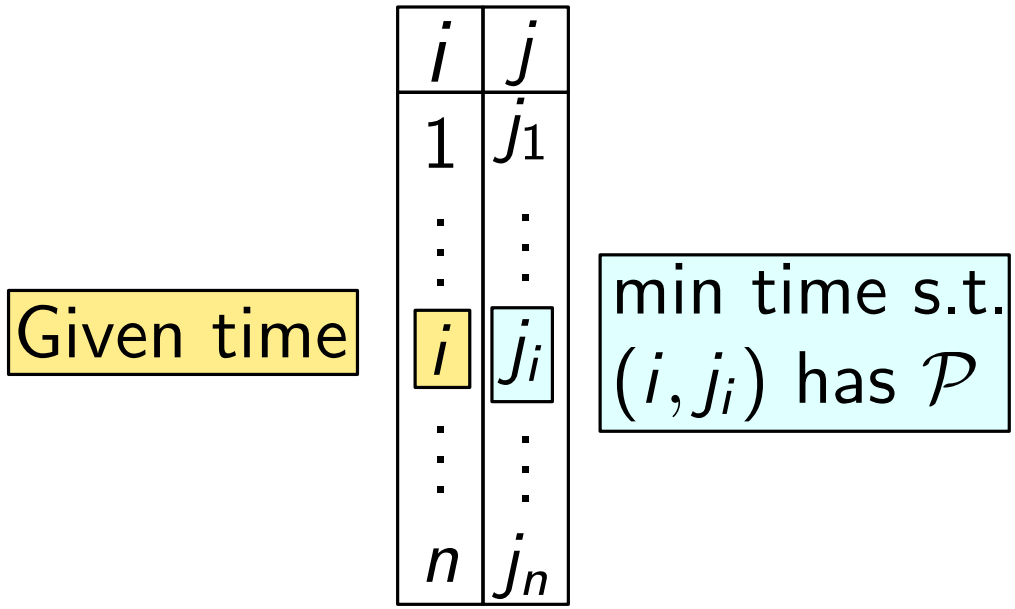

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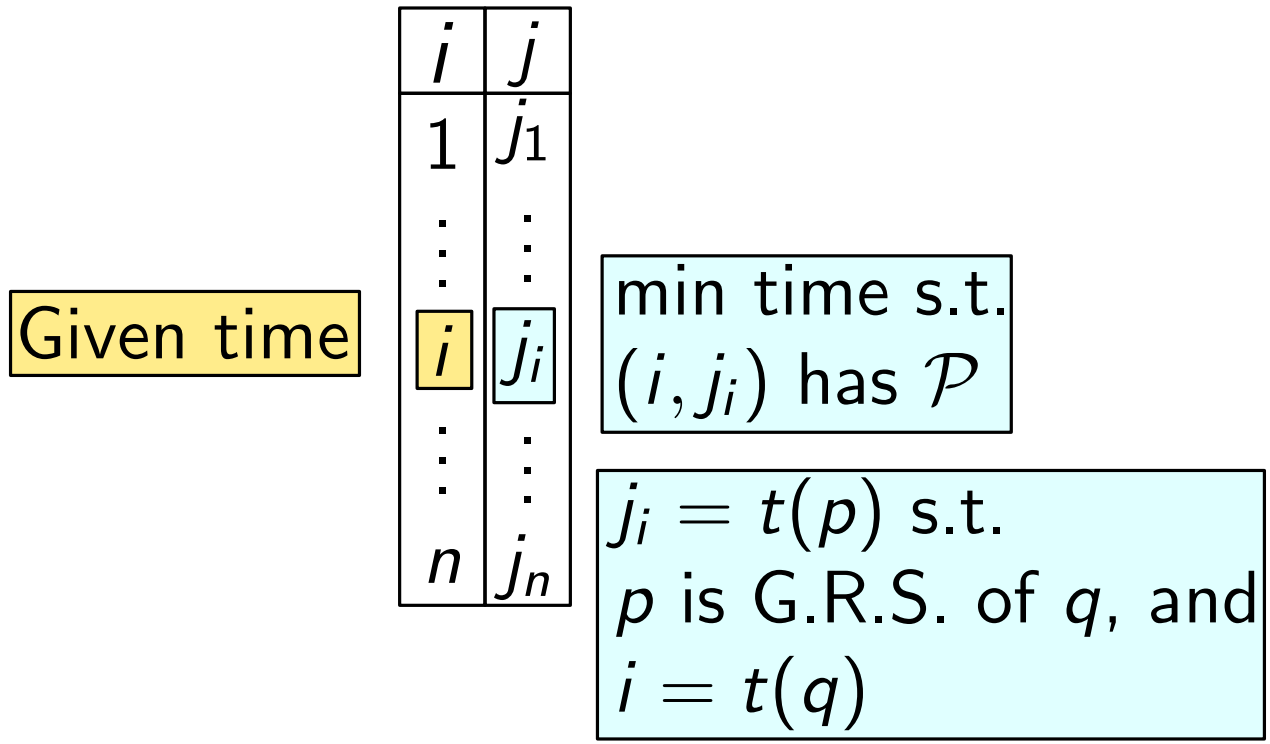

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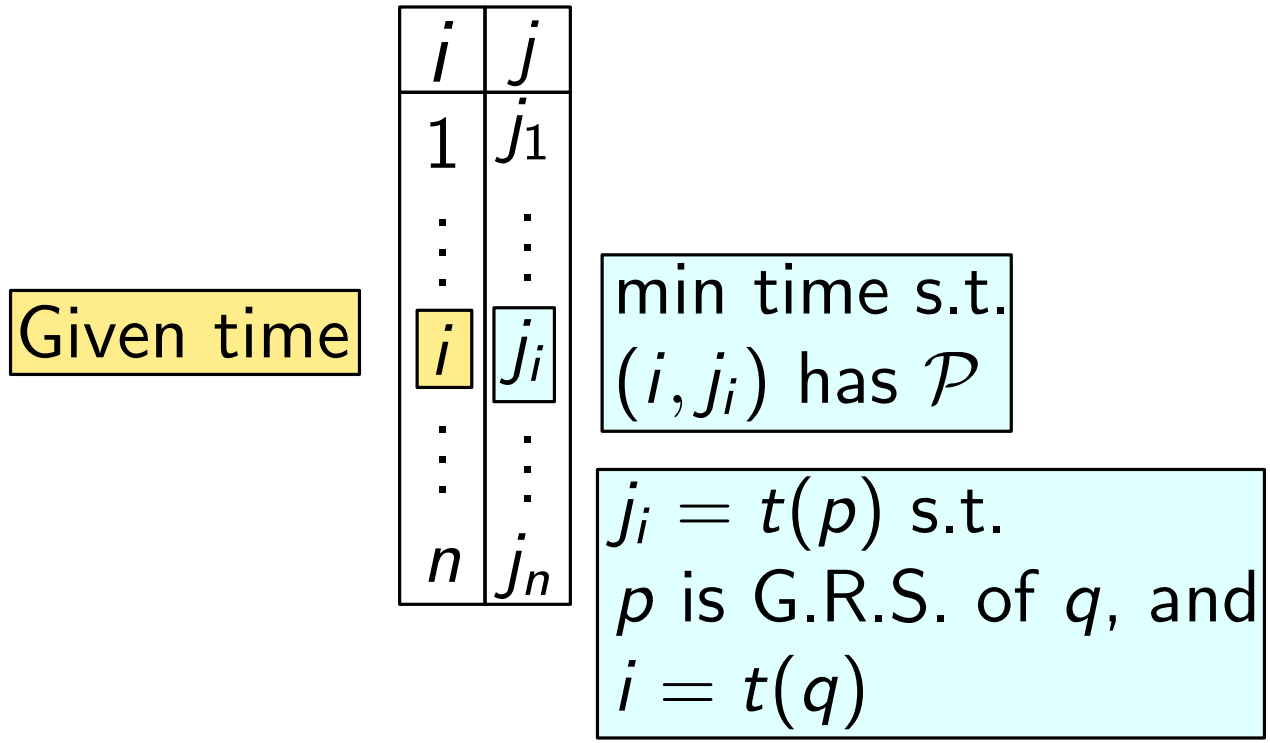
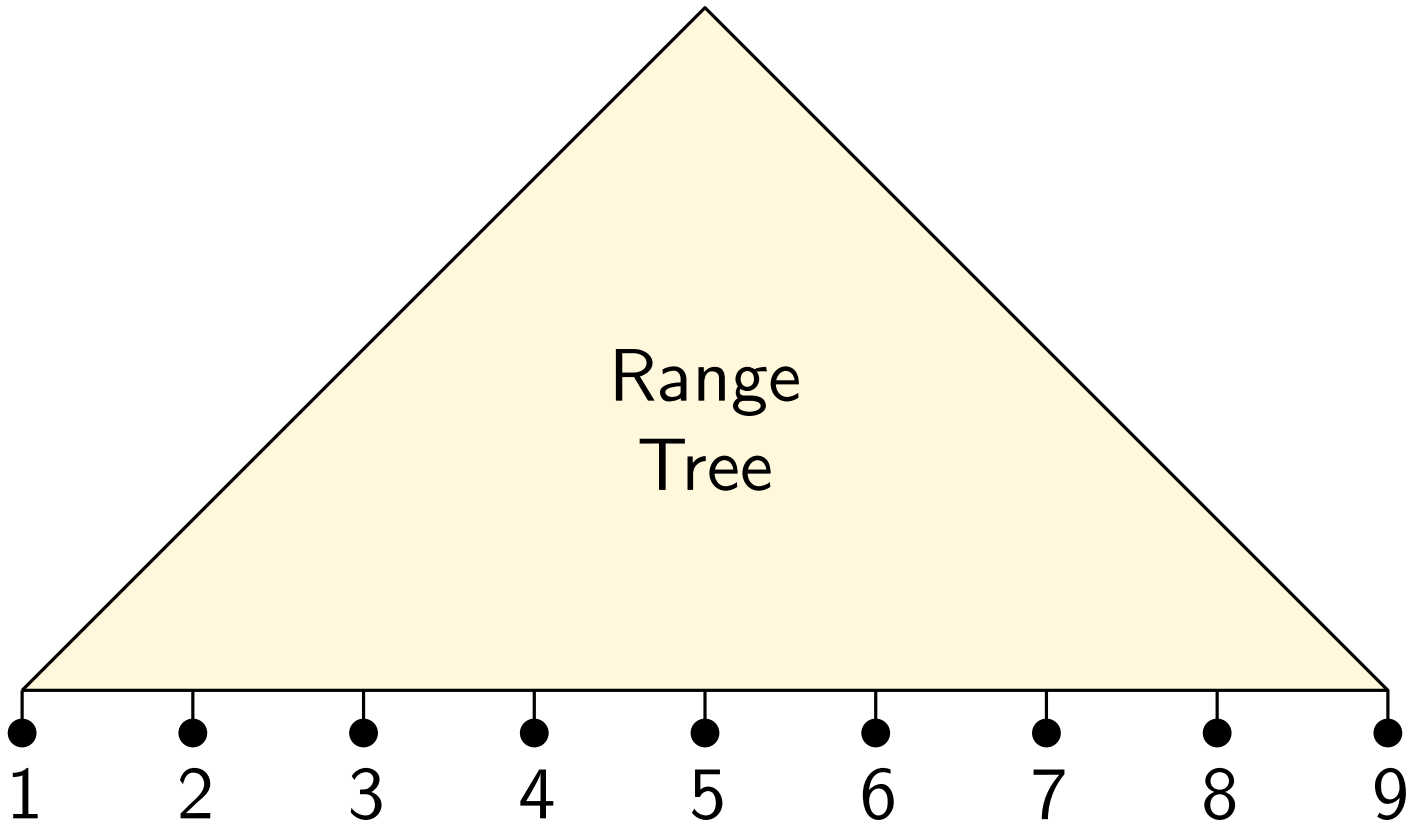


$\therefore$ can preprocess for some time-windowed decision problems by solving n instances of G.R.S.

Generalized Range Successor

Consider the objects in time-order

Generalized Range Successor



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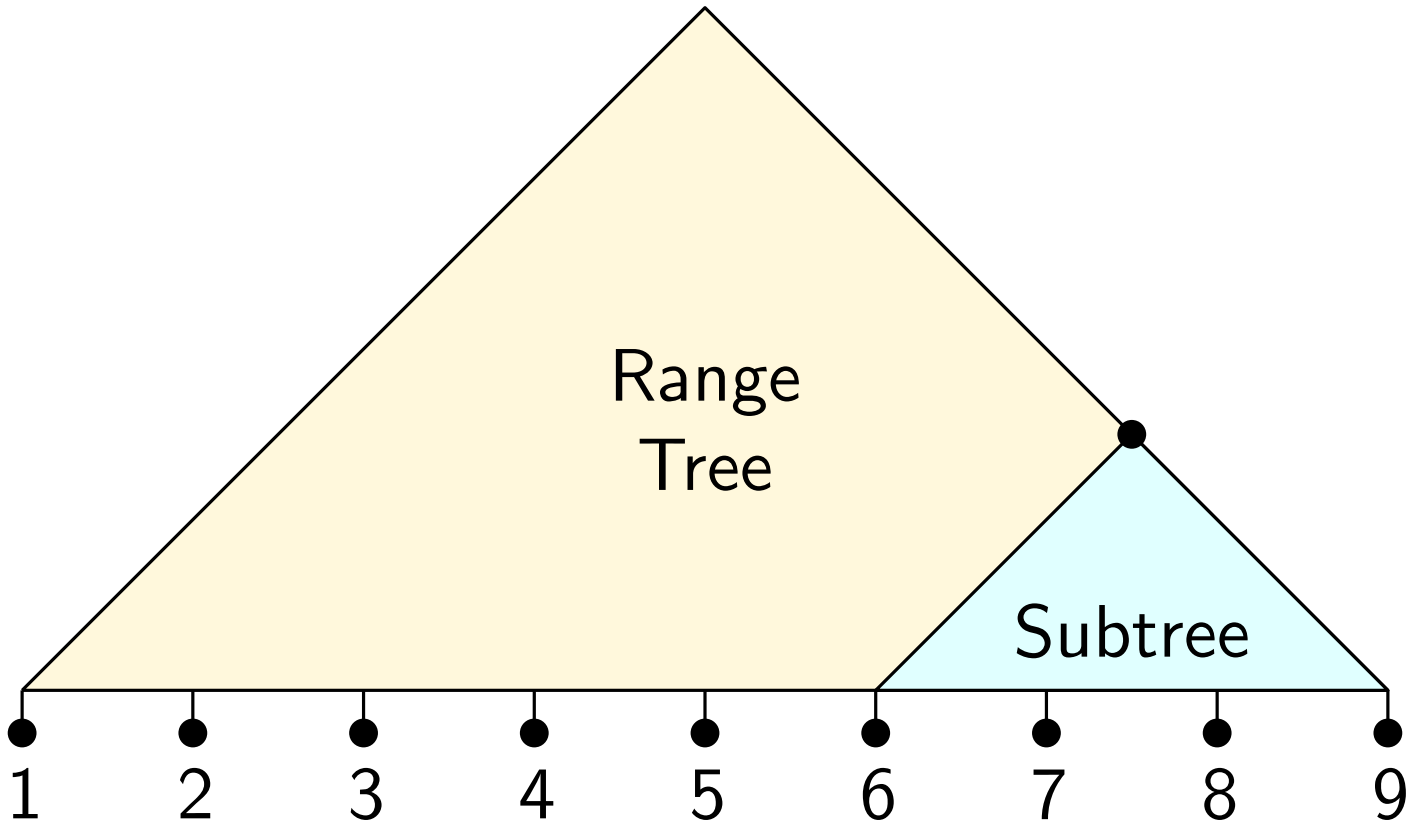

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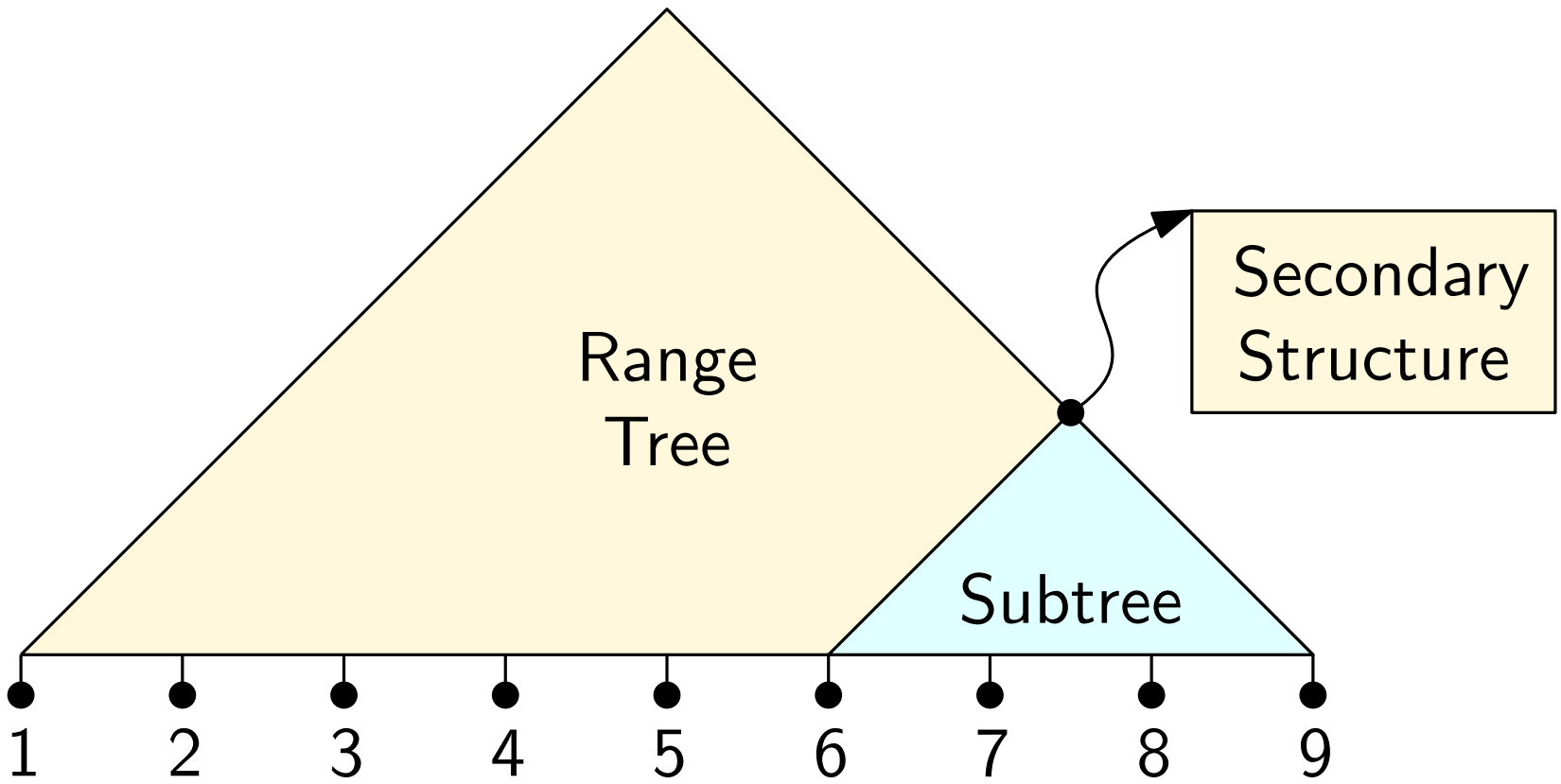

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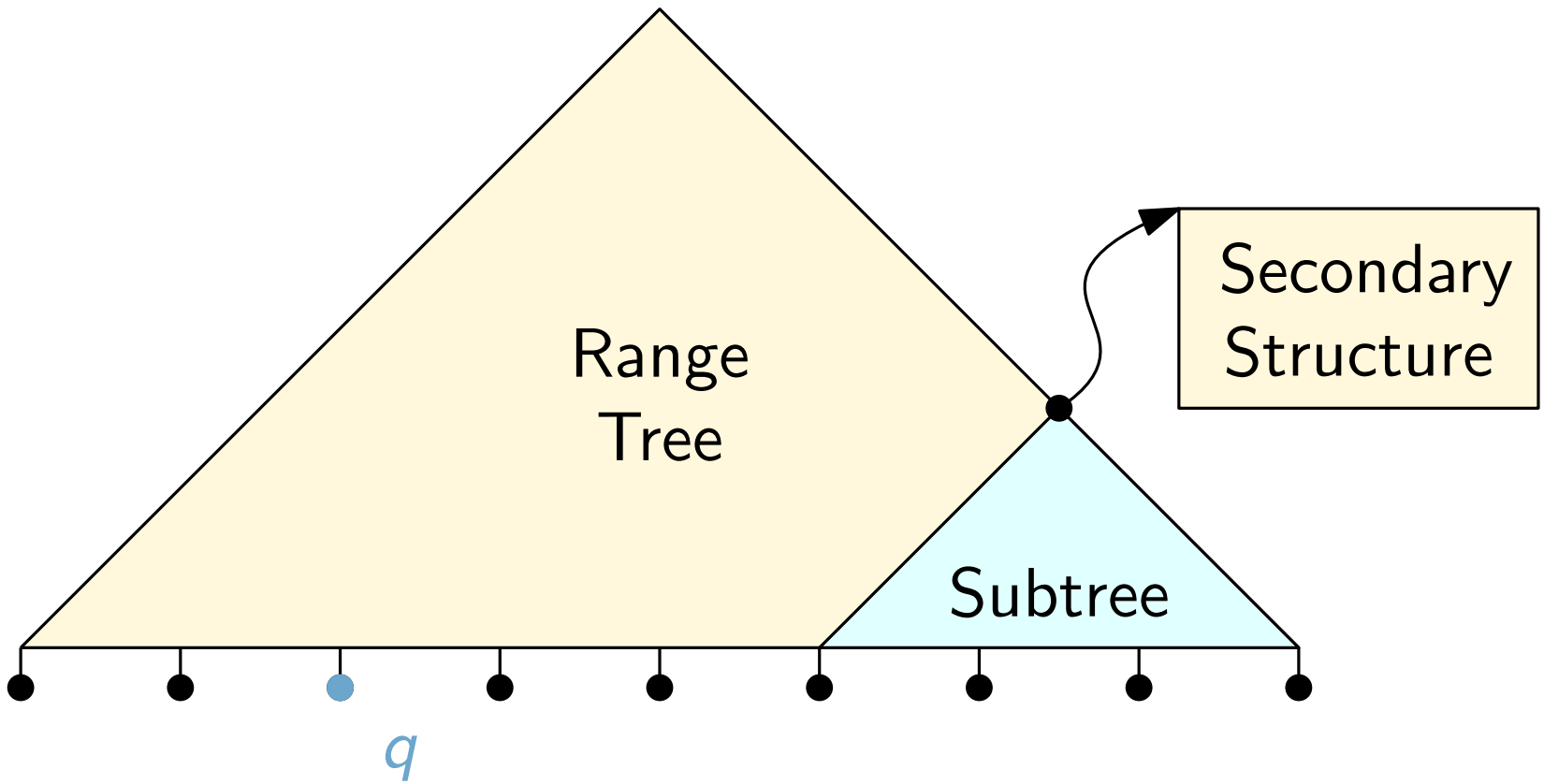

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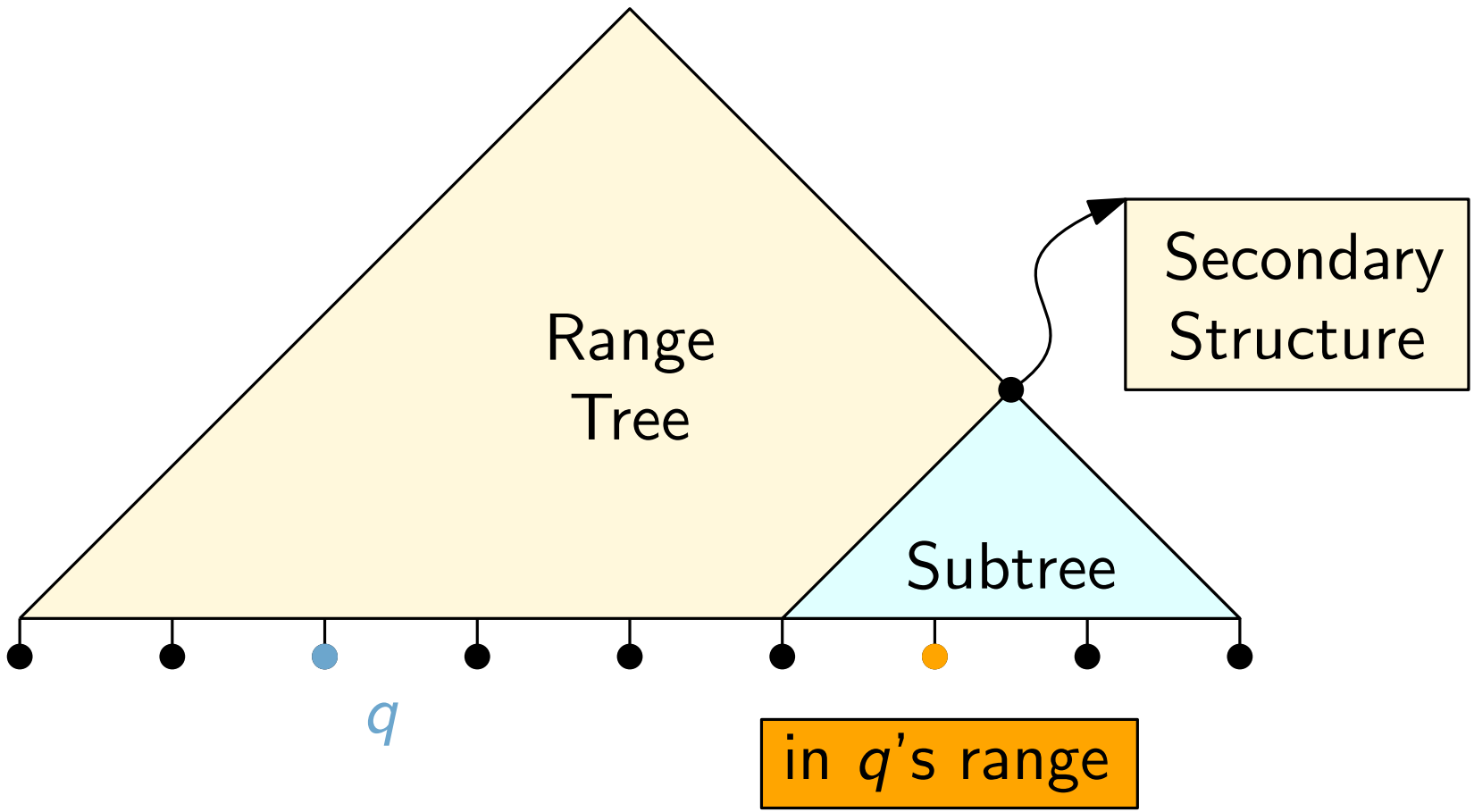
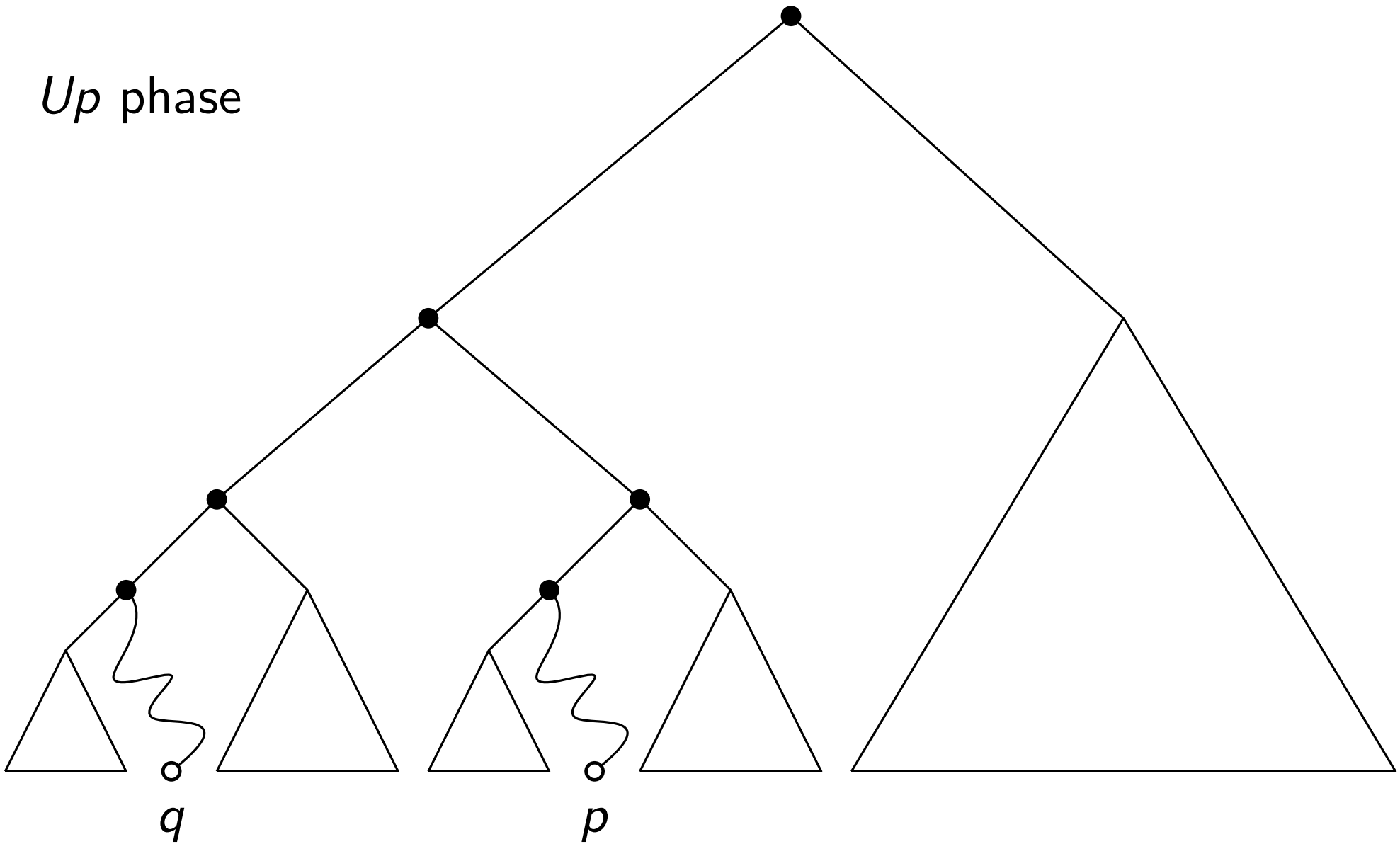


Avoiding Binary Search



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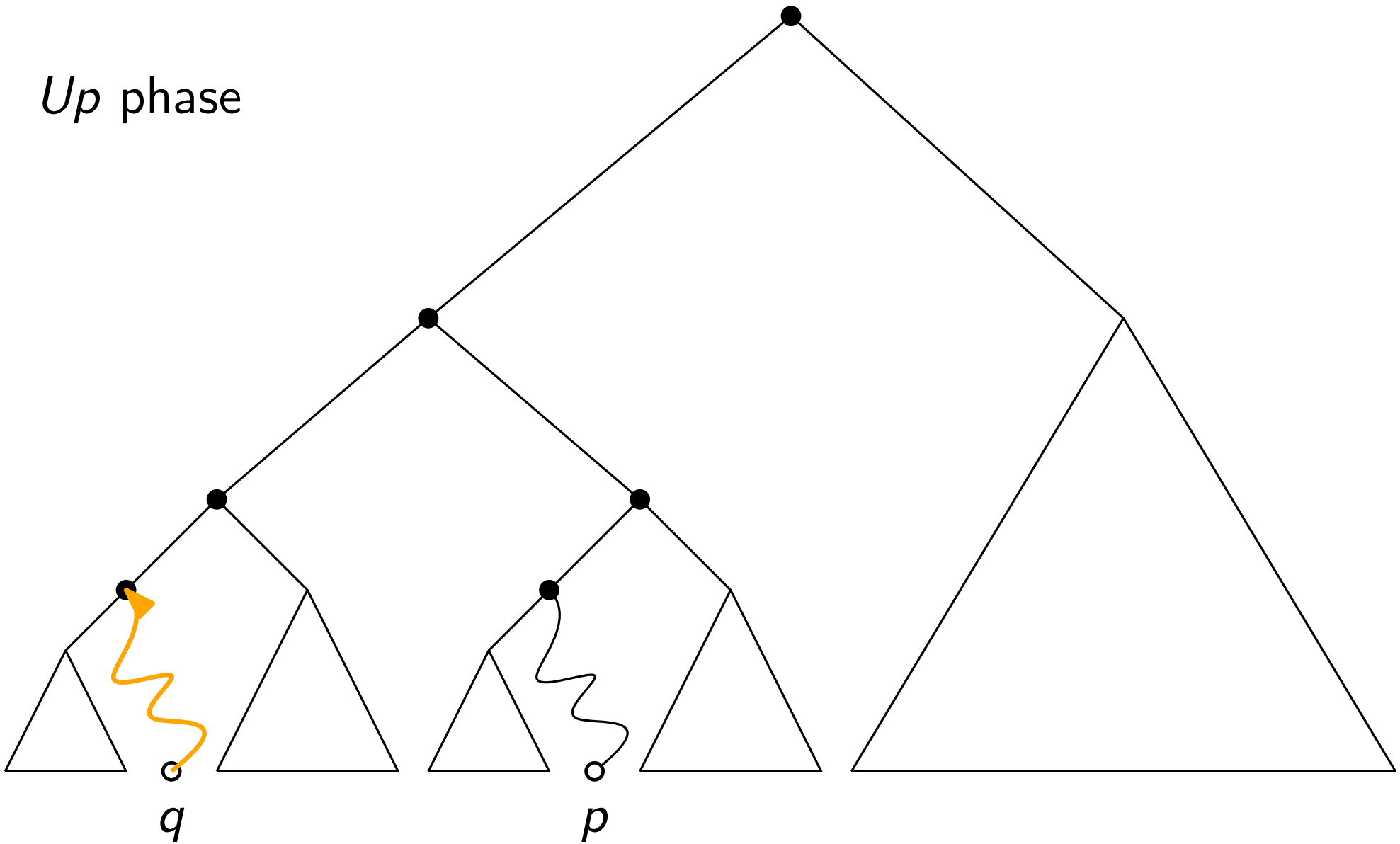

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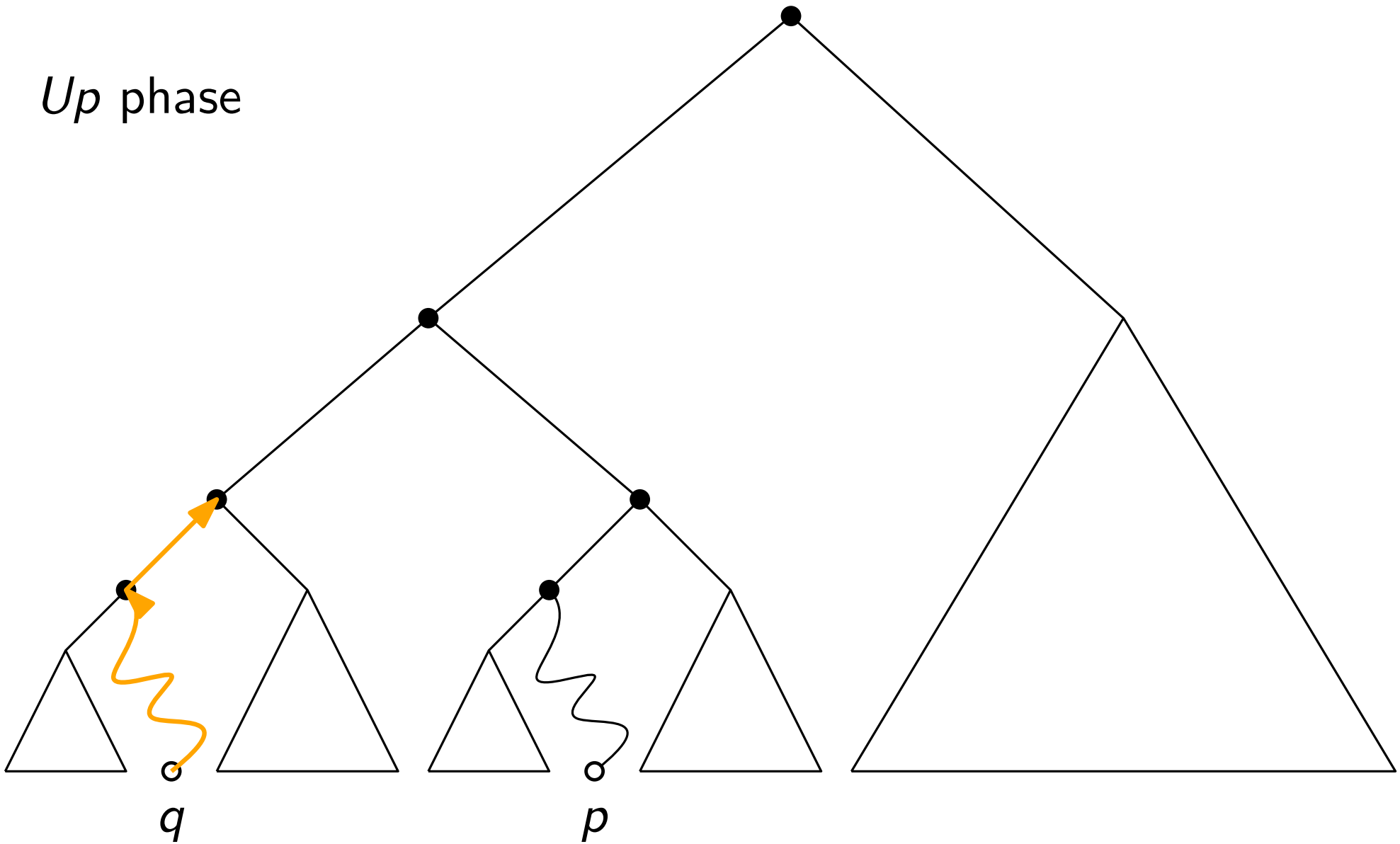

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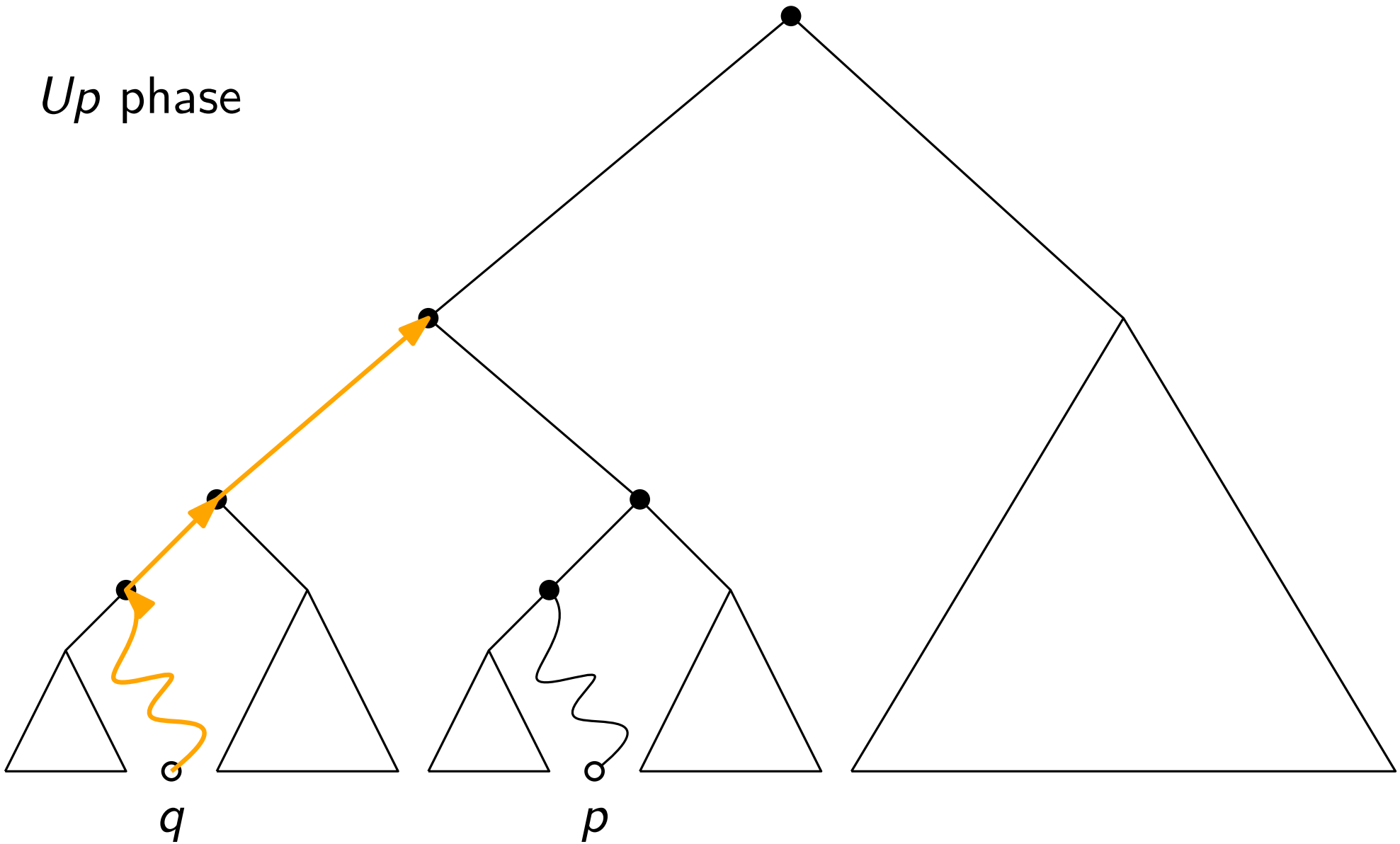

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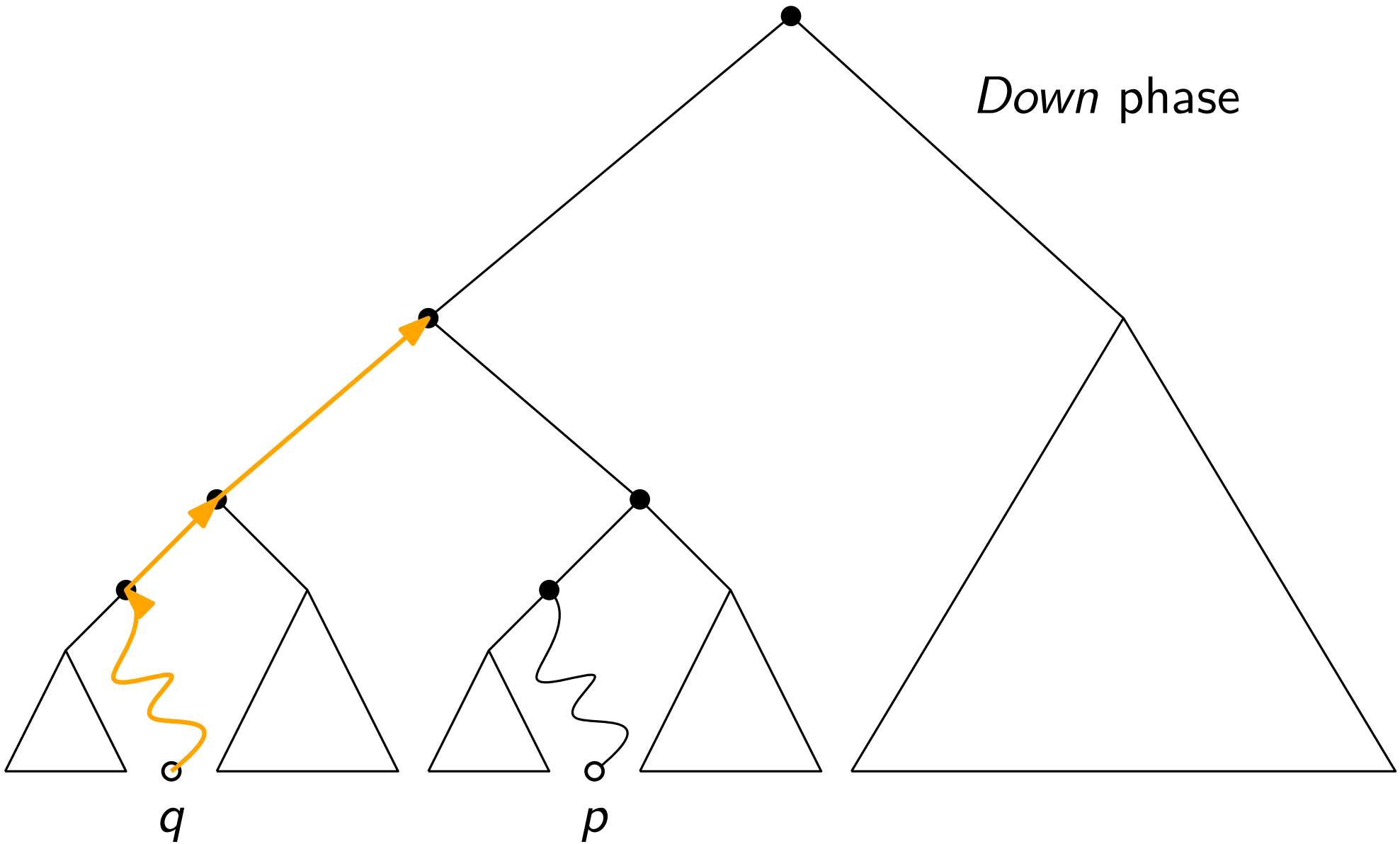

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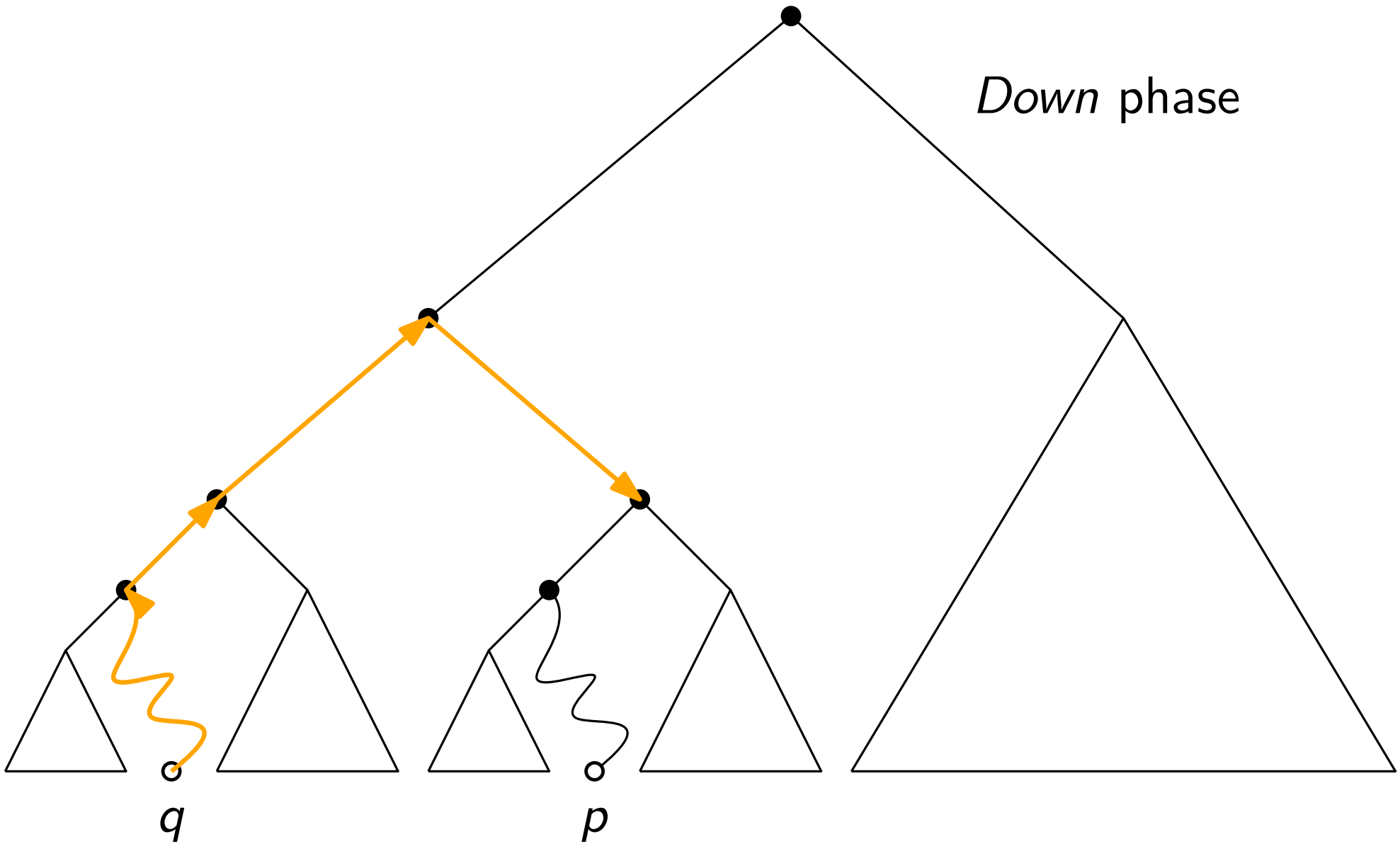

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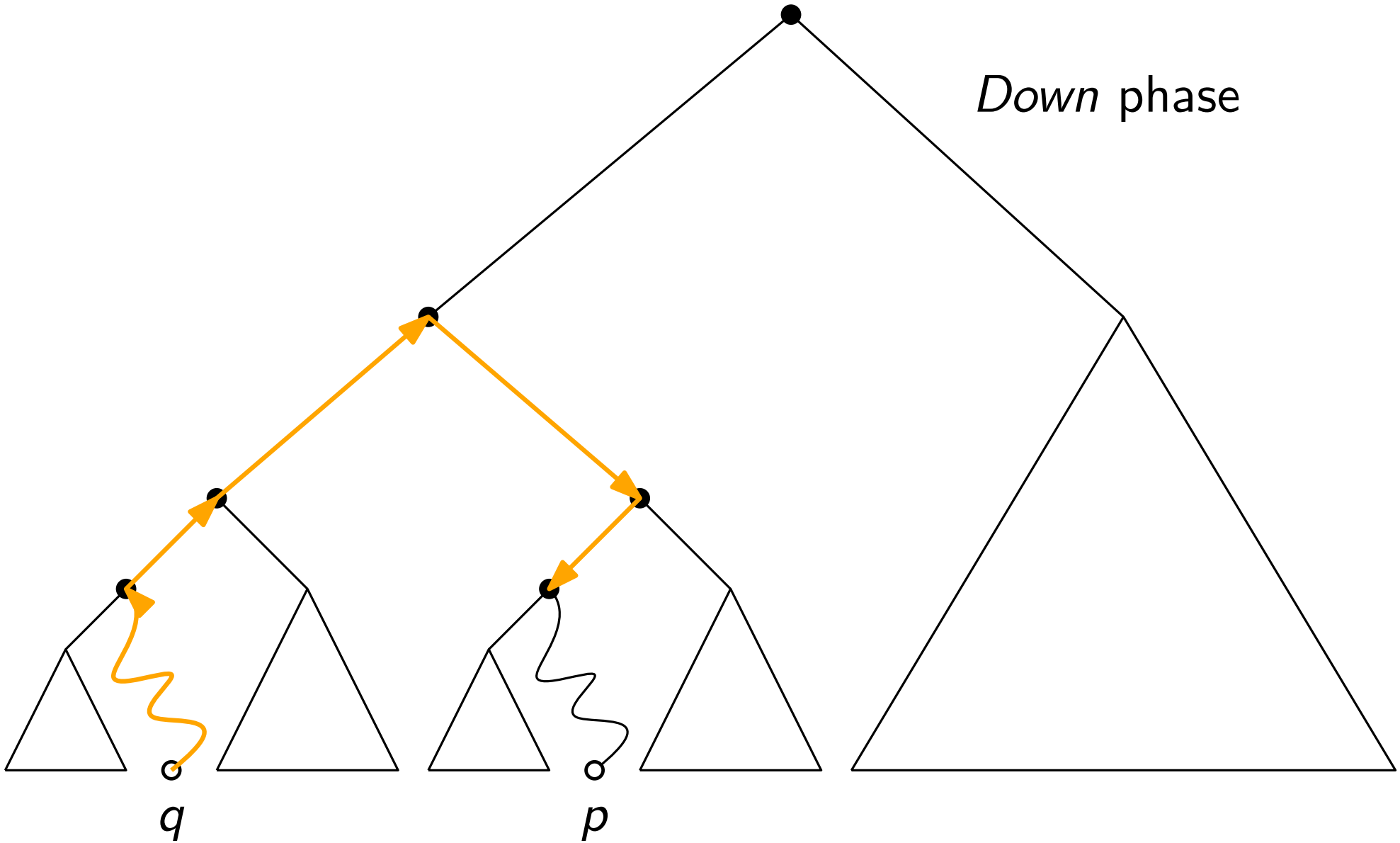

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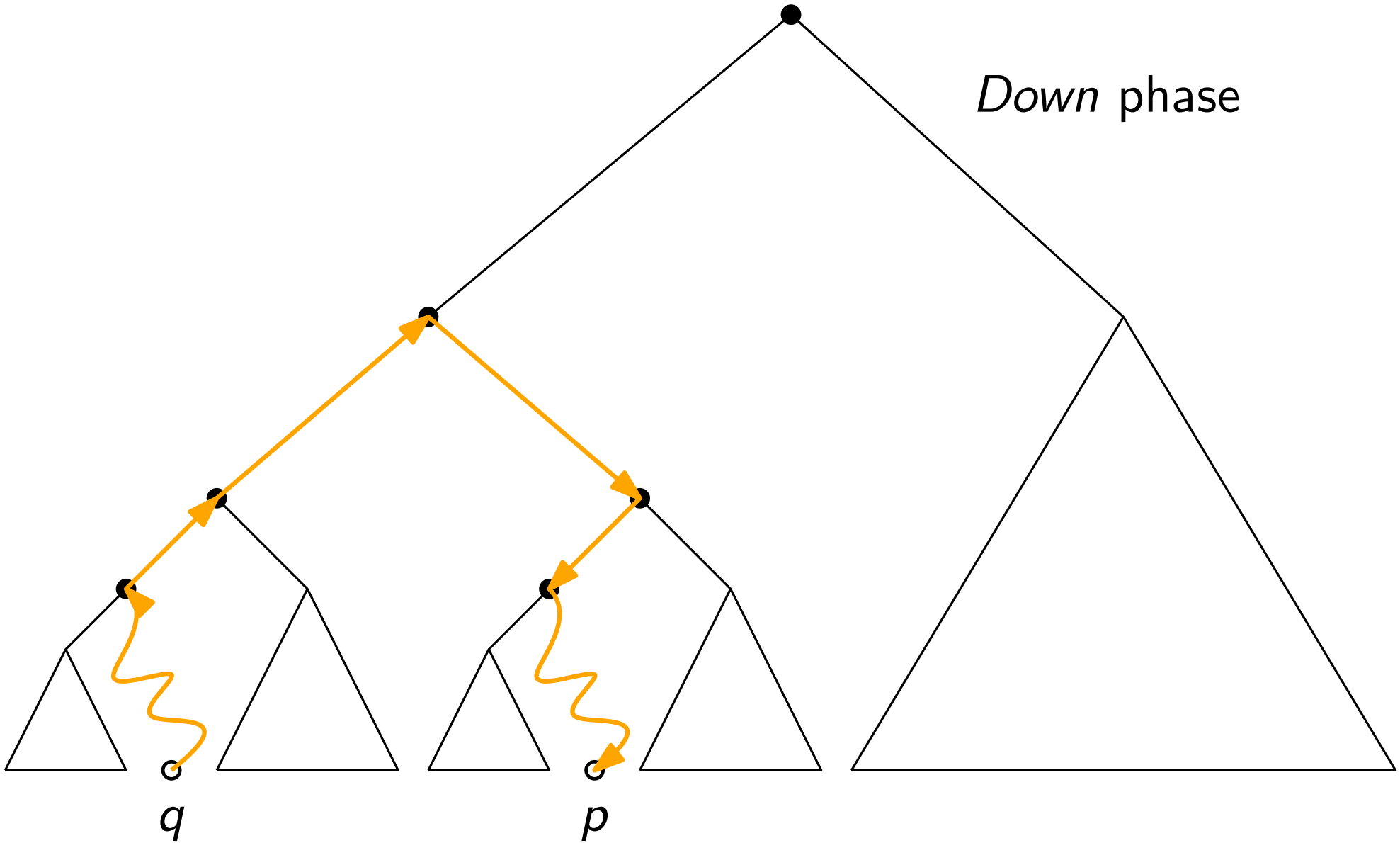

Generalized Range Successor: Conclusion

2D diameter decision

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2D diameter decision

- Point location in unit disk intersection $\rightarrow$ binary search $O(\log n) \cdot O(n \log n) = O(n \log^2 n)$

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3D diameter decision

Generalized Range Successor: Conclusion

2D diameter decision

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3D diameter decision

- Point location in unit ball intersection $\rightarrow$ 2D point location
  $O(\log n) \cdot O(n \log n) = O(n \log^2 n)$

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Orthogonal segment intersection detection

Generalized Range Successor: Conclusion

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3D diameter decision

- Point location in unit ball intersection $\rightarrow$ 2D point location
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Orthogonal segment intersection detection

- Orthogonal intersection detection $\rightarrow$ point location
  $O(\log \log n) \cdot O(n \log n) = O(n \log n \log \log n)$

New Results

Approach #1: Generalized Range Successor

Problem	Preprocessing Time
2D diameter decision	$O(n \log^2 n)$
$\rightarrow$ improved to	$O(n \log n)$
3D diameter decision	$O(n \log^2 n)$
2D orth. seg. intersection detection	$O(n \log n \log \log n)$

Approach #2: FIFO Update Sequence

Problem	Preprocessing Time
2D convex hull area decision	$O(n \log n \log \log n)$
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2D width decision	$O(n \log^8 n)$

New Results

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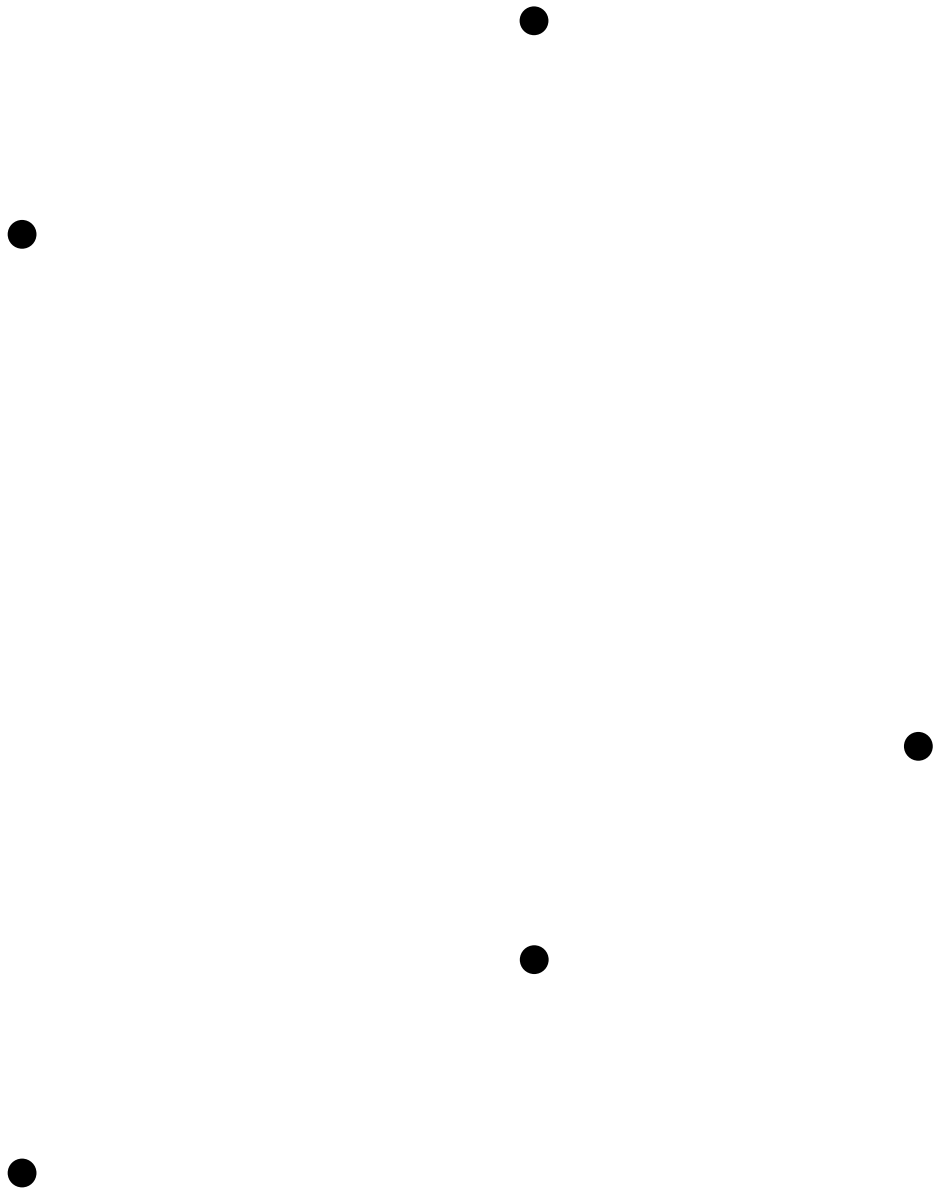
Time-Windowed $\rightarrow$ Dynamic

CH area > 1?

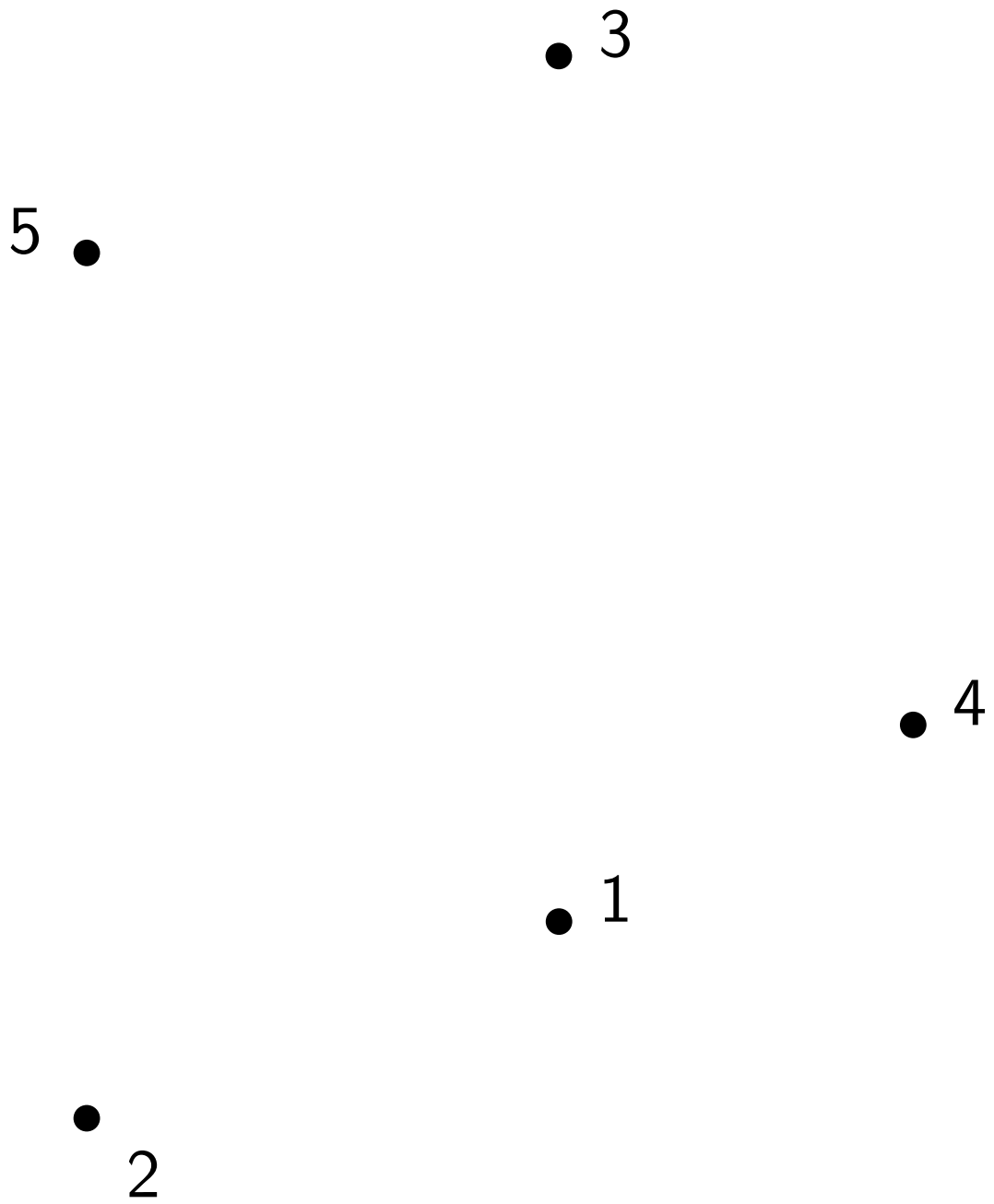
Time-Windowed $\rightarrow$ Dynamic

CH area > 1?



Time-Windowed → Dynamic

CH area > 1?



Time-Windowed $\rightarrow$ Dynamic

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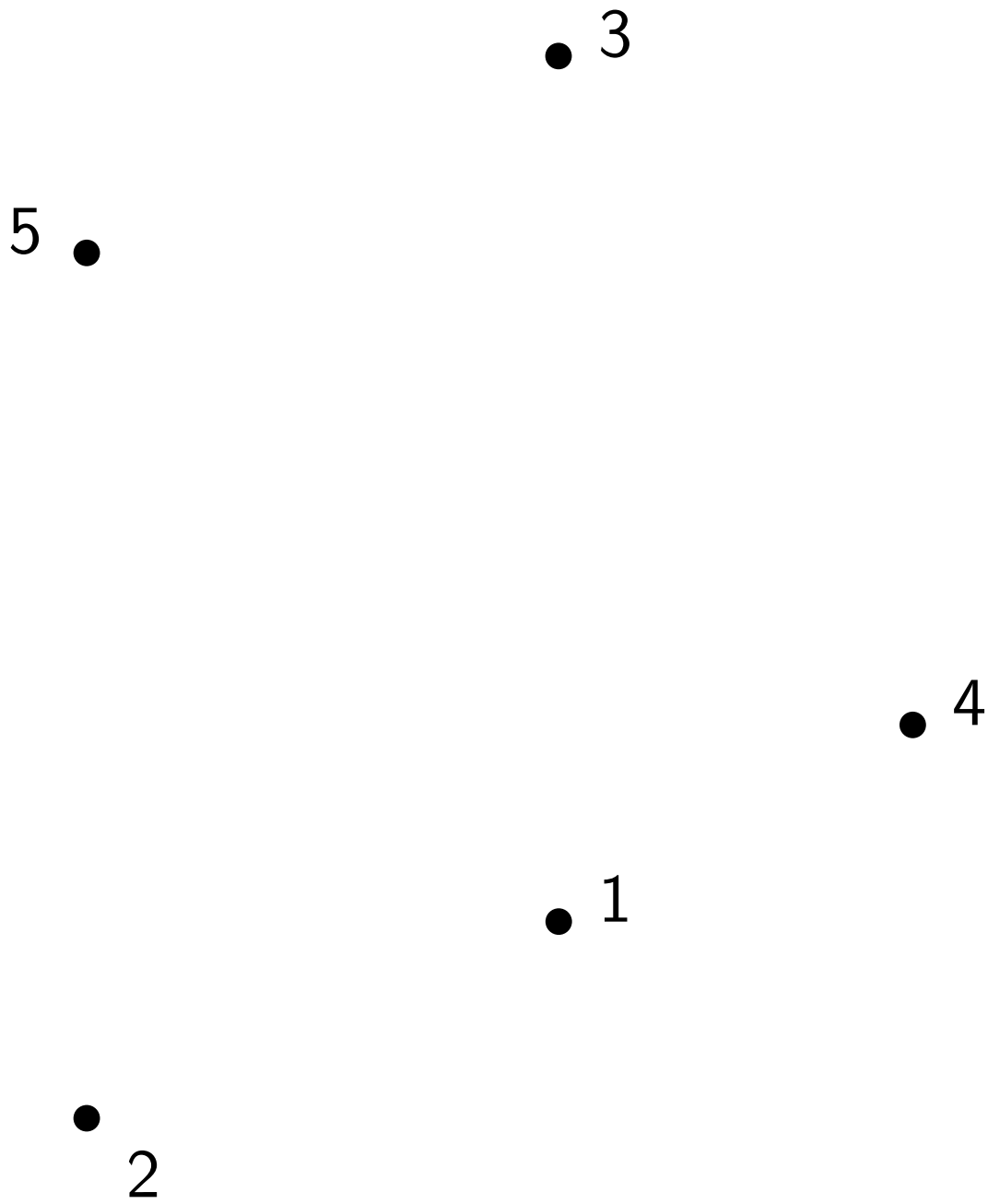
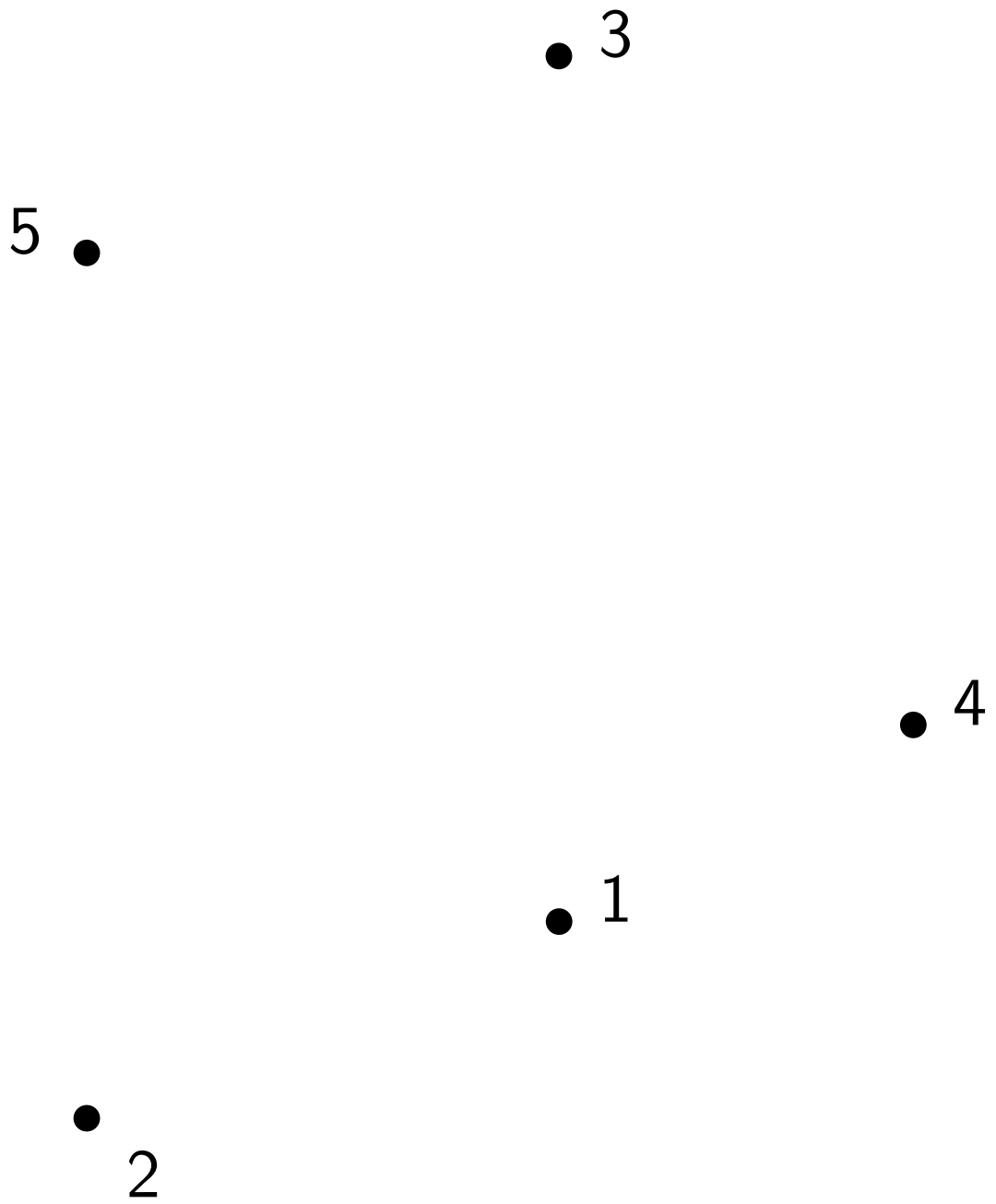


i
1
2
3
4
5

Time-Windowed $\rightarrow$ Dynamic

CH area > 1?



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1	
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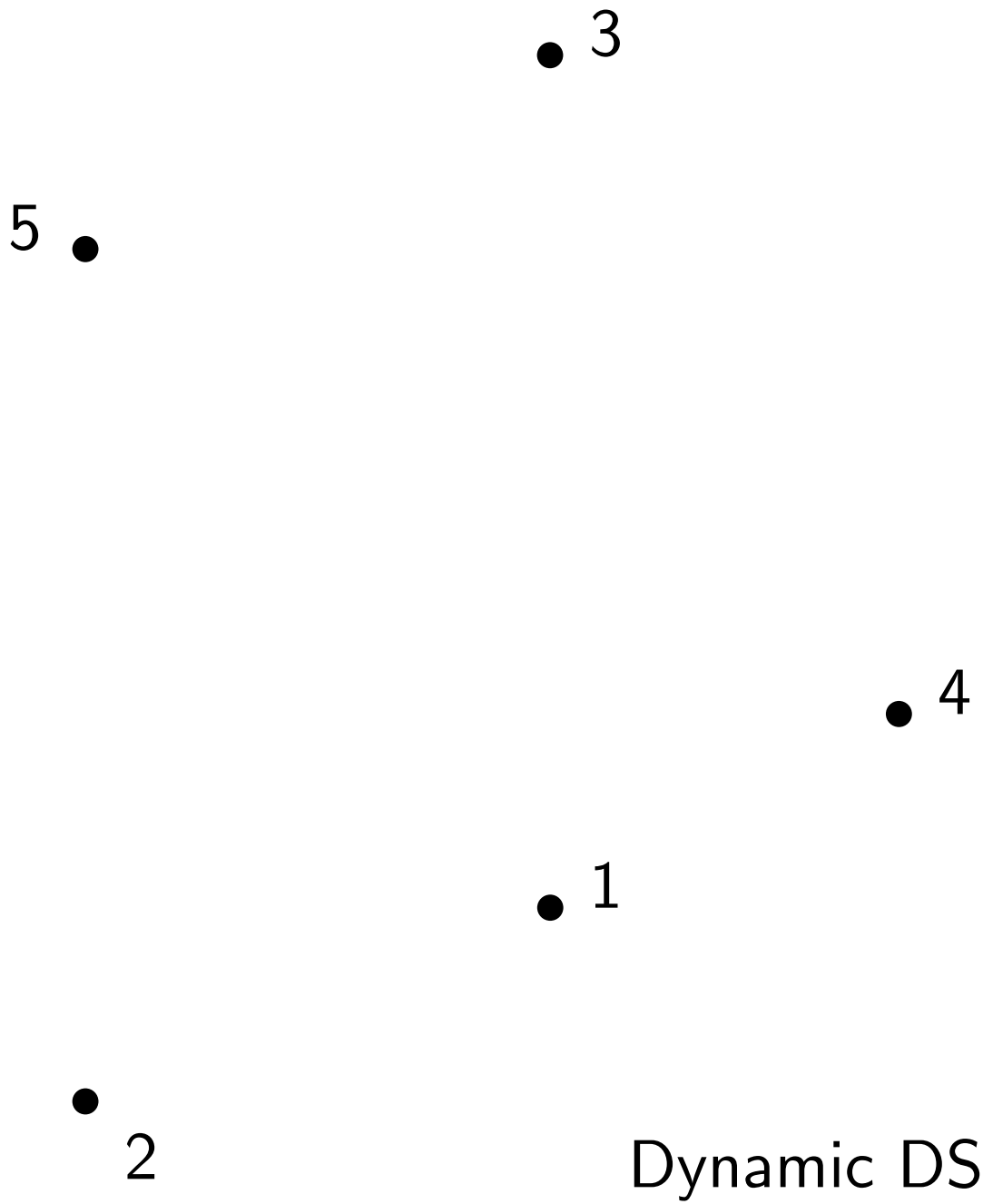
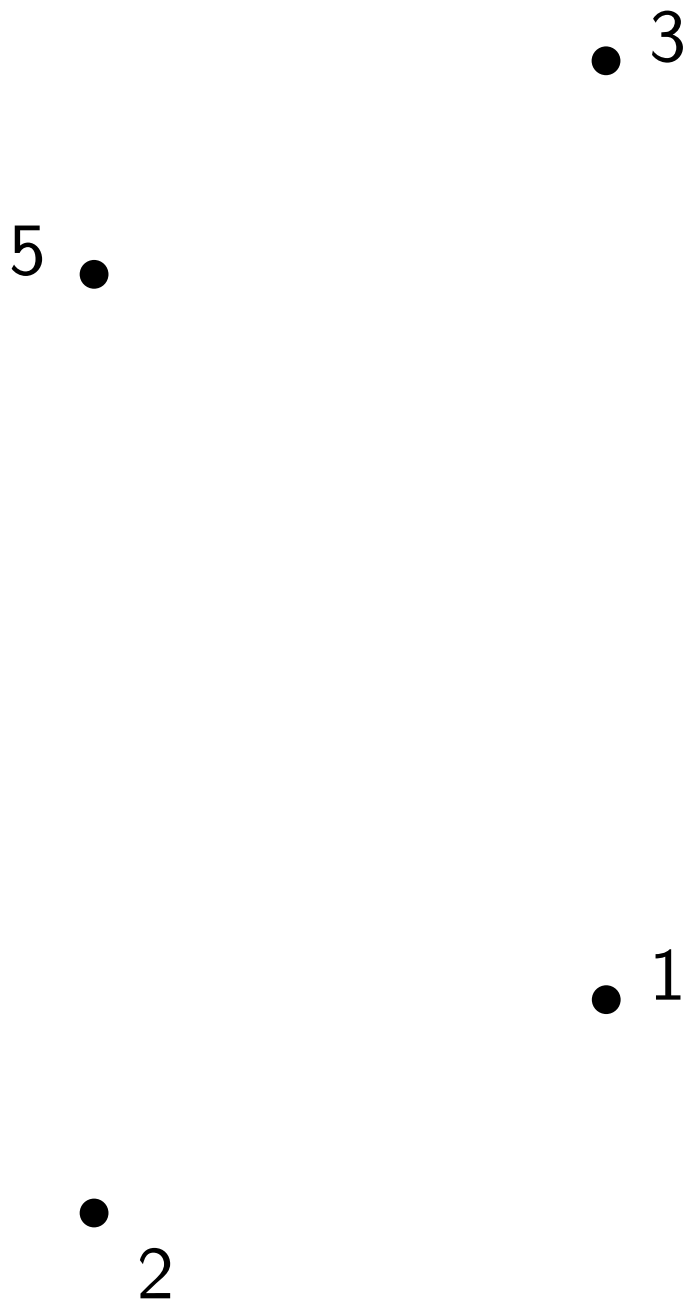


Dynamic DS

Time-Windowed $\rightarrow$ Dynamic

CH area > 1?



i	j
1	
2	
3	
4	
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Dynamic DS : Has property? No

Time-Windowed → Dynamic

CH area > 1?

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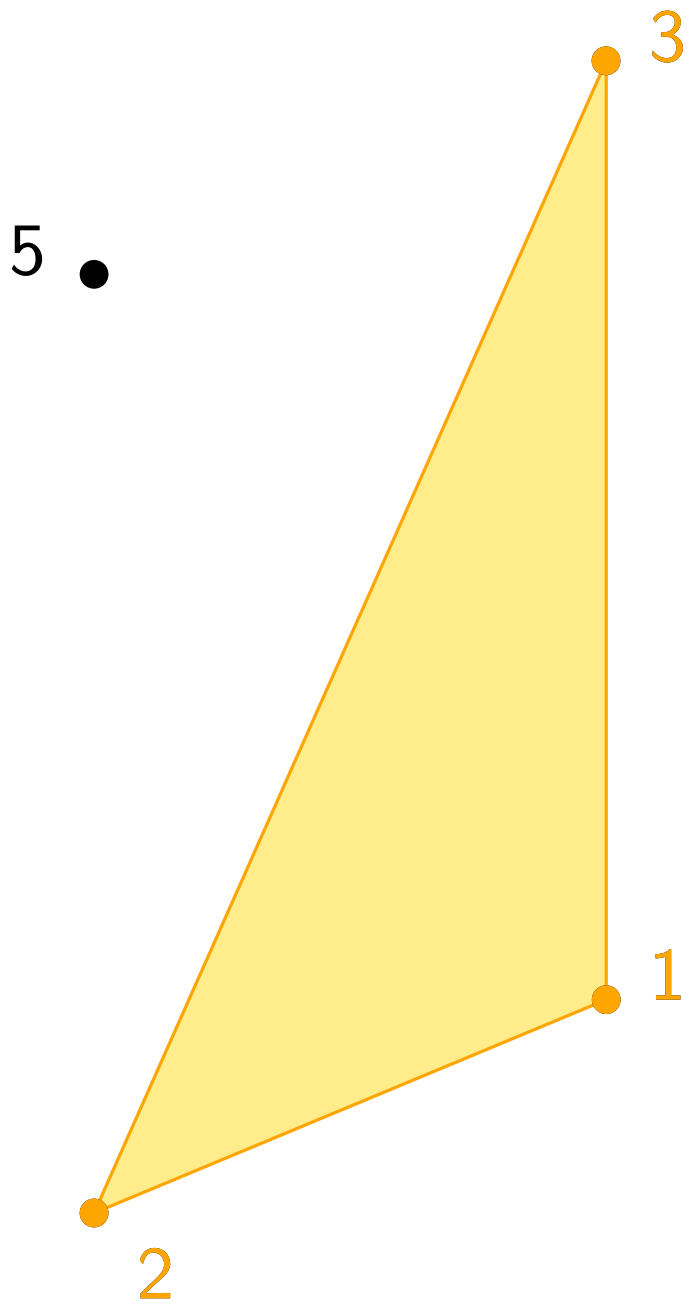

i	j
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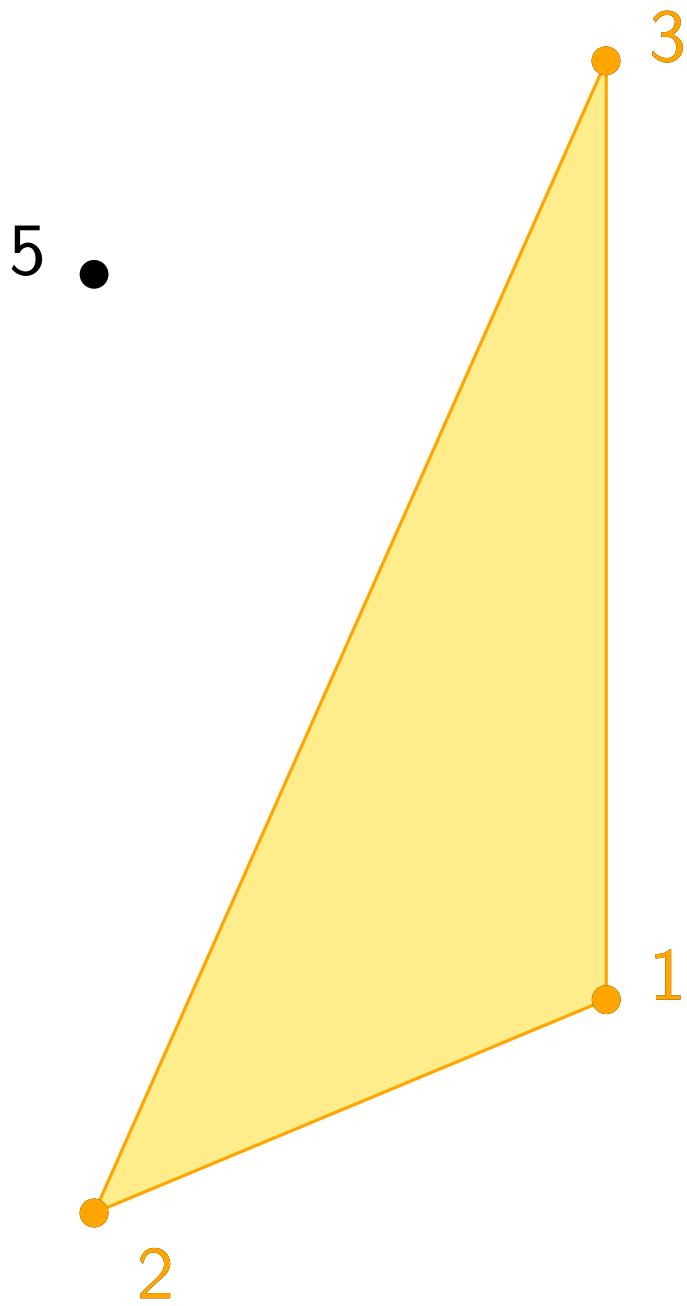

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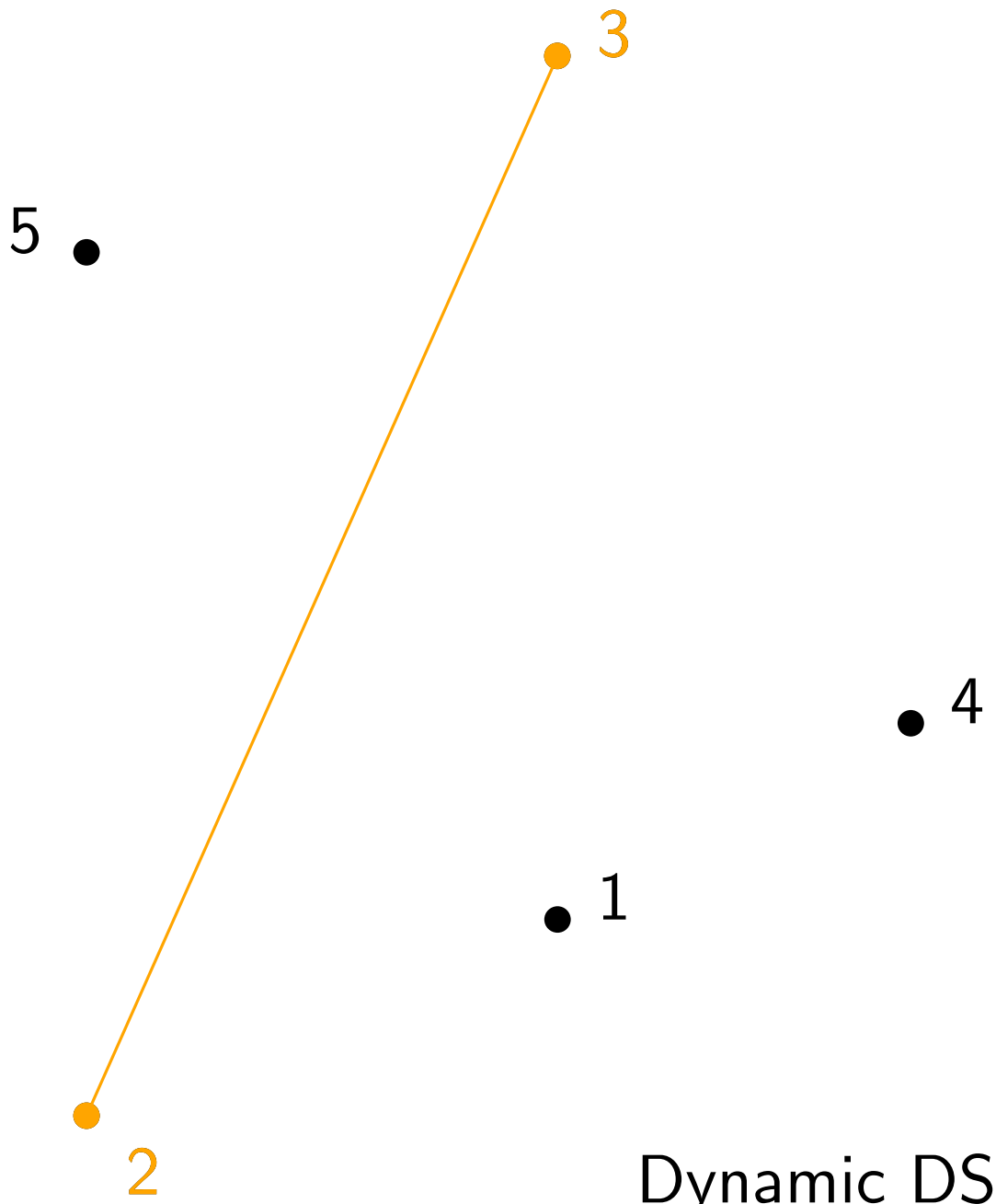

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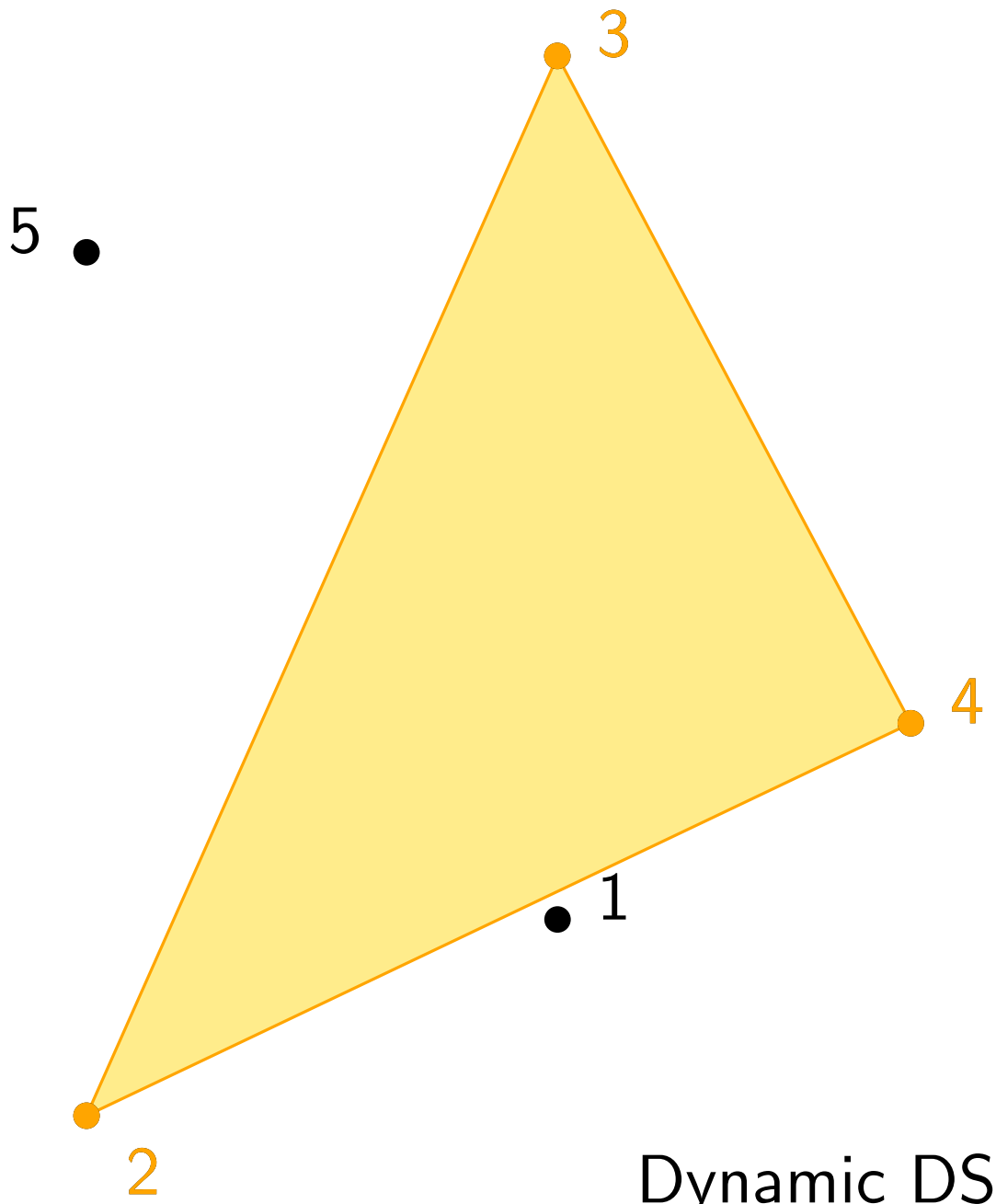

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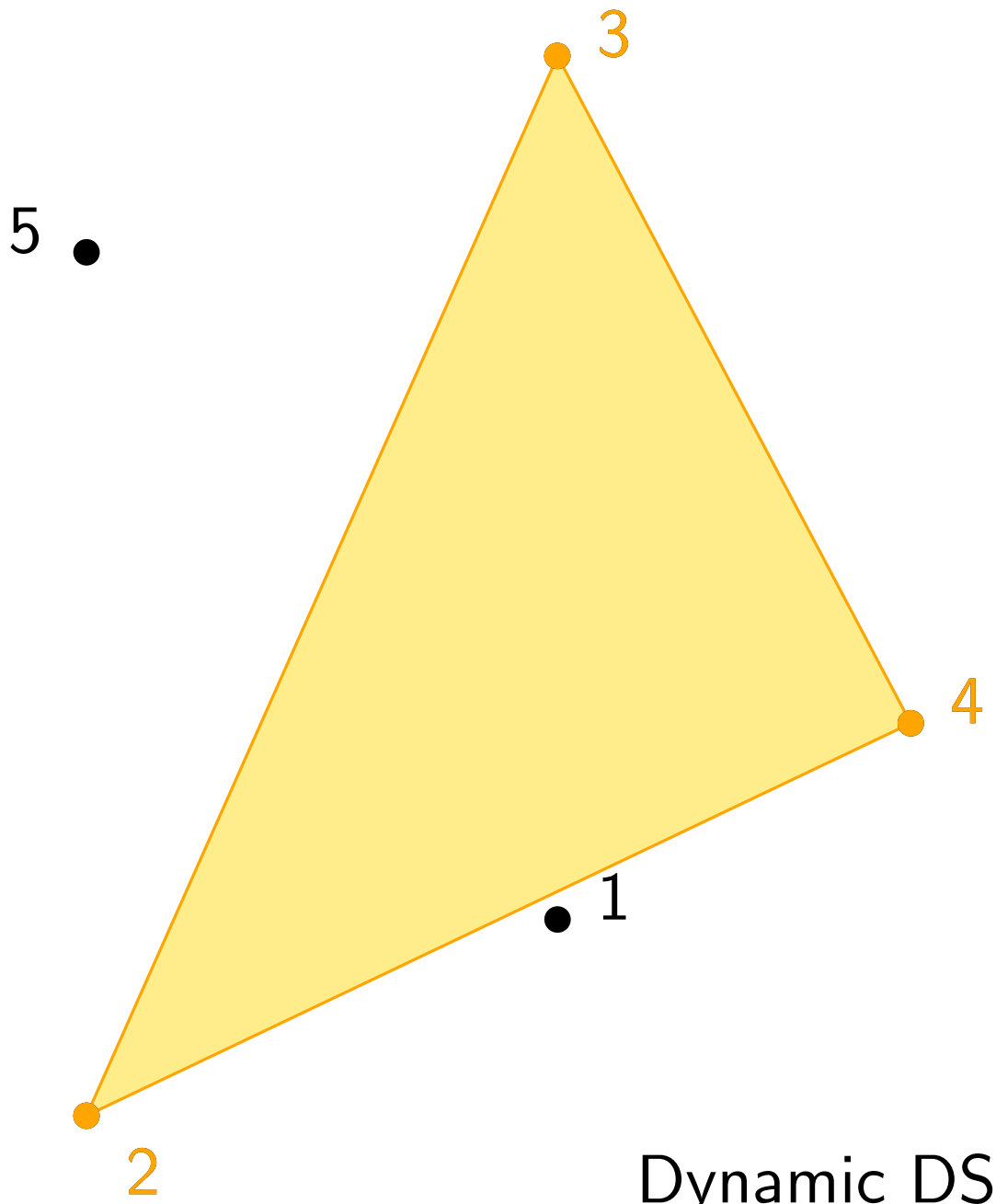

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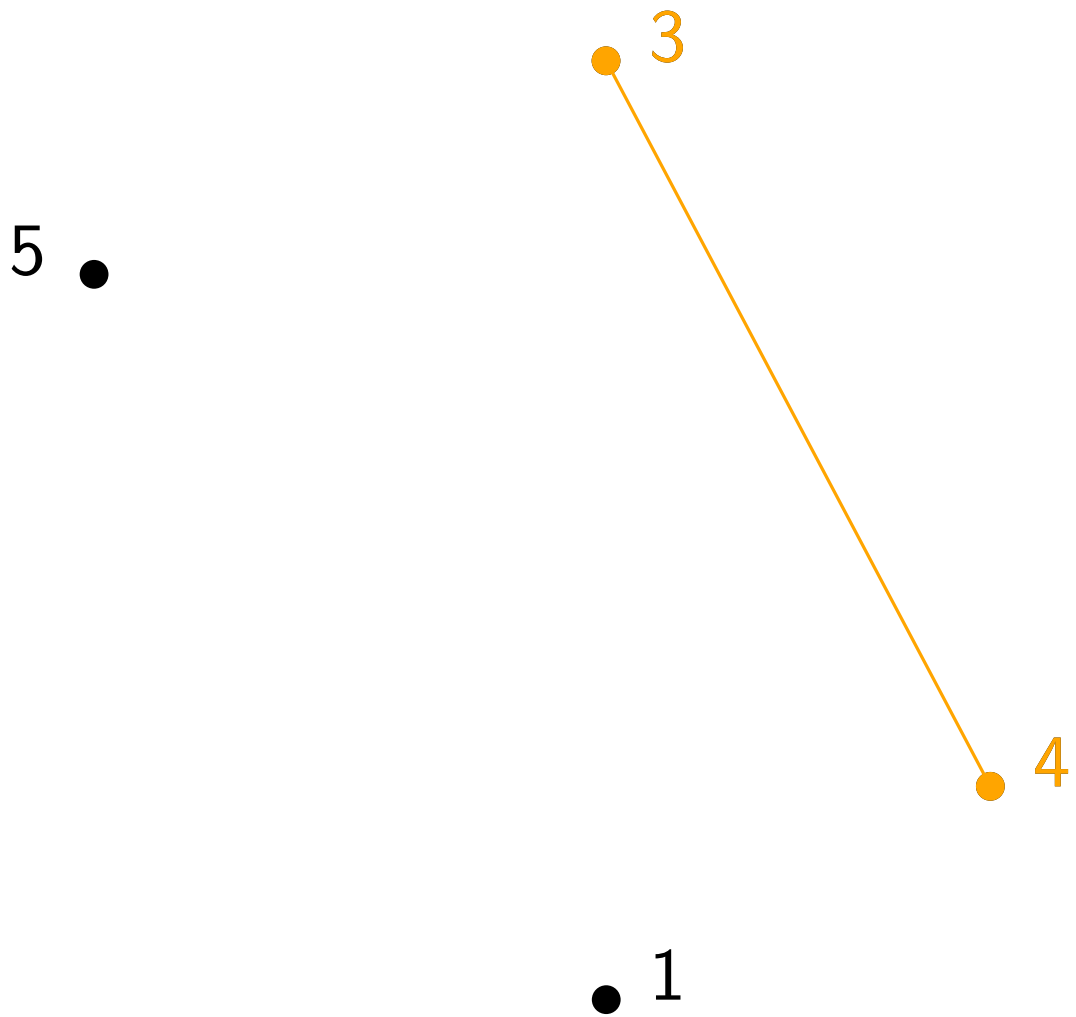

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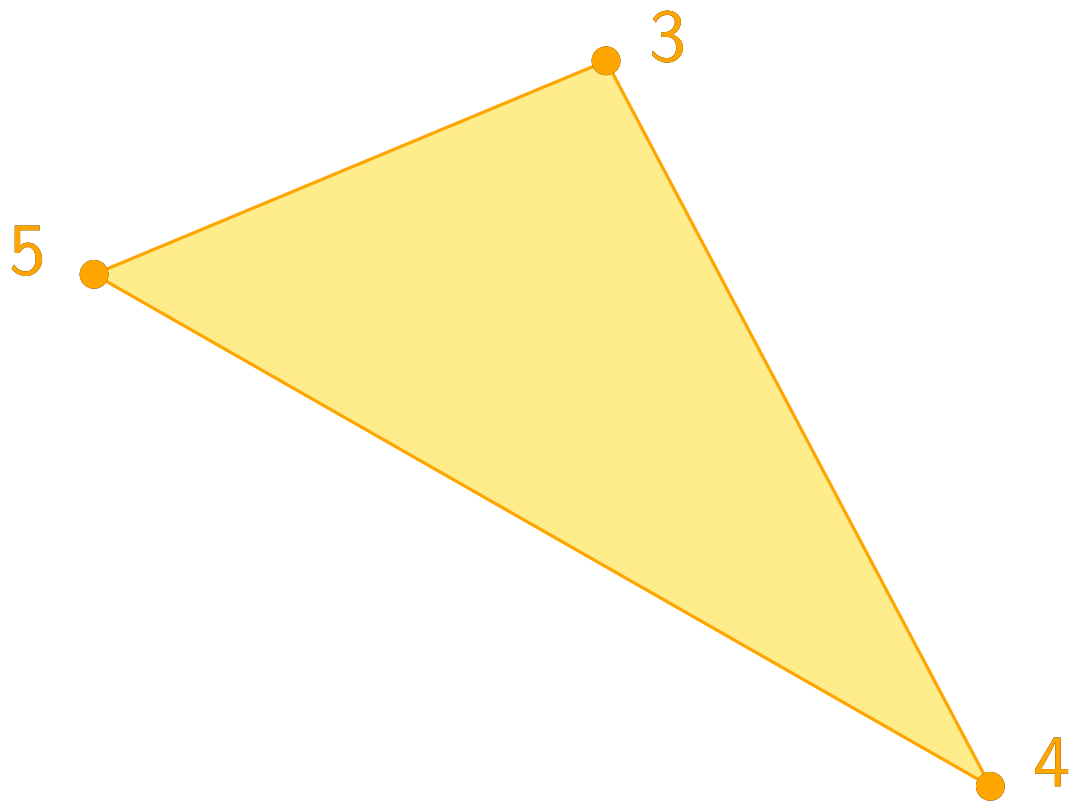


i	j
1	3
2	4
3	
4	
5	

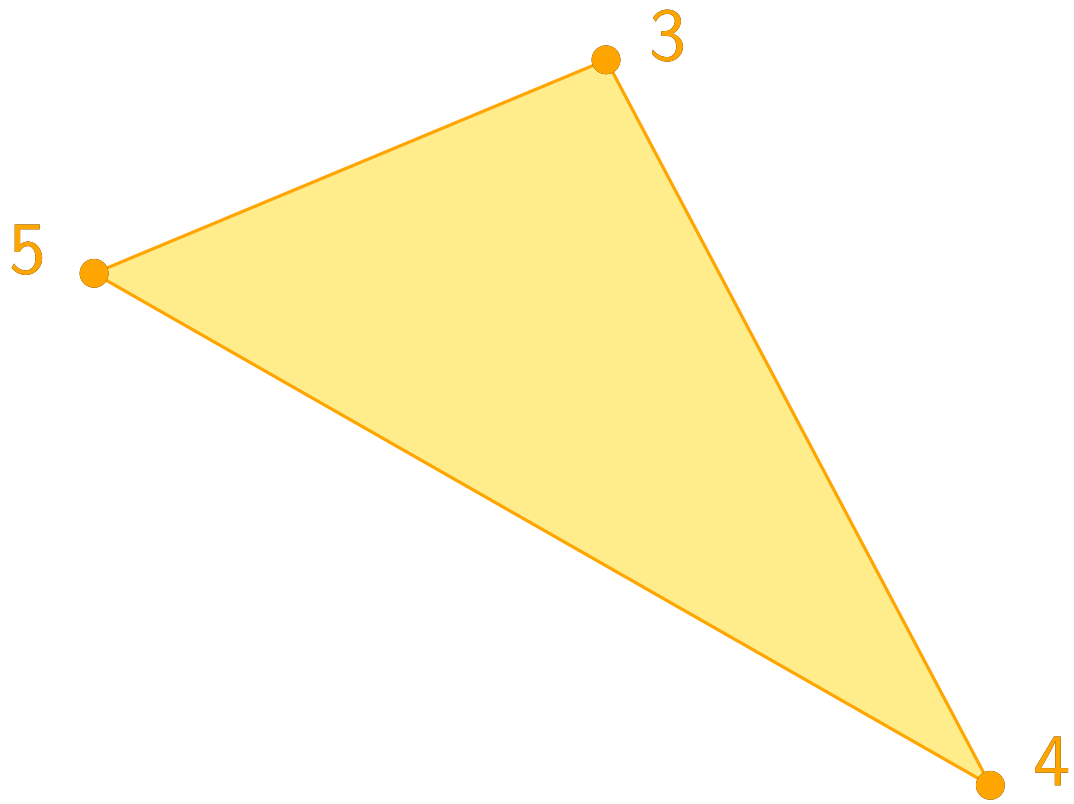
1

2

Dynamic DS : Has property? Yes

Time-Windowed $\rightarrow$ Dynamic

CH area > 1?



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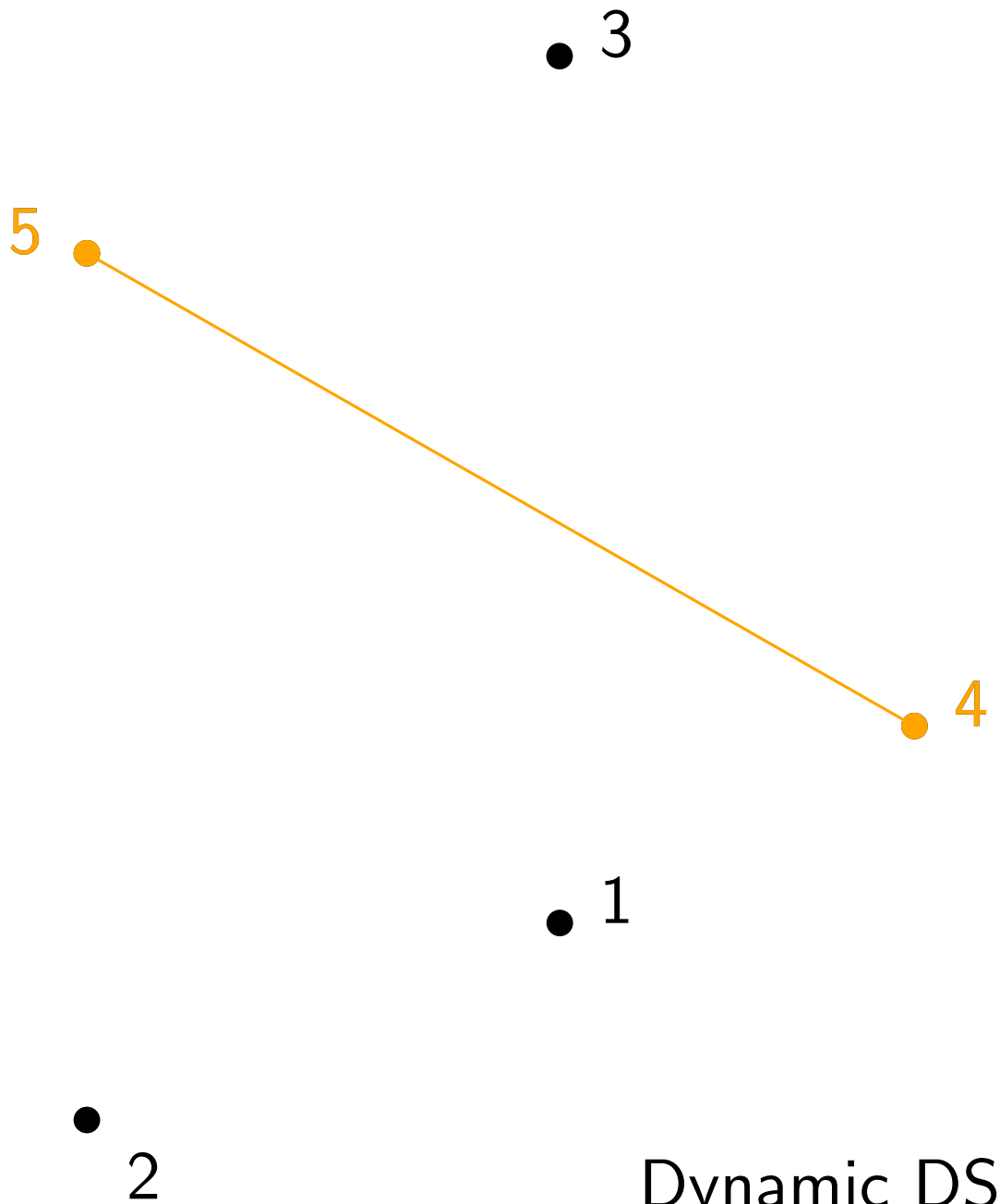

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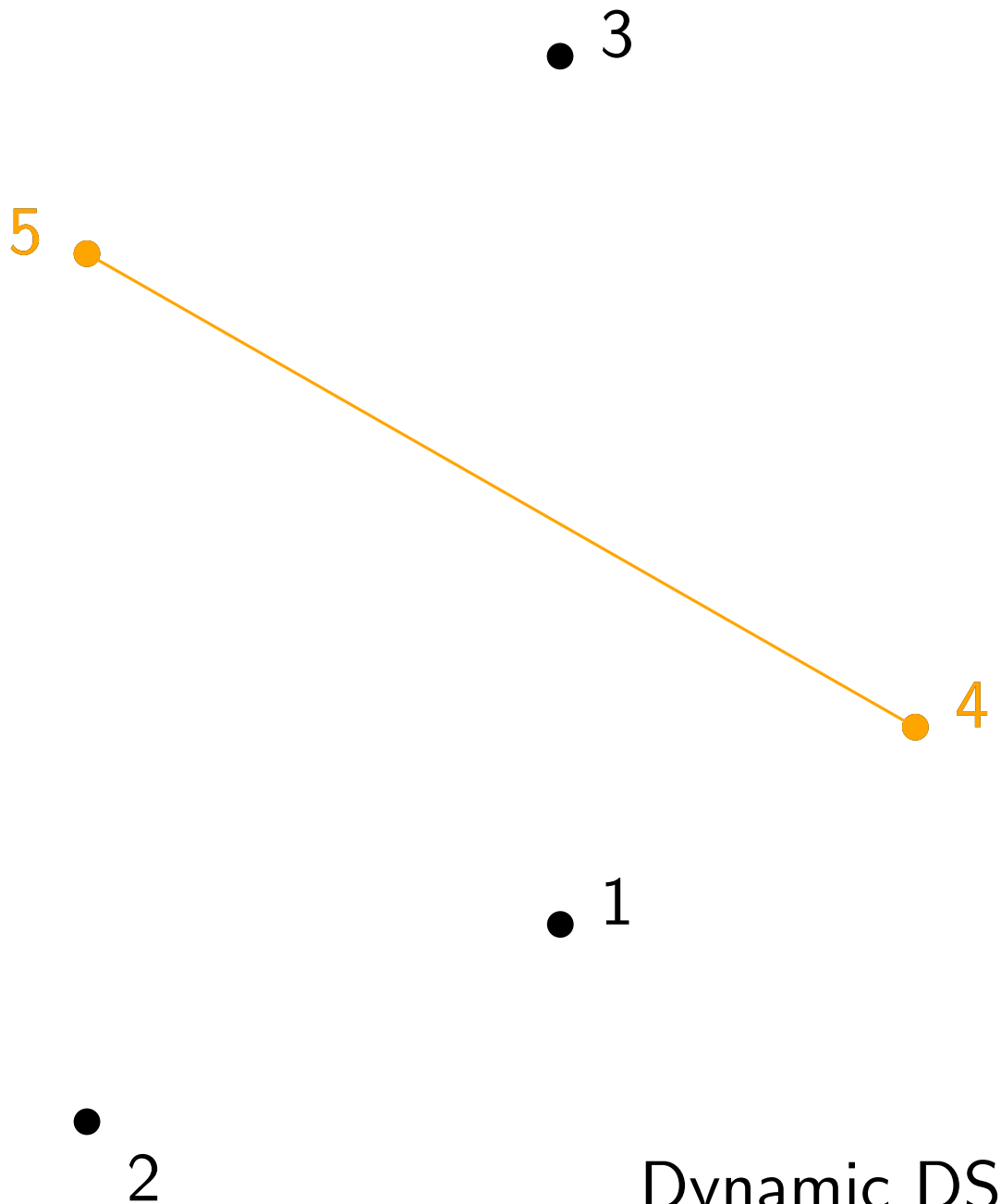

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1	3
2	4
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5	

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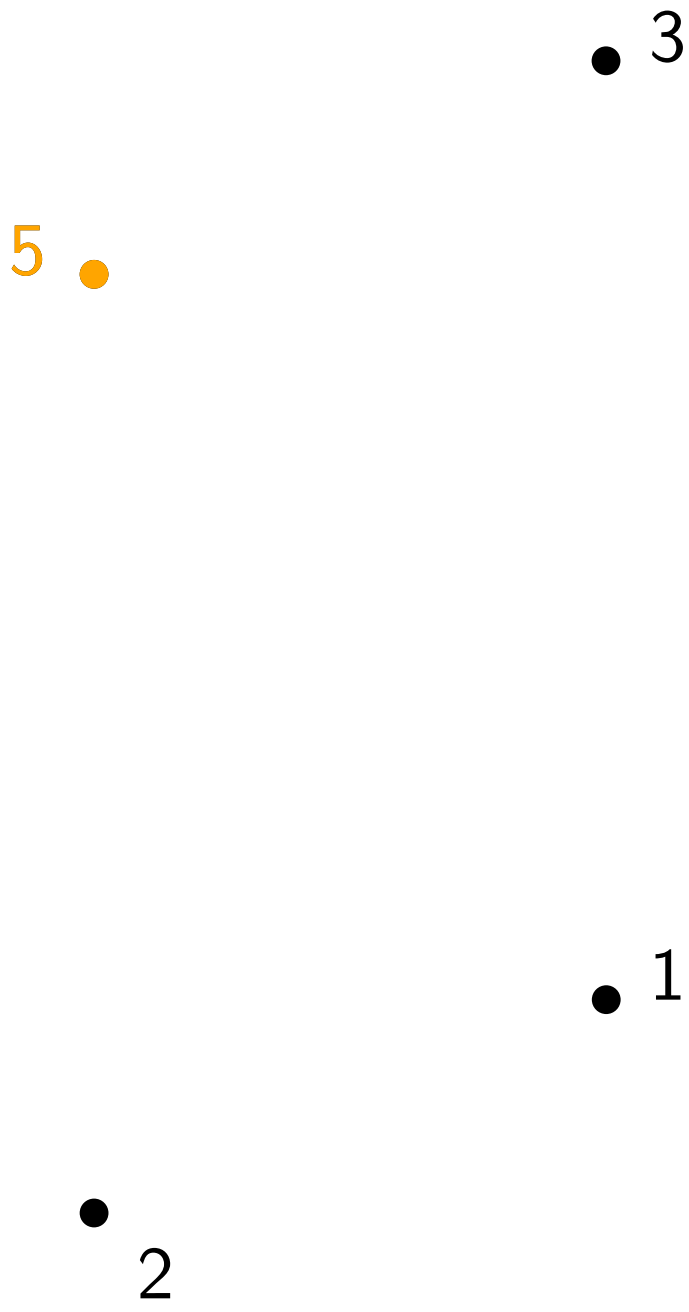

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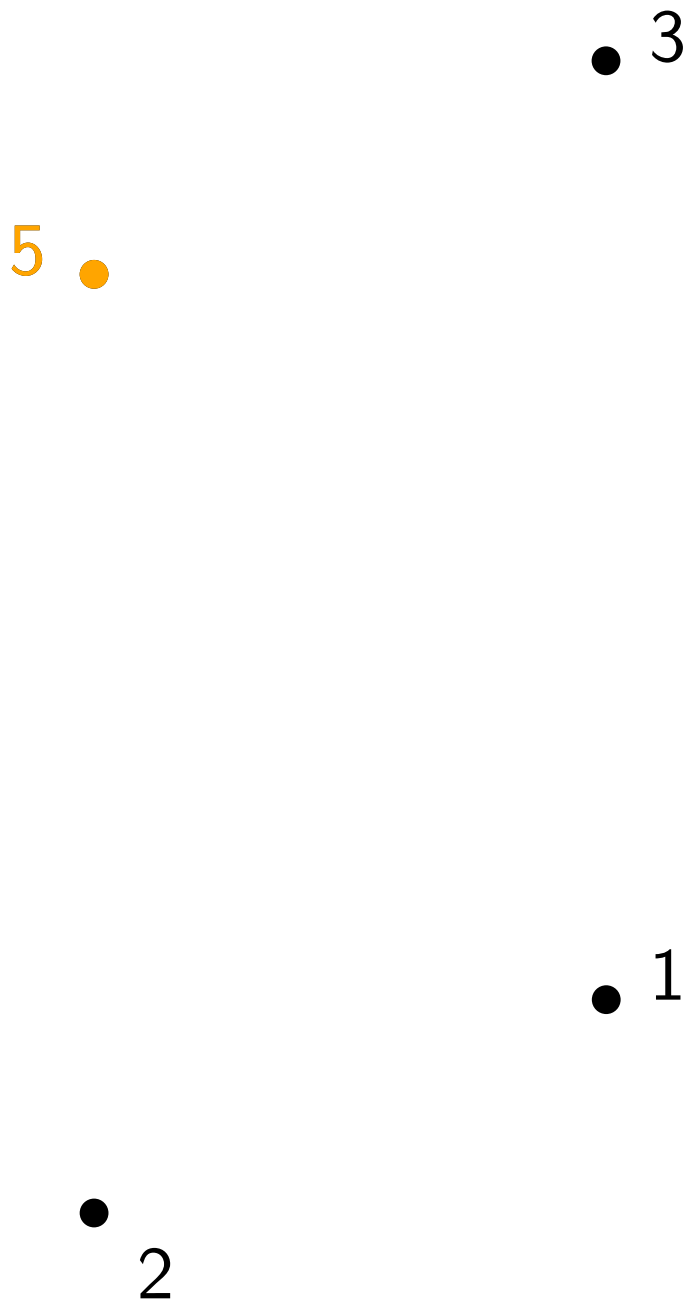

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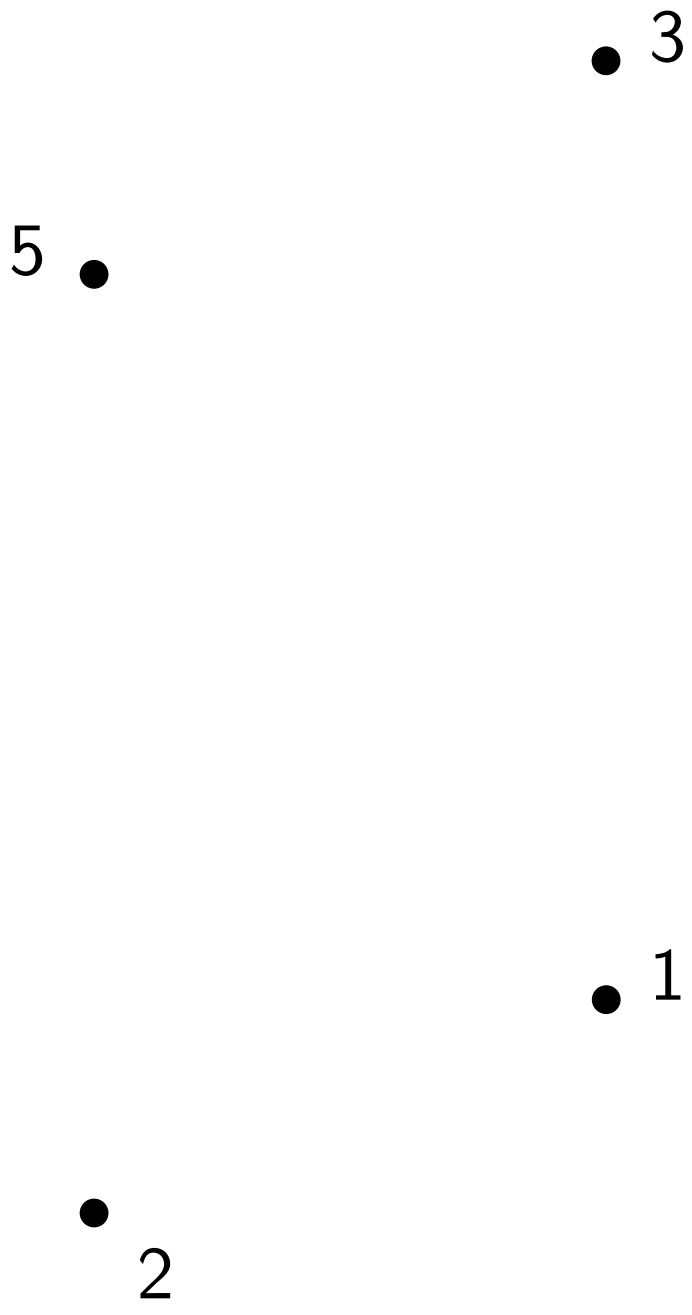

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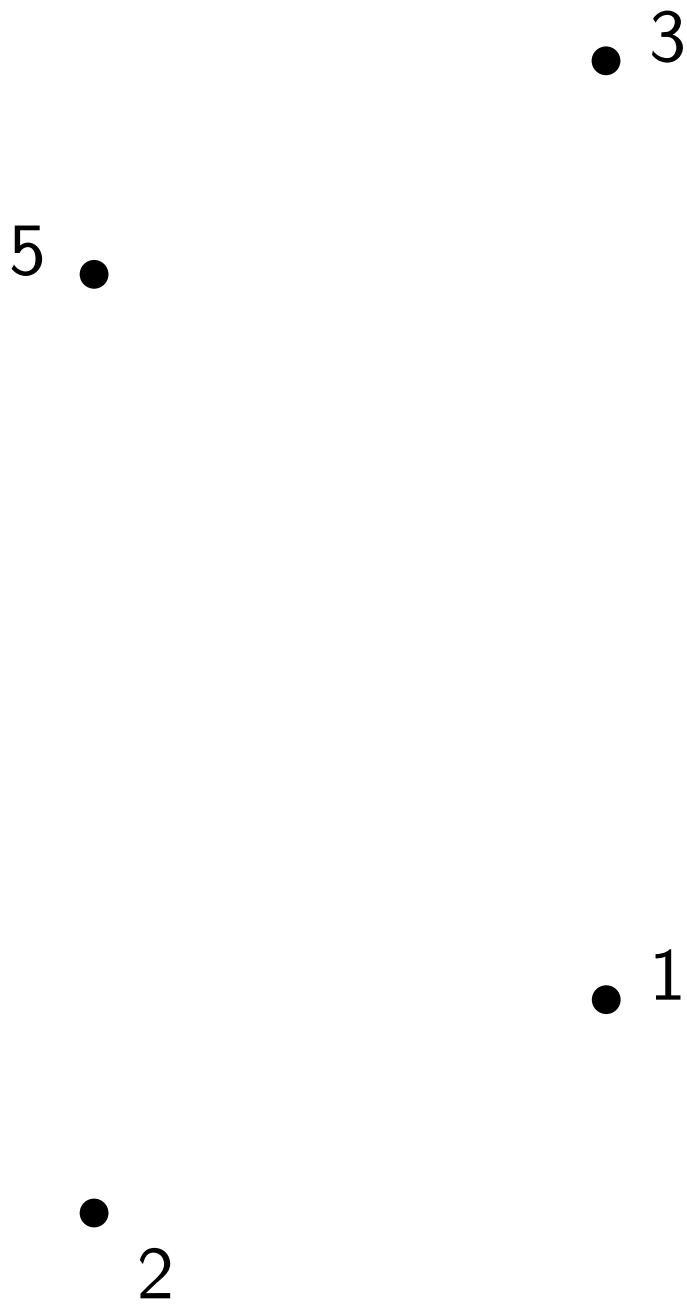

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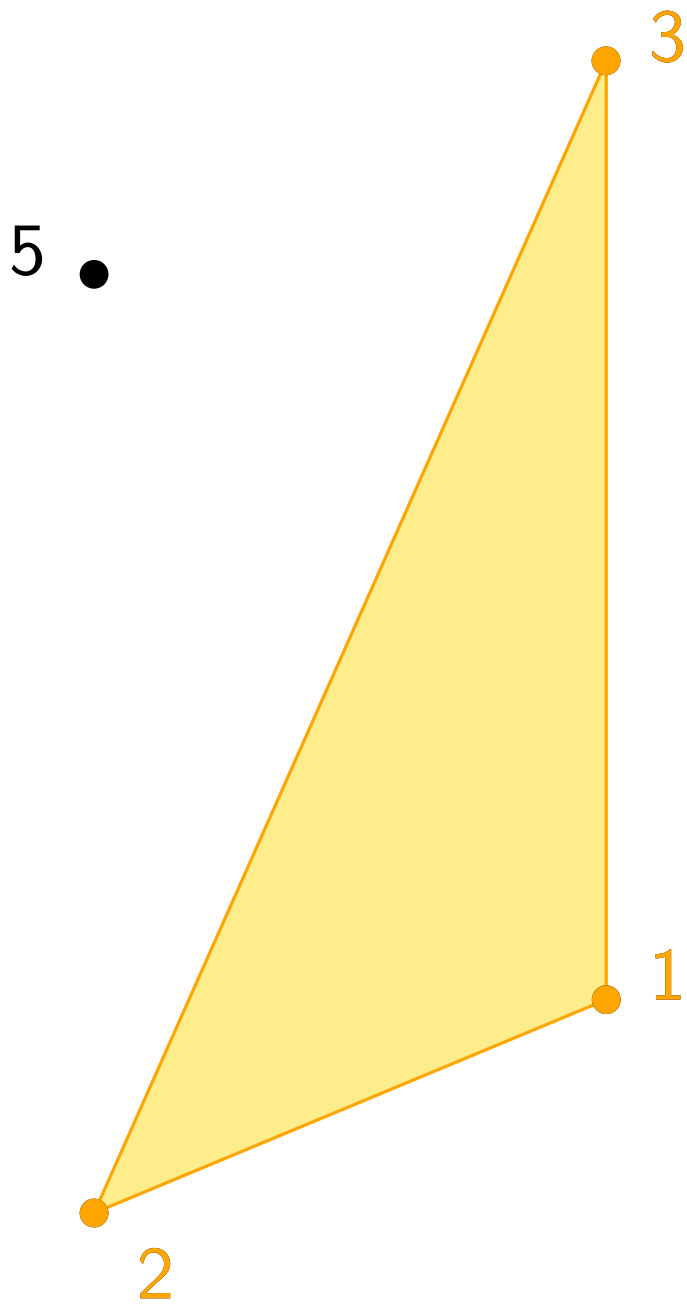

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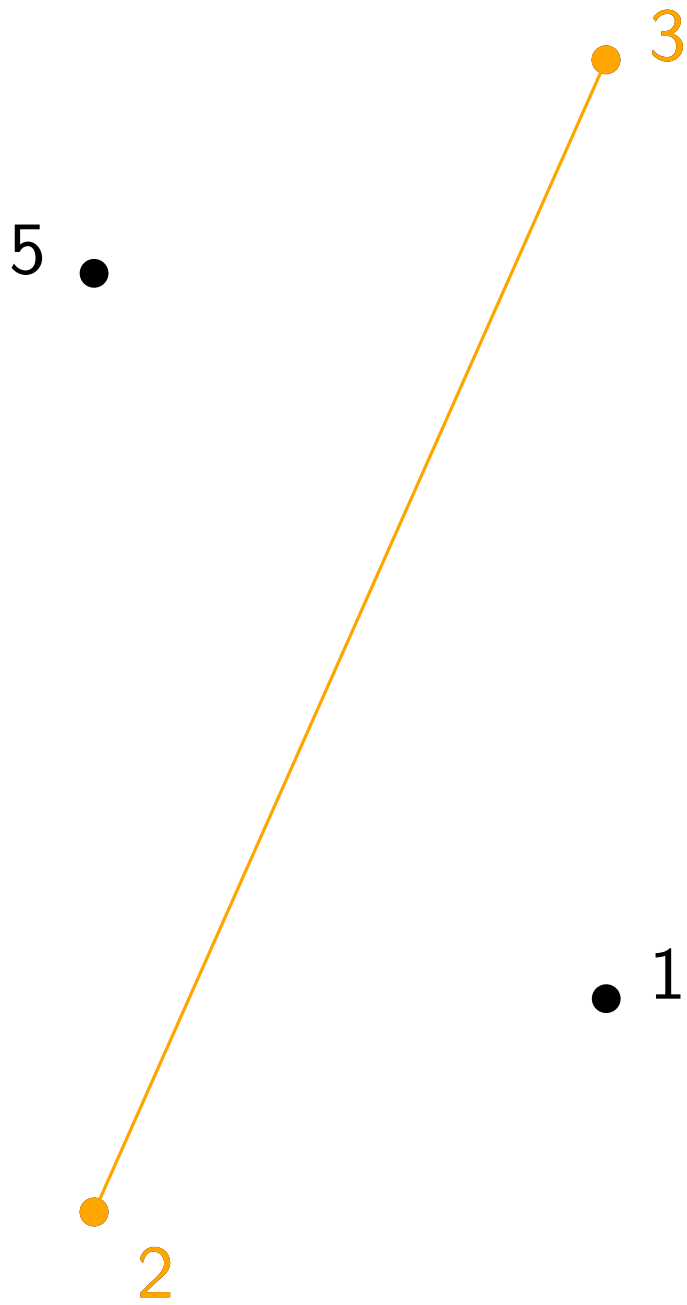

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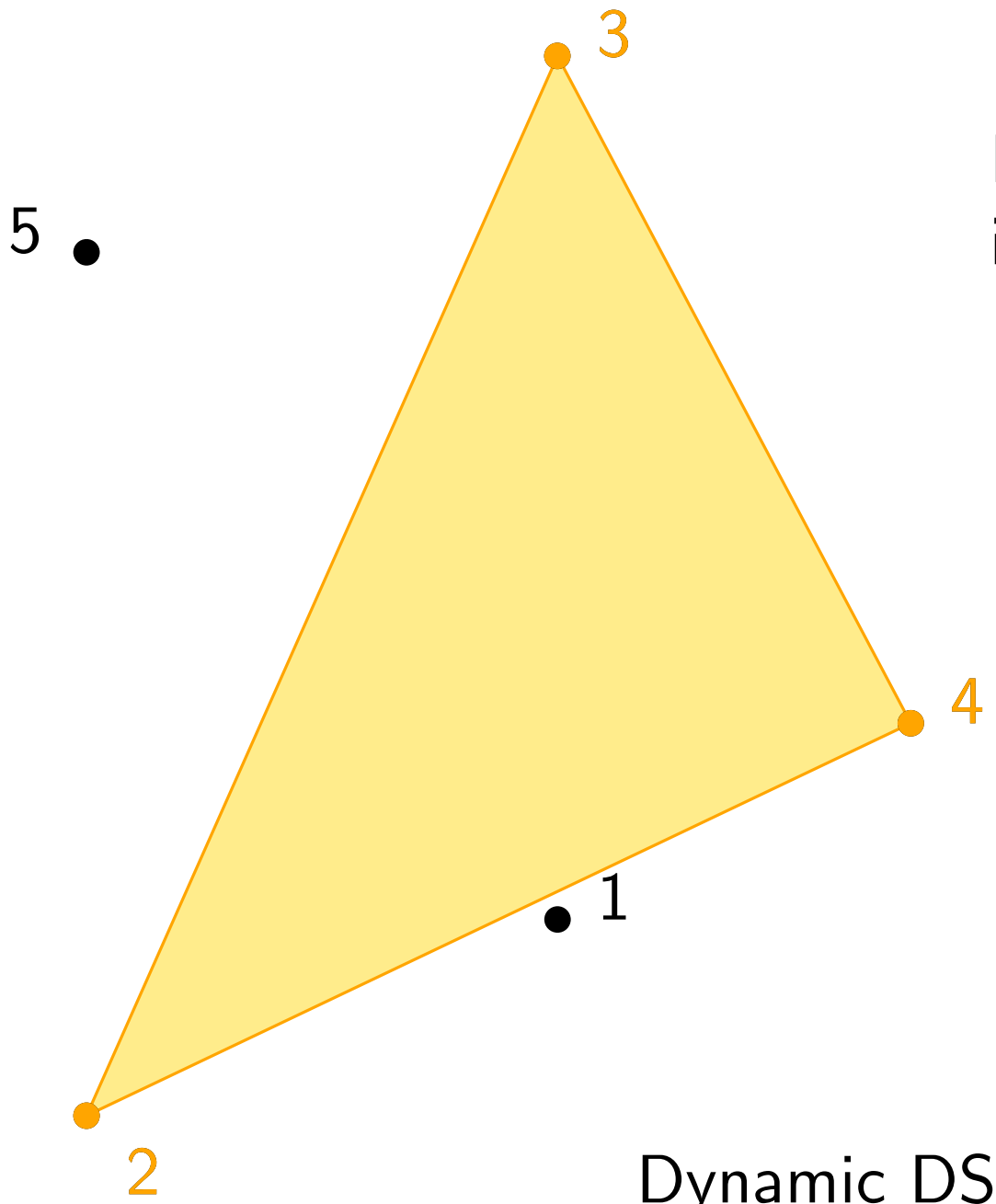

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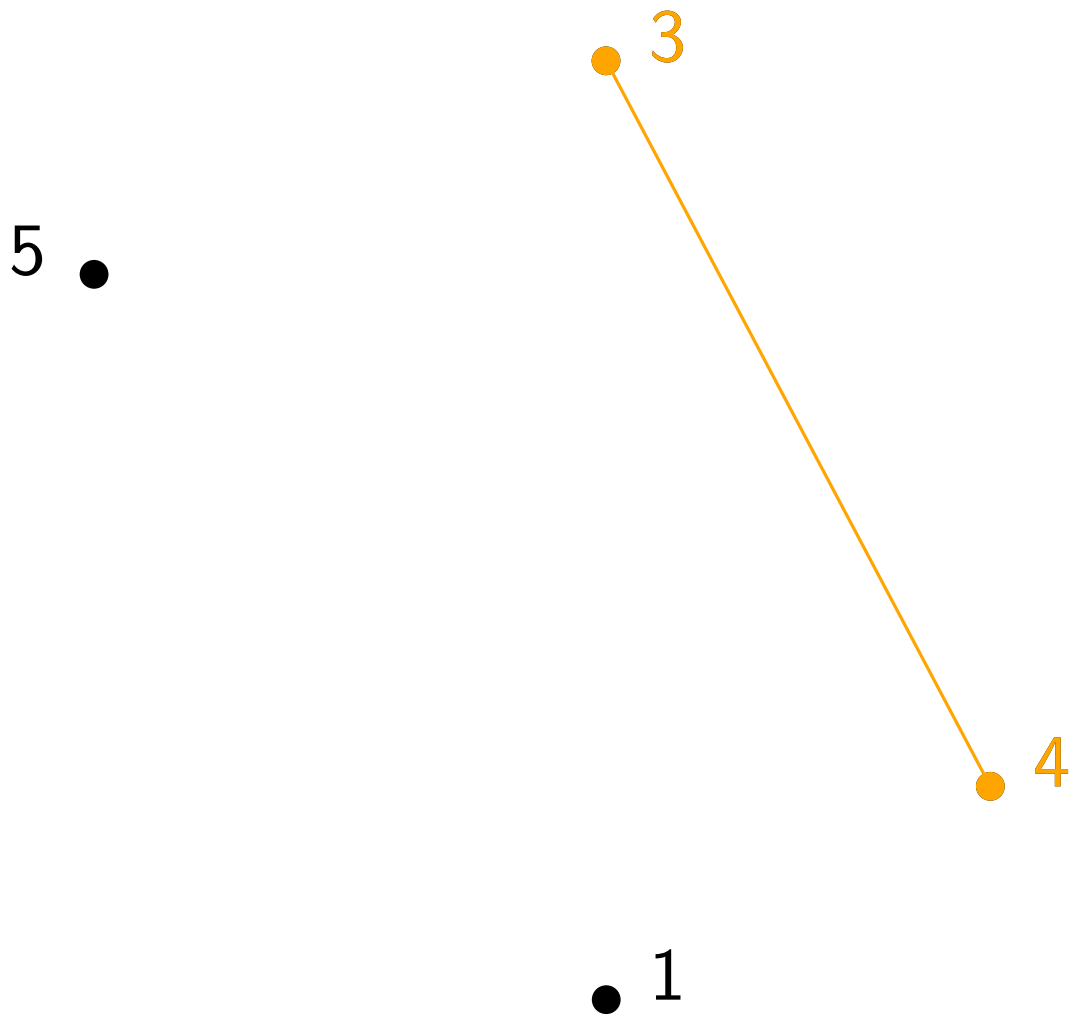

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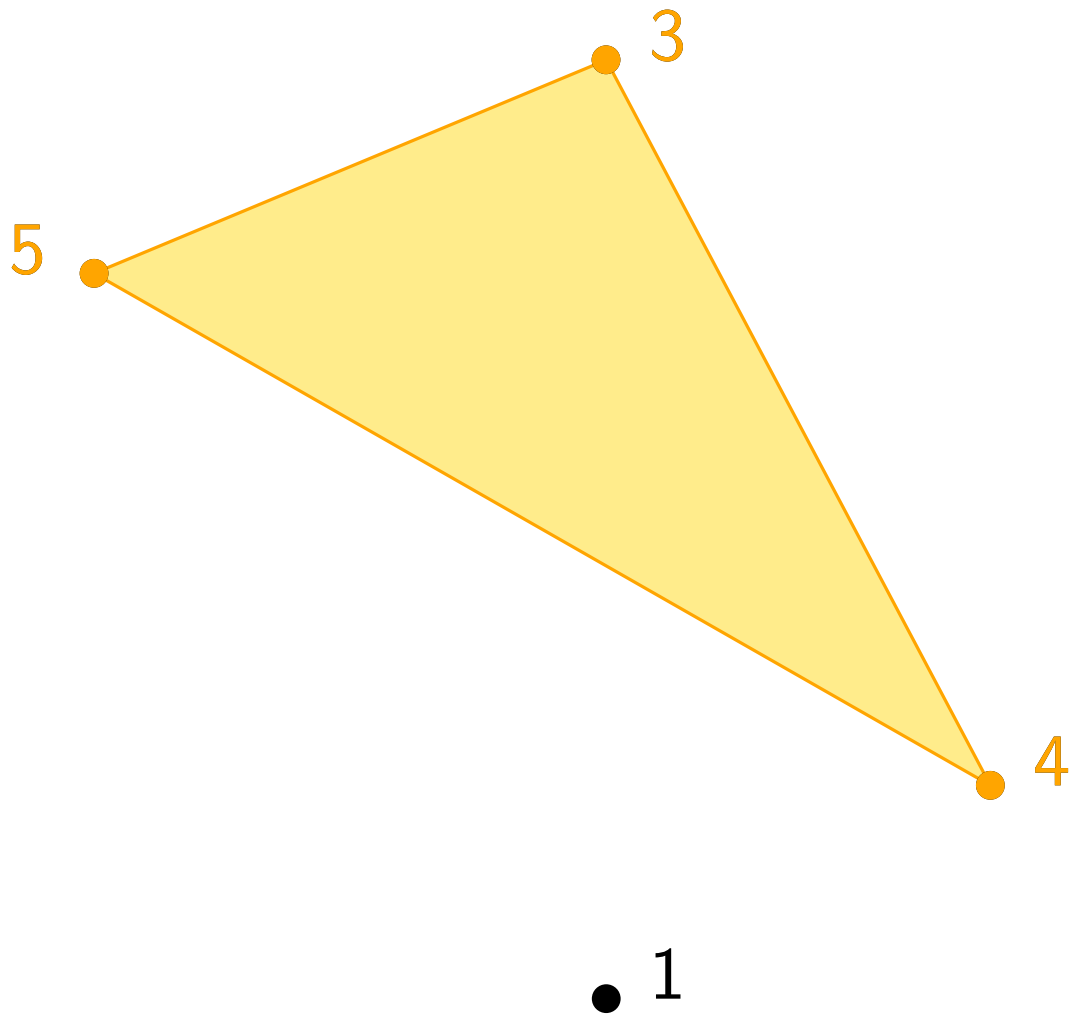

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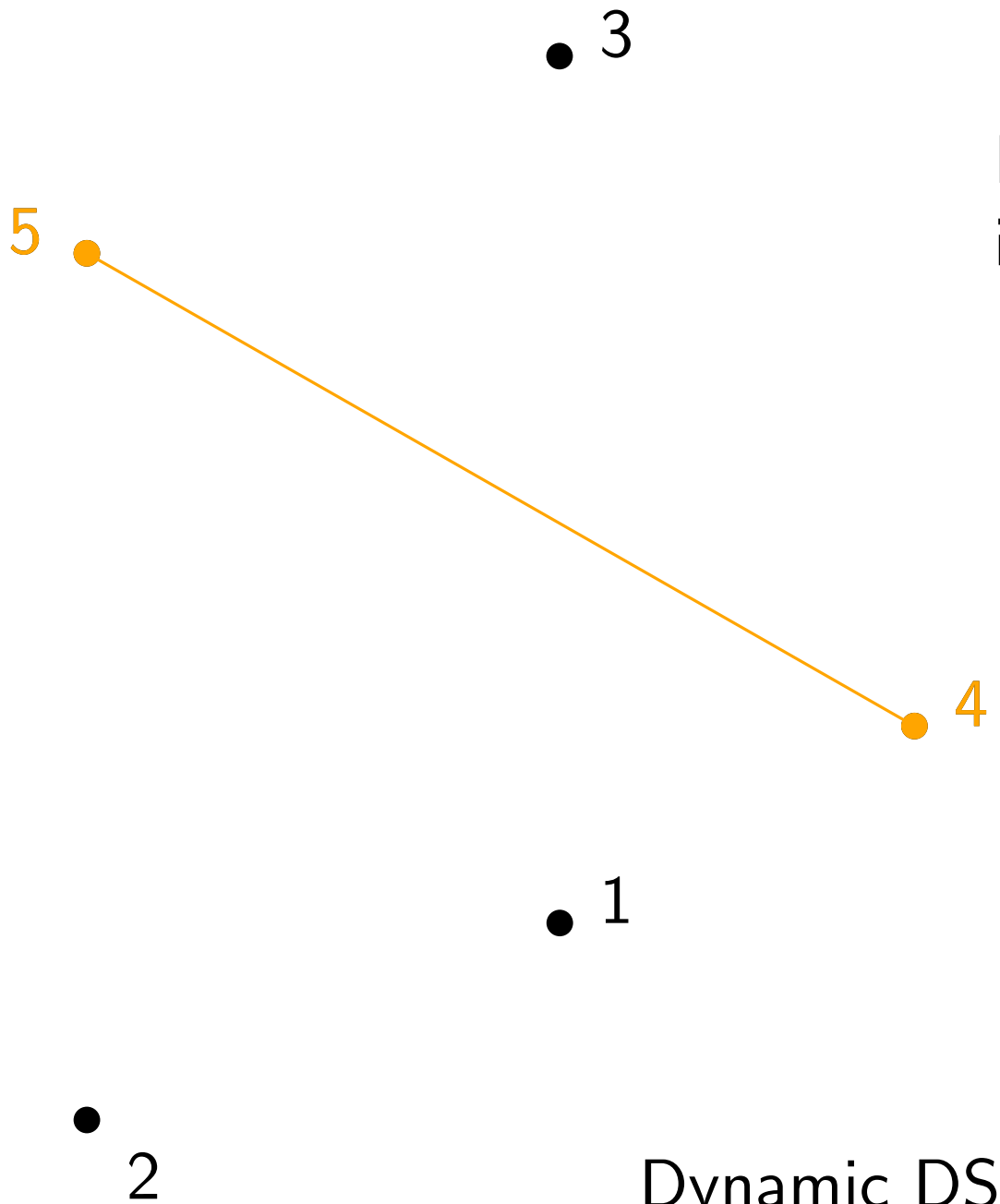

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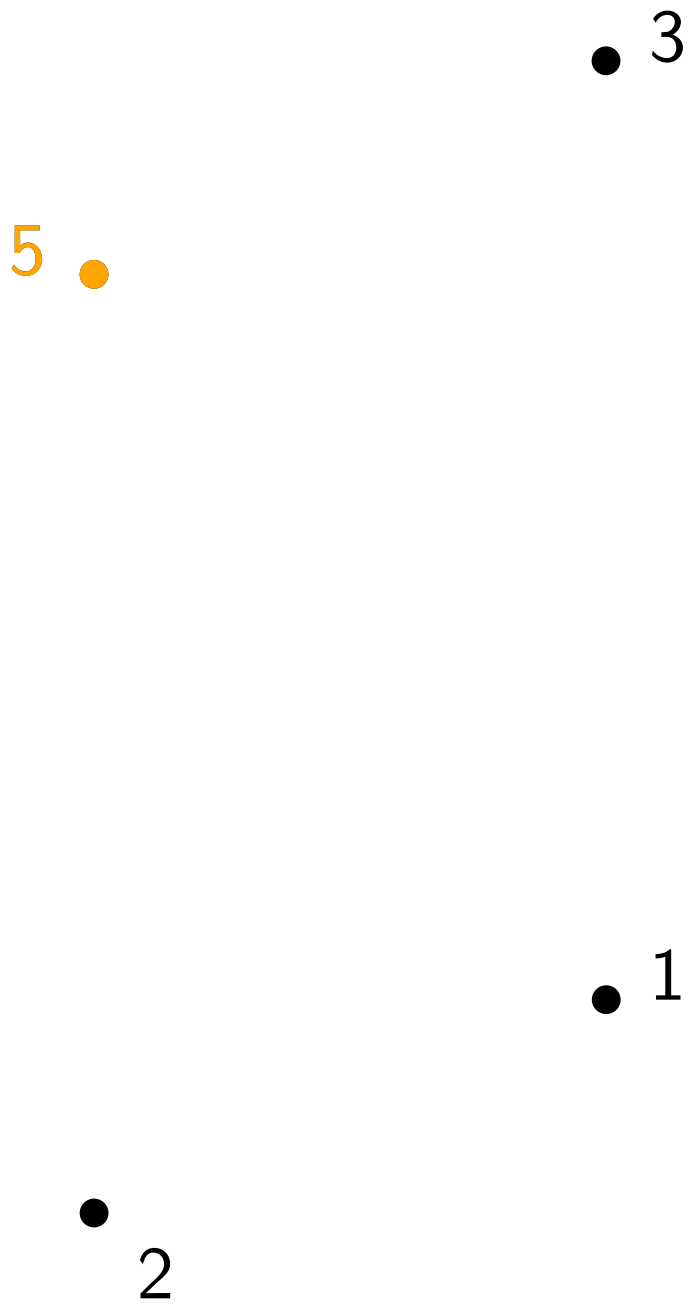

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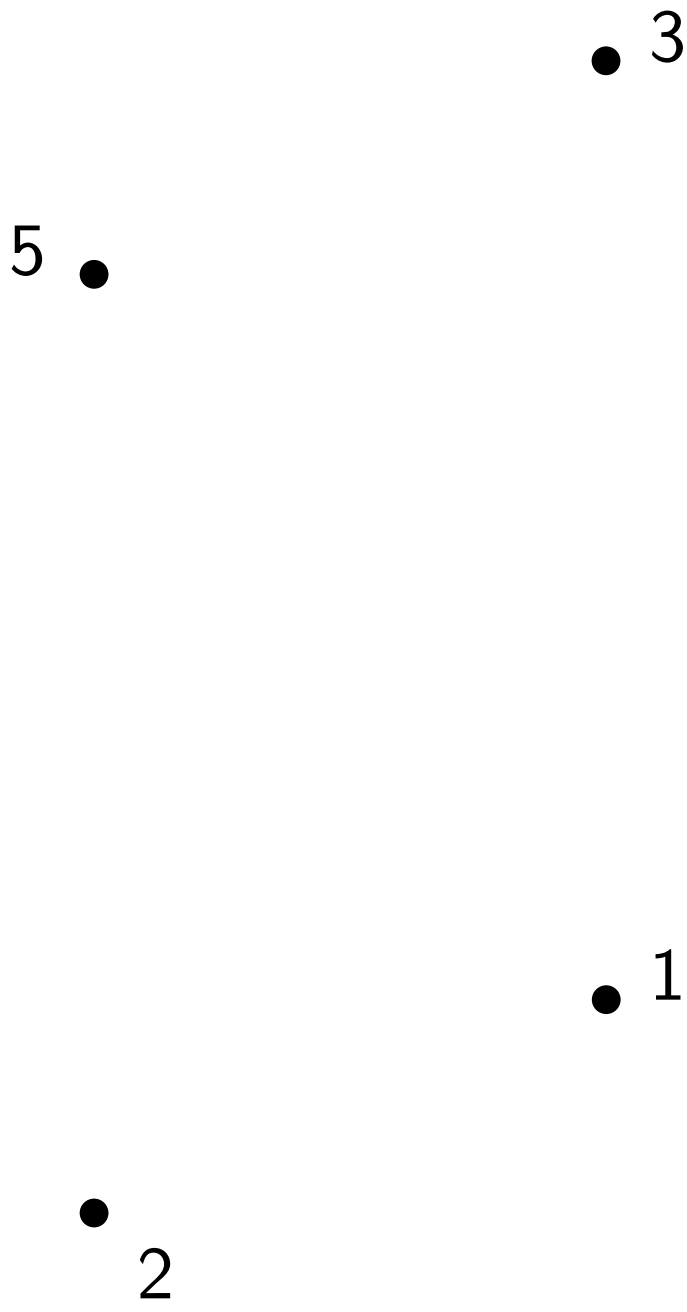

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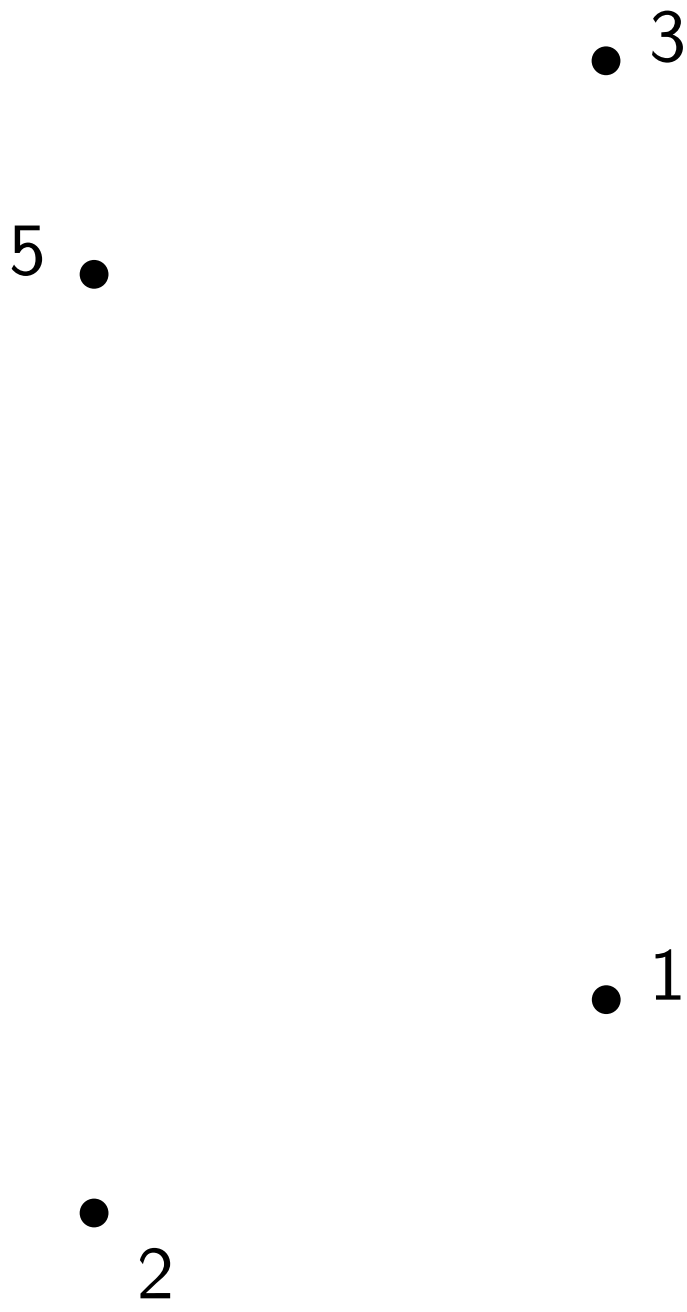

Notice the order of insertion/deletion

First-in, first-out: like a queue

i	j
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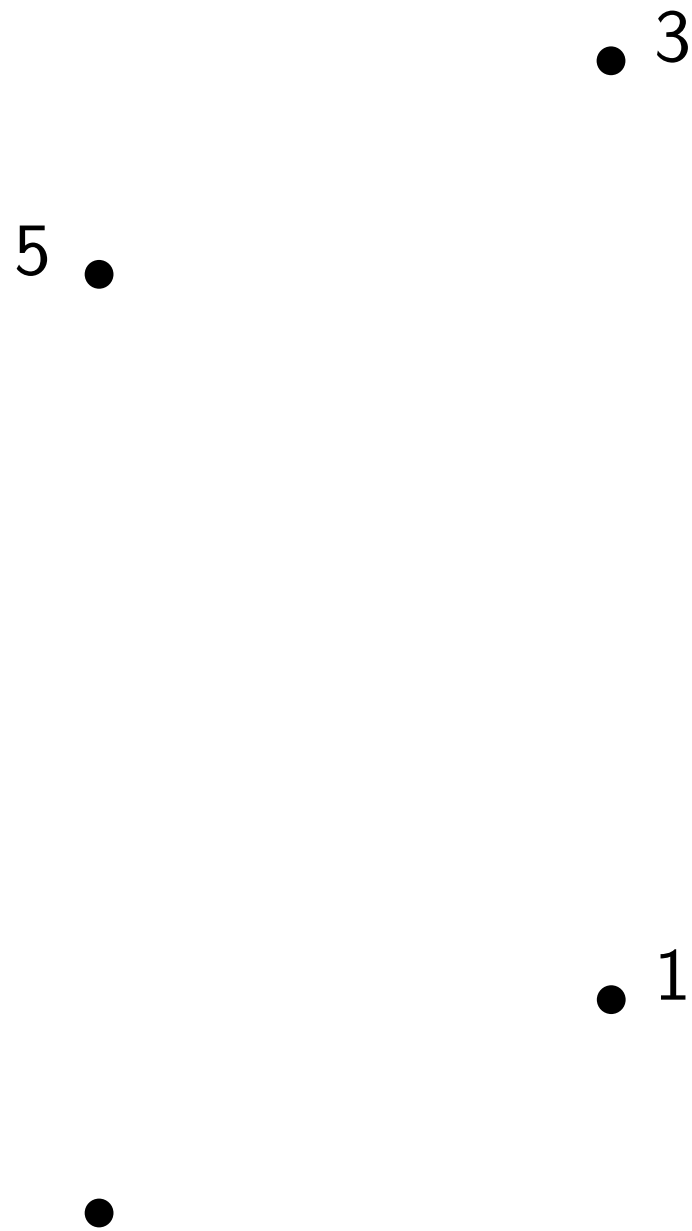
5	6

Dynamic DS : Has property? No

Time-Windowed → Dynamic

CH area > 1?



Notice the order of insertion/deletion

First-in, first-out: like a queue

FIFO update sequence

i	j
1	3
2	4
3	5
4	6
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Dynamic DS : Has property? No

Dynamic Convex Hull Complexity

Generally, $O(n^2)$ changes over $O(n)$ updates

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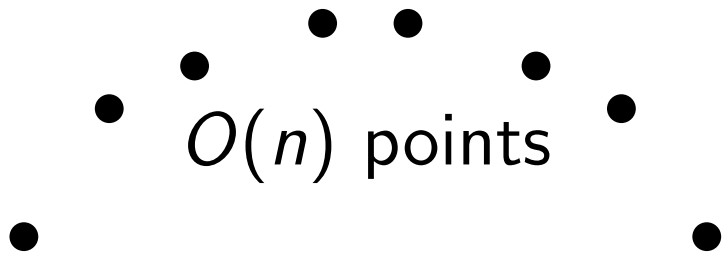

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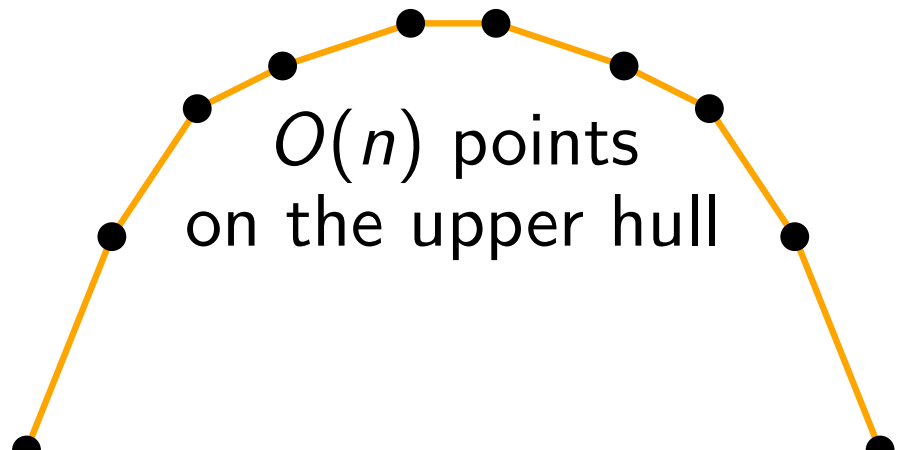

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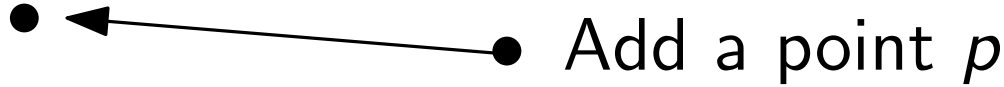

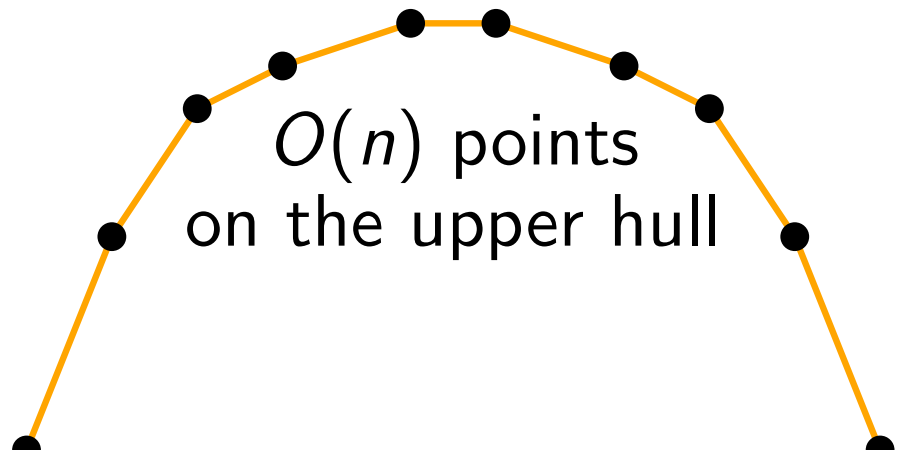

Dynamic Convex Hull Complexity

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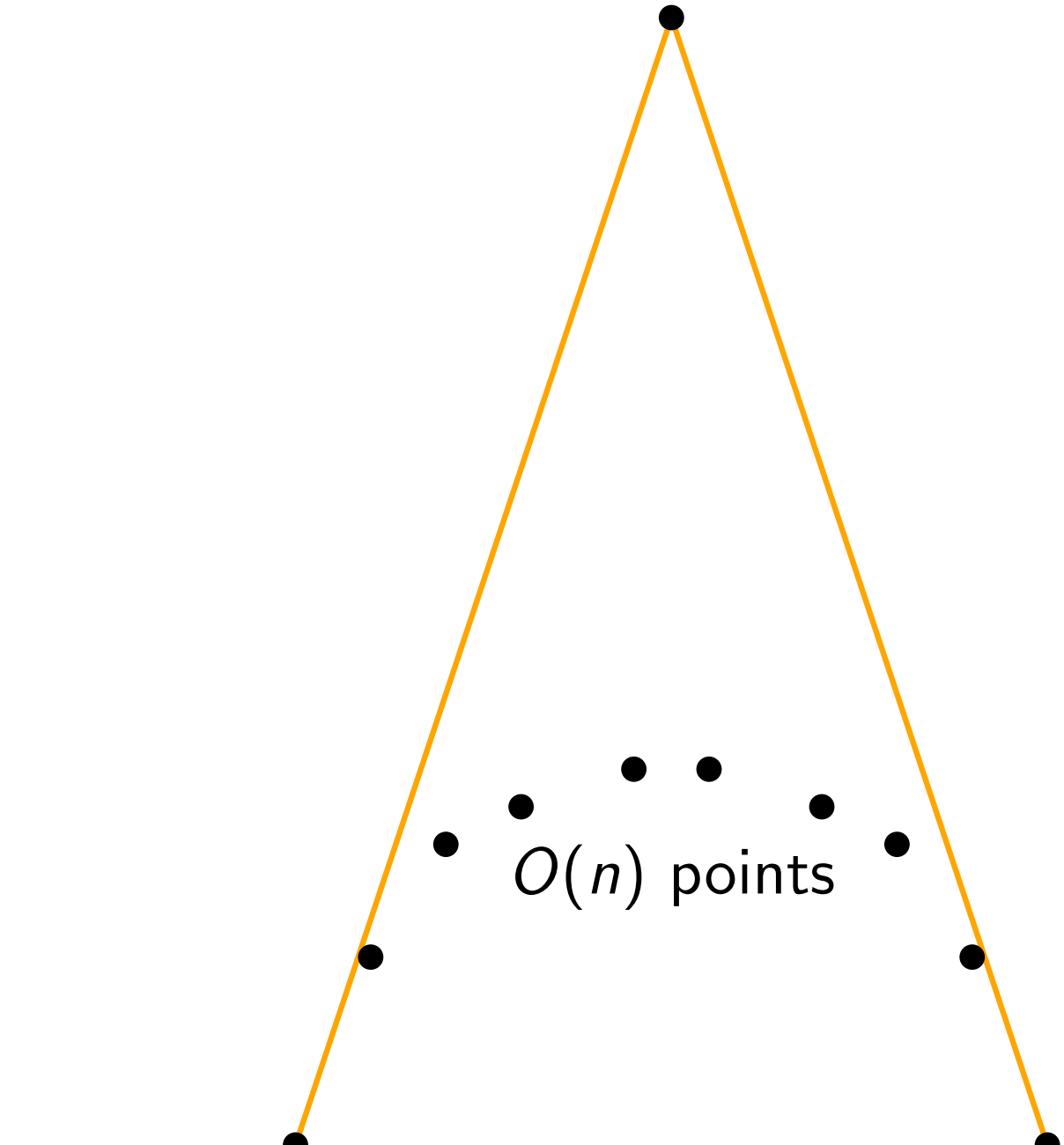
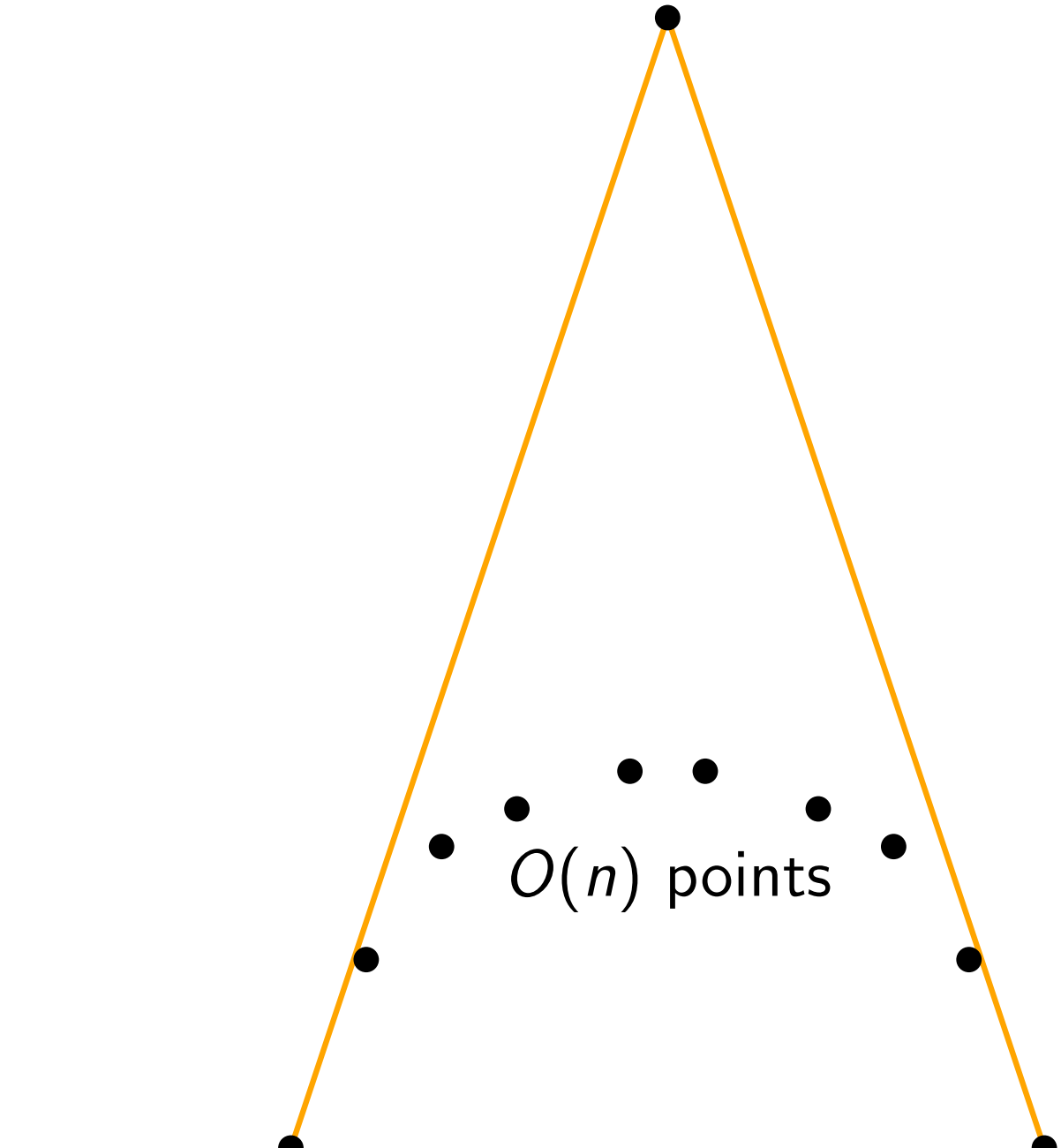


- Add a point p
- $O(n)$ changes

Dynamic Convex Hull Complexity

Generally, $O(n^2)$ changes over $O(n)$ updates



- Add a point p
- $O(n)$ changes
- Add/remove p O(n) times

Dynamic Convex Hull Complexity

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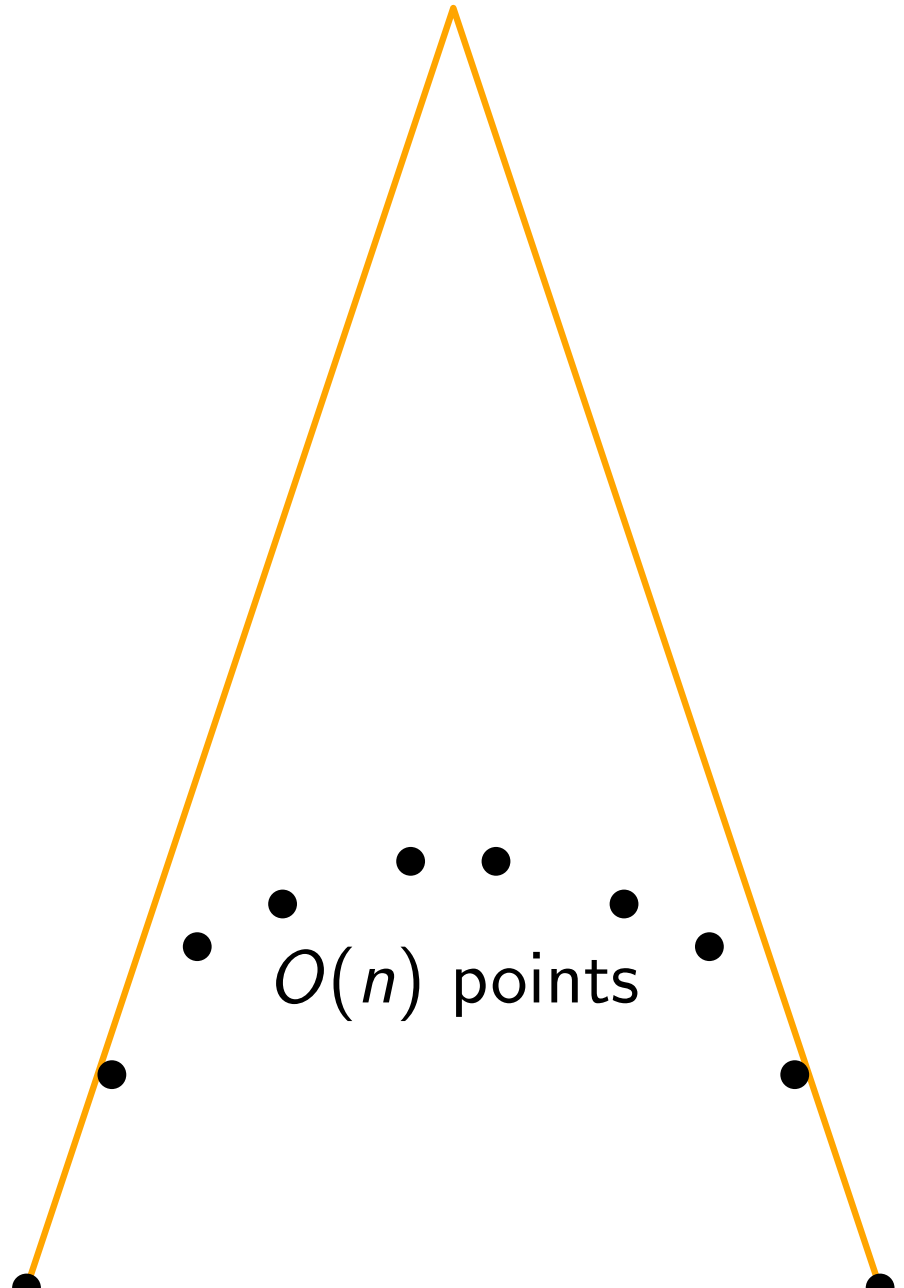

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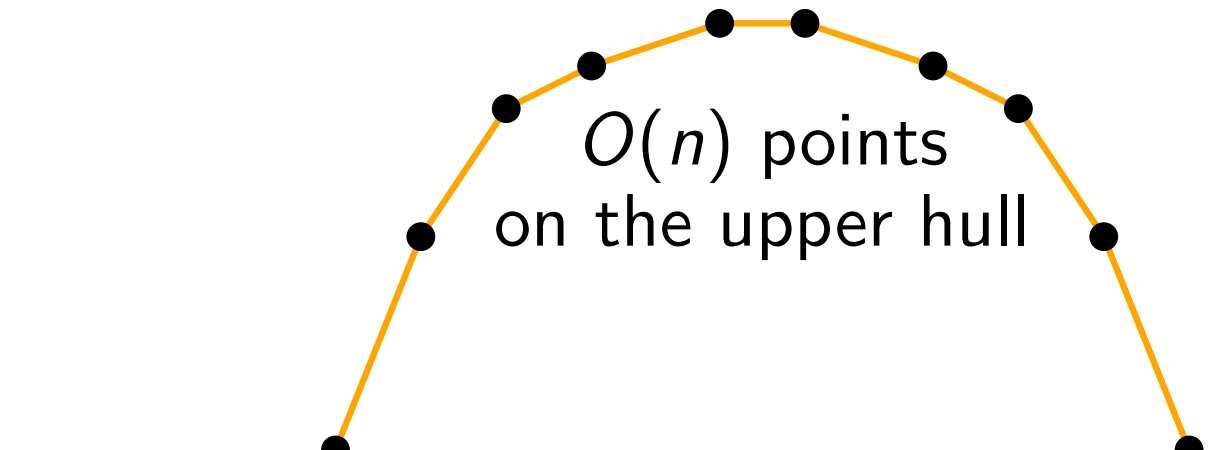

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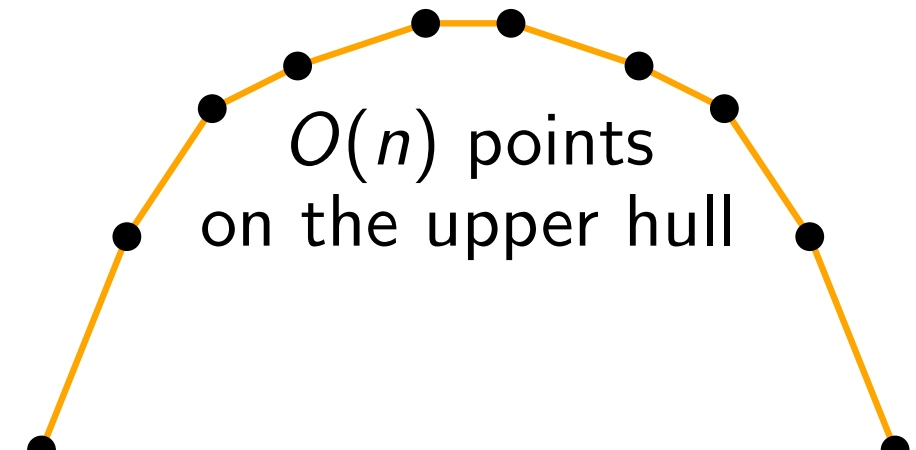

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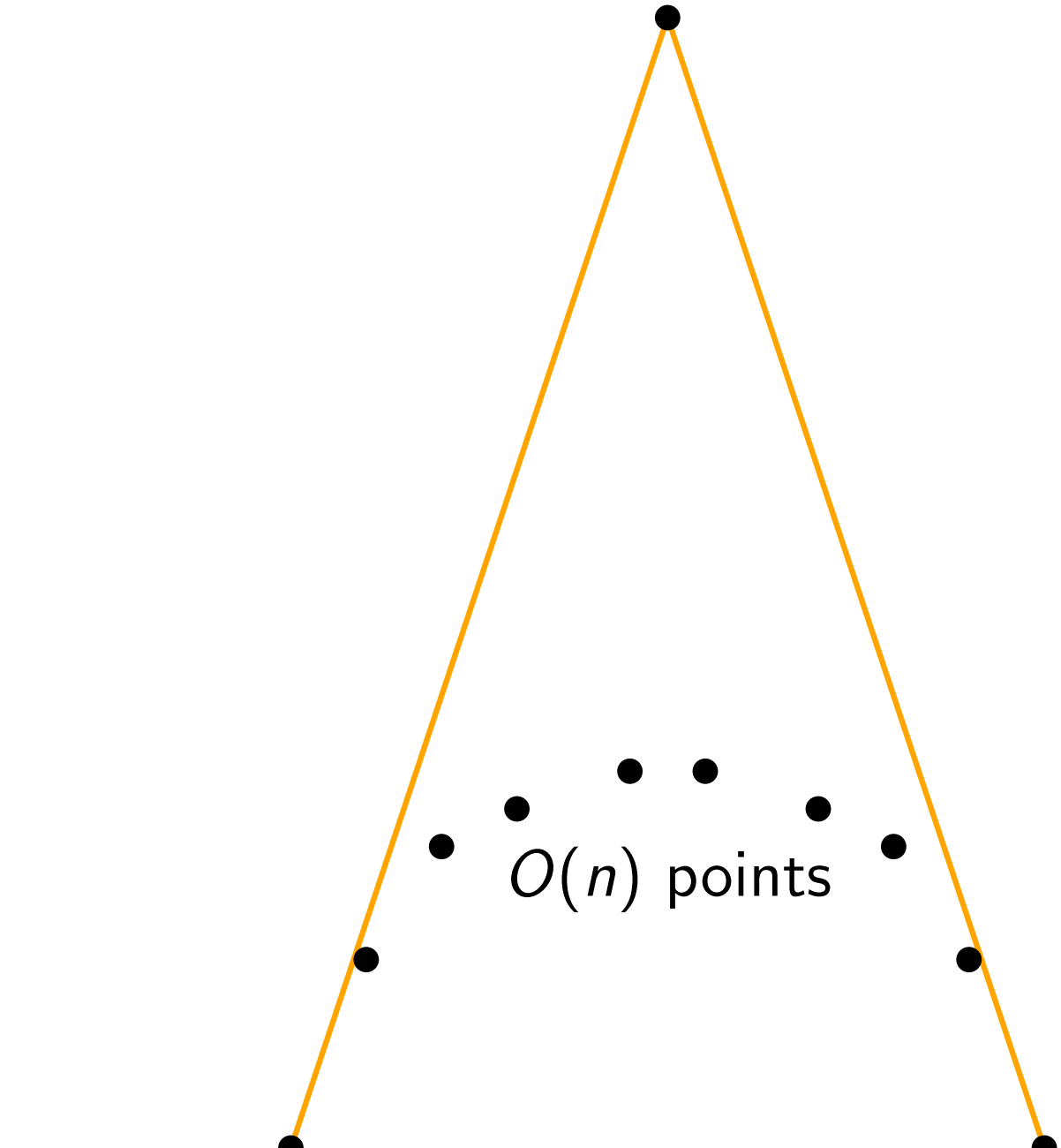

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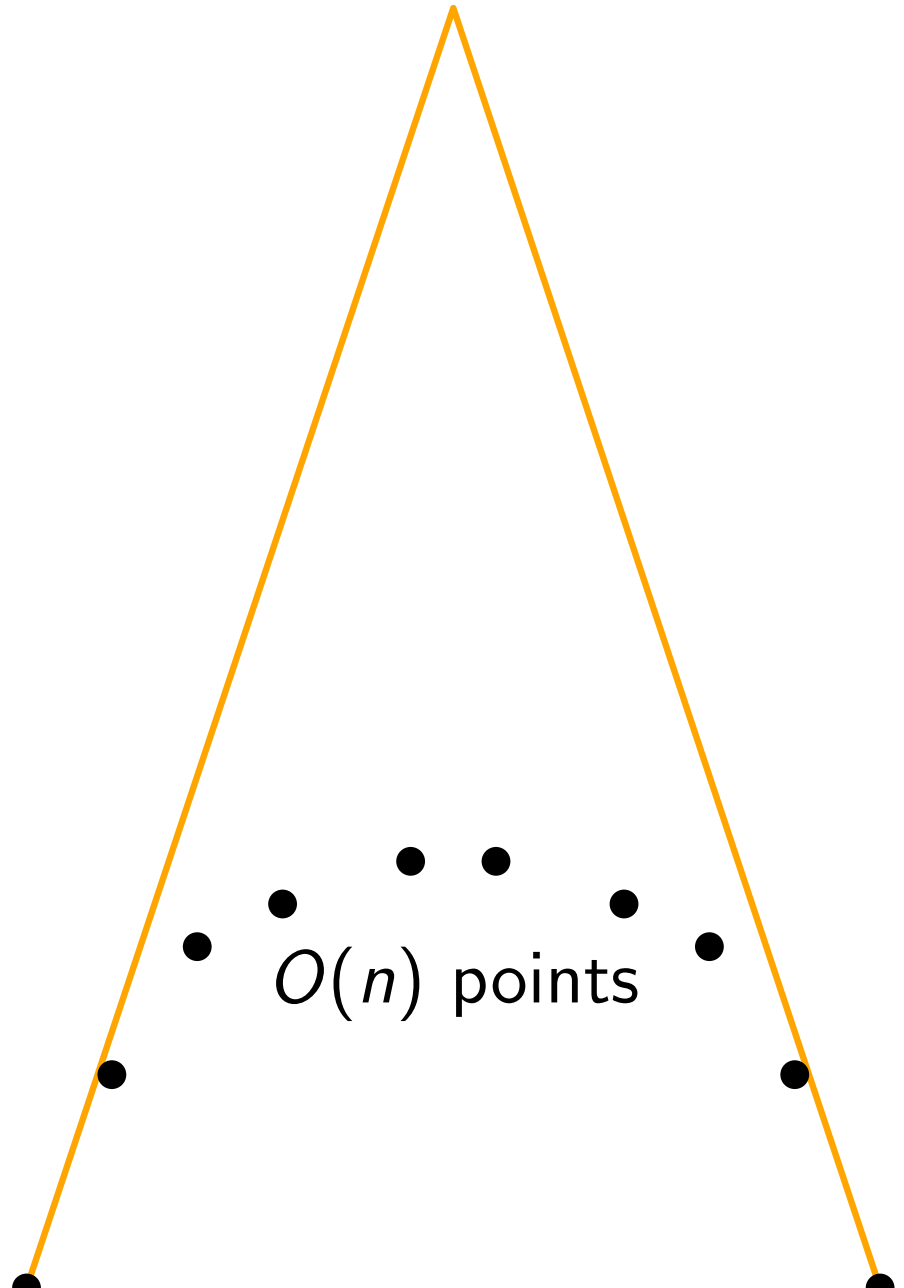

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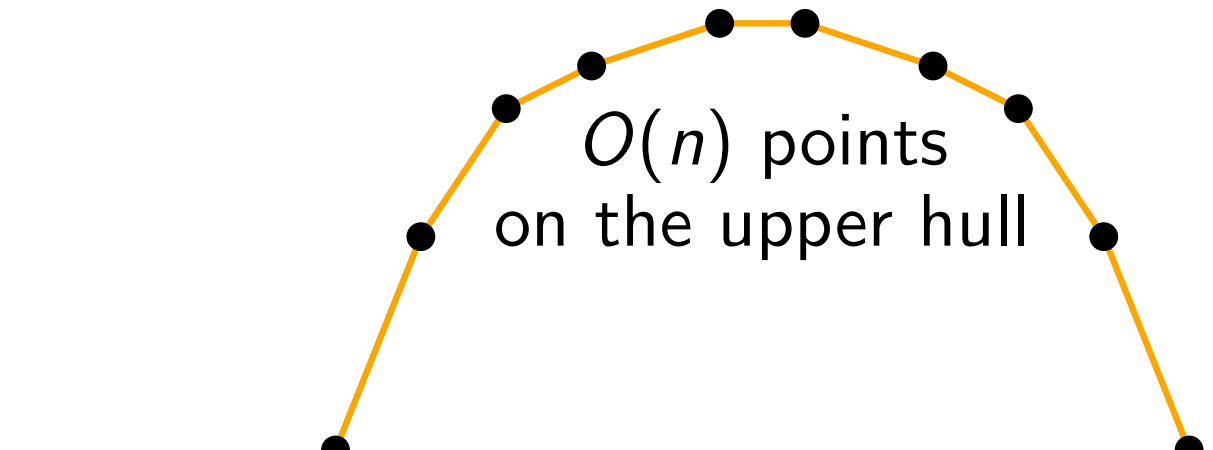

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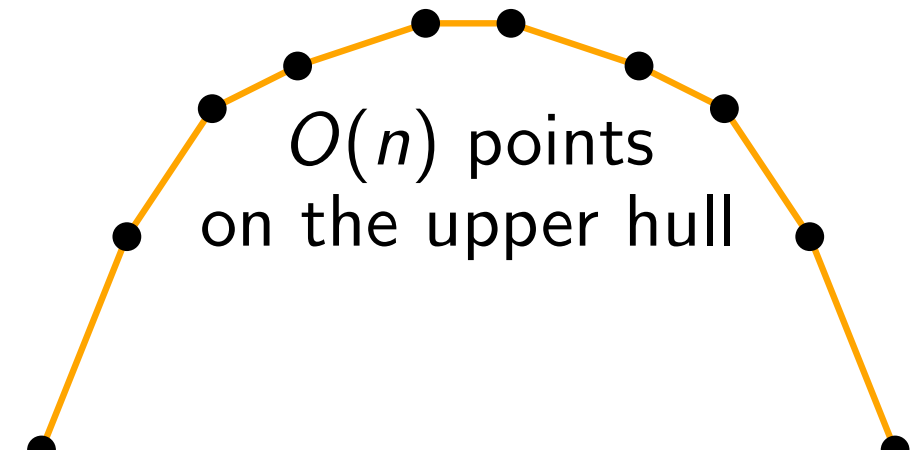

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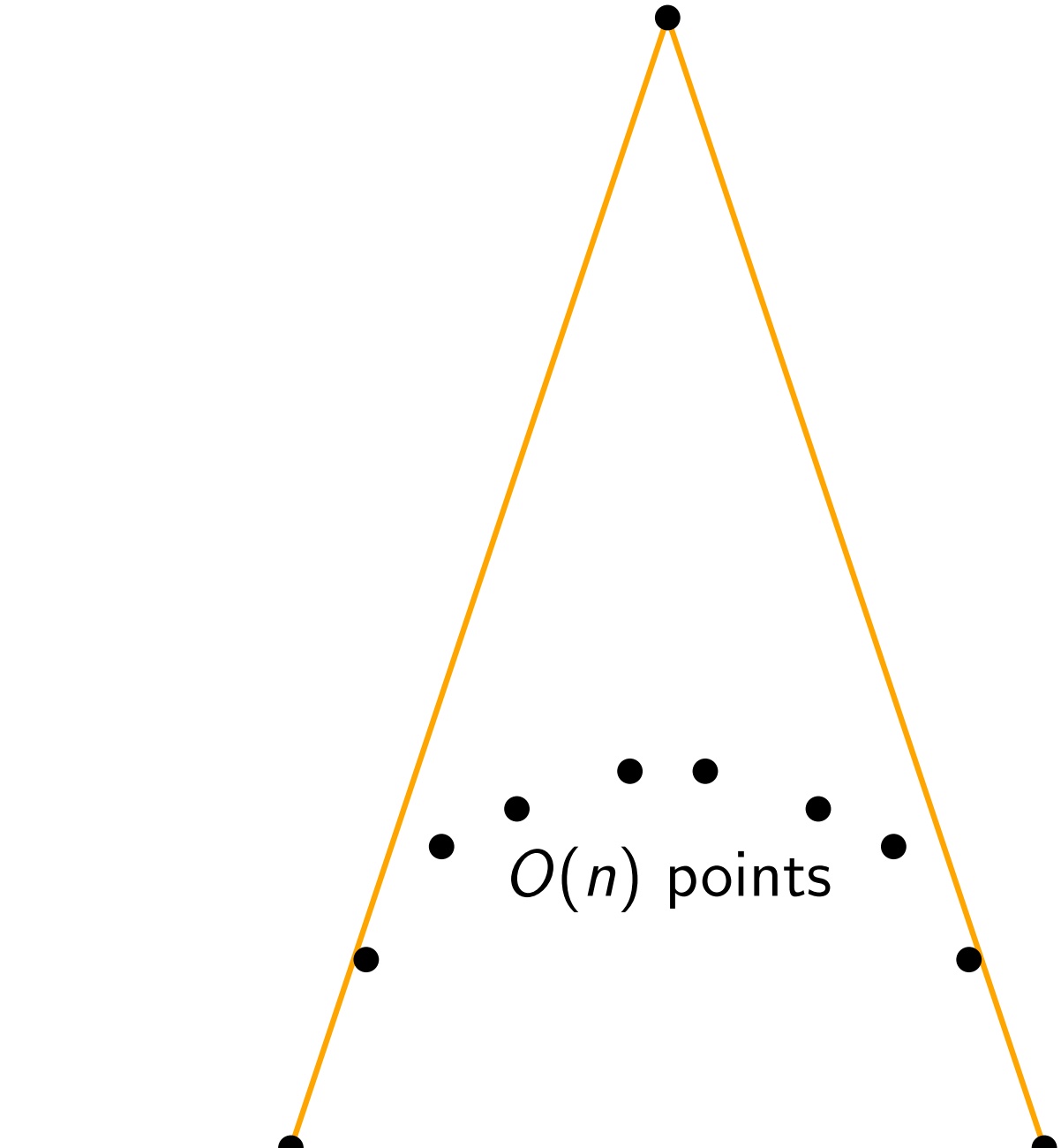

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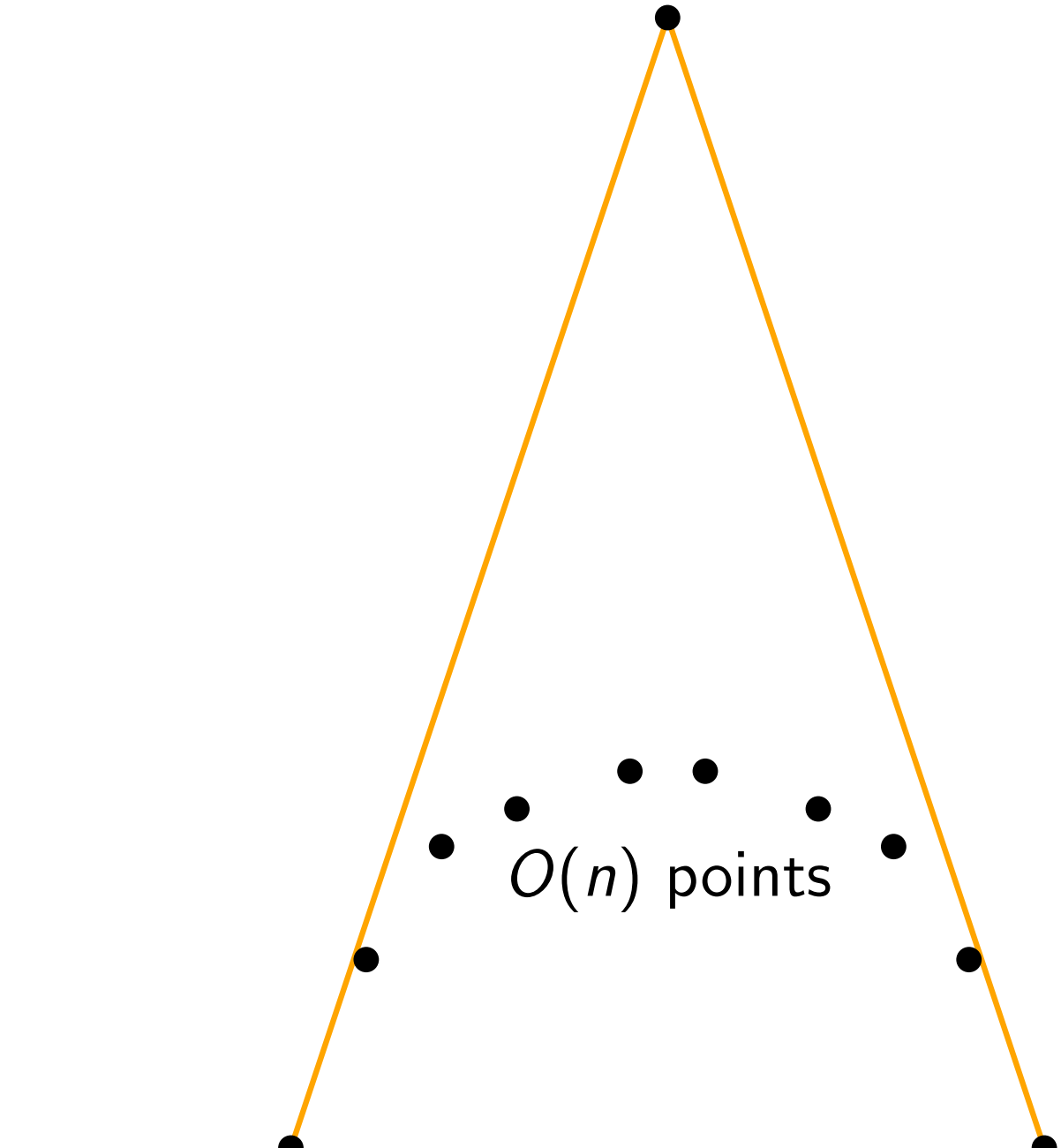

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Lemma: # of structural changes to upper hull after n FIFO updates is $O(n \log n)$.

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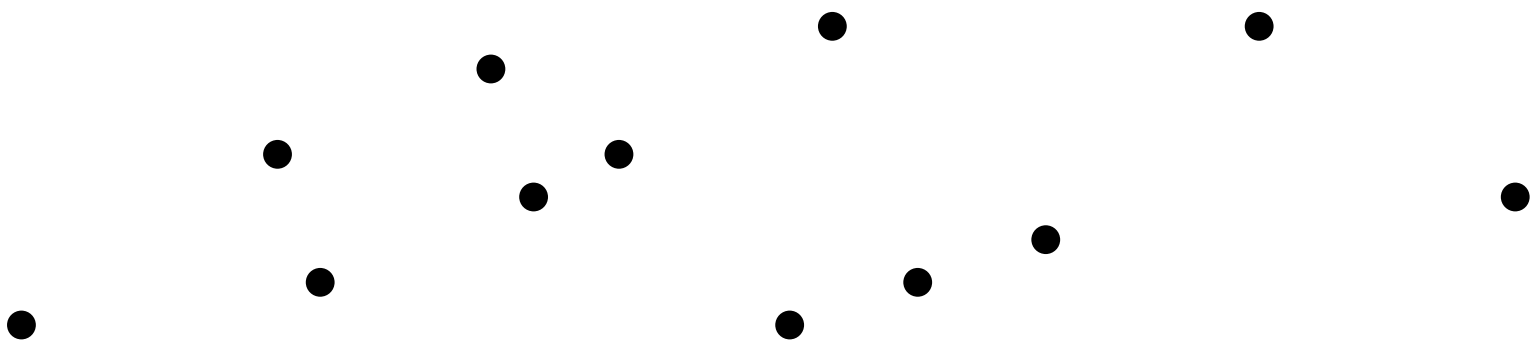

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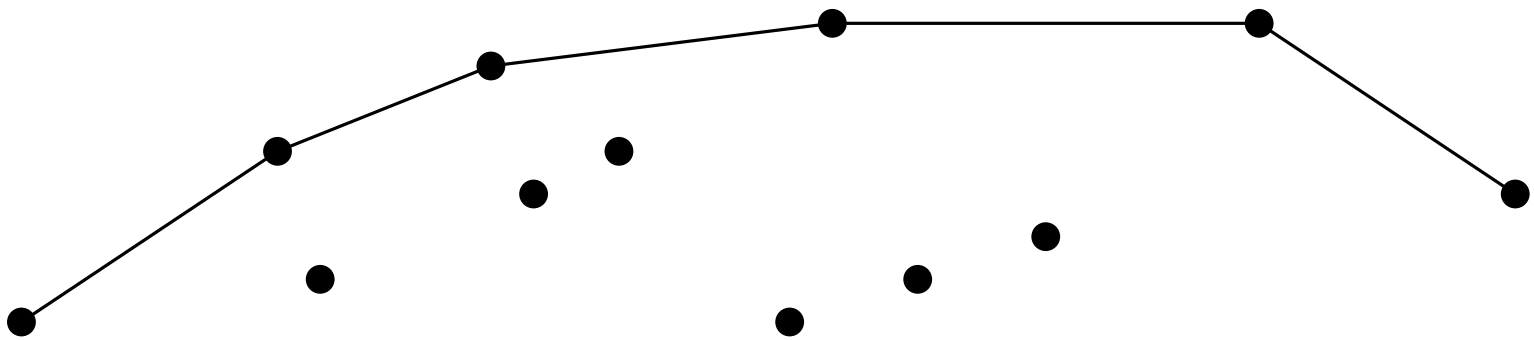

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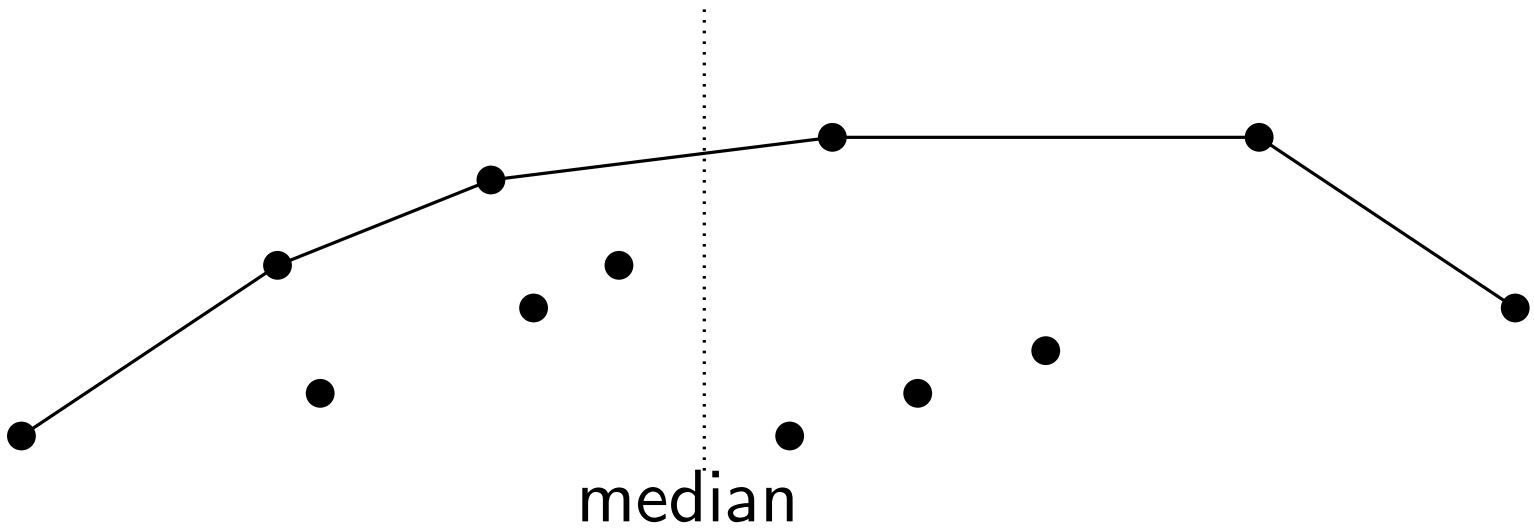

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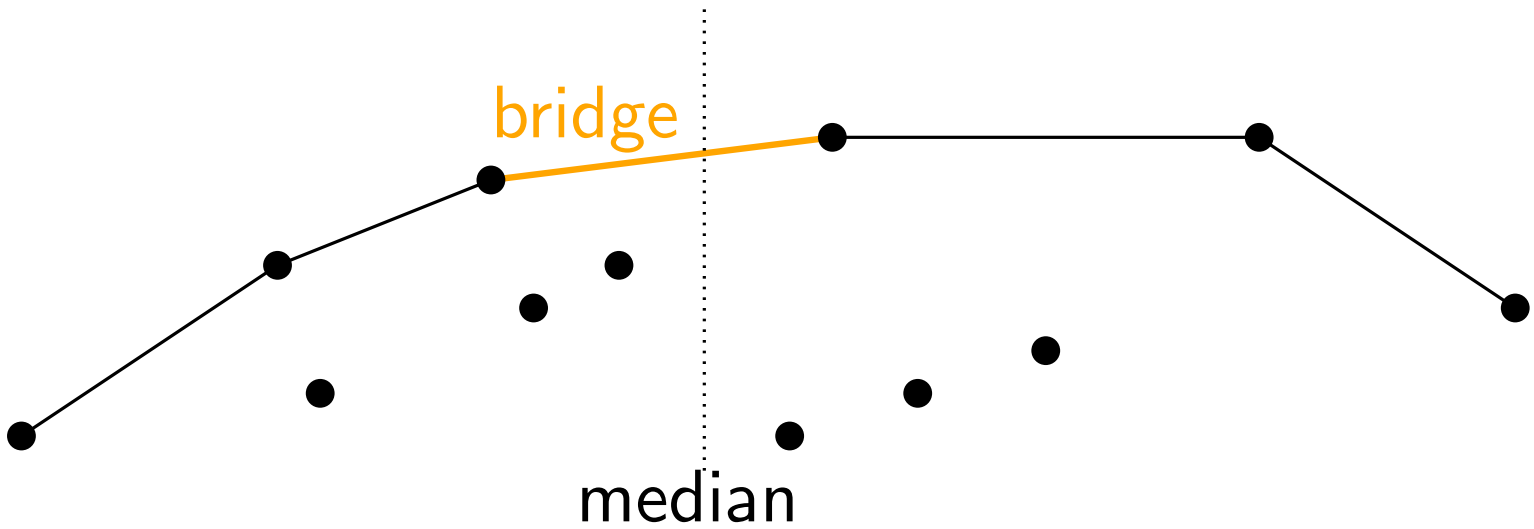

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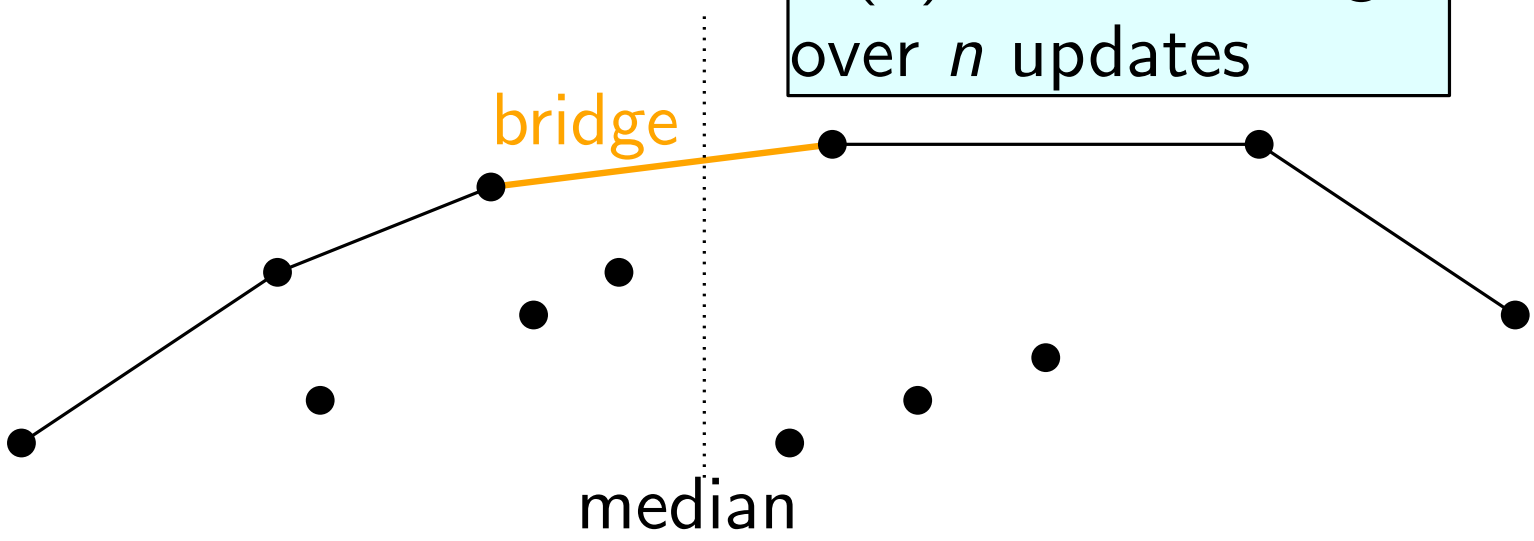

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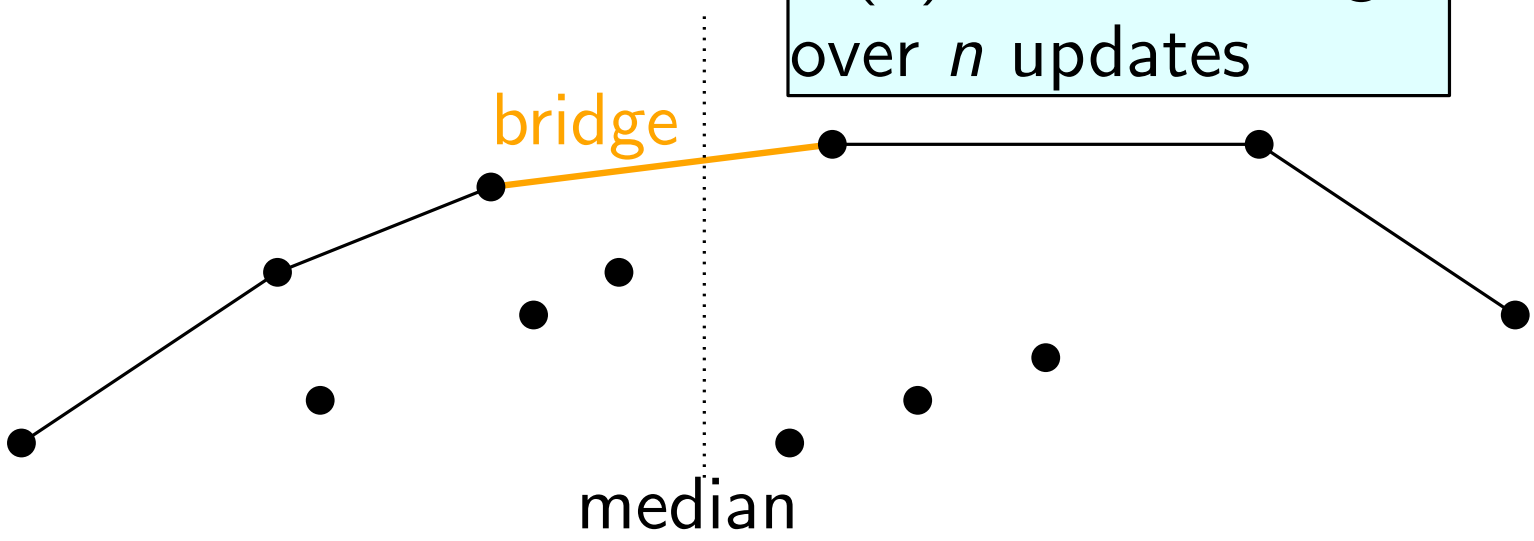

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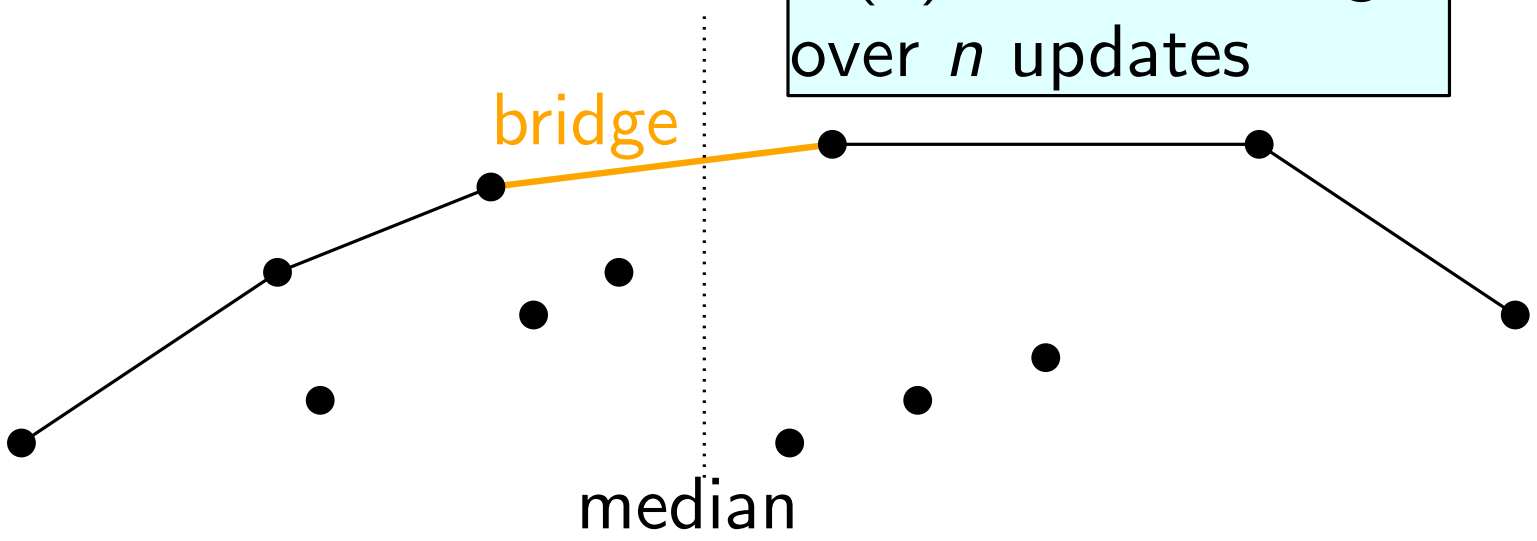

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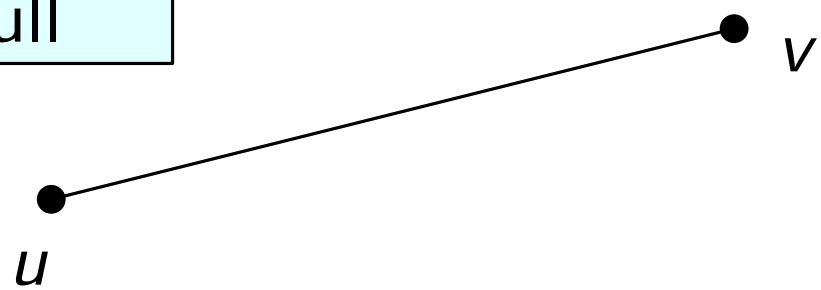

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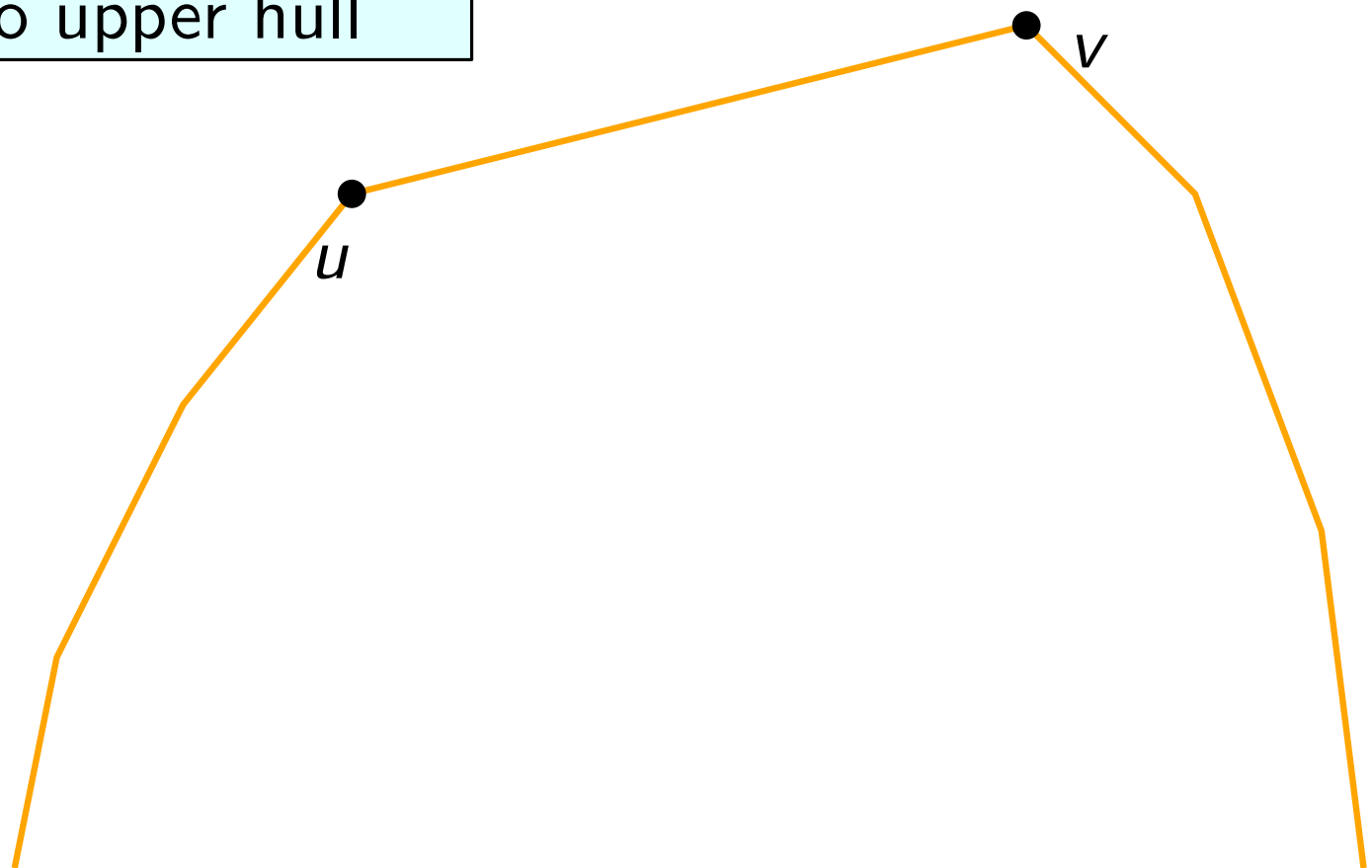

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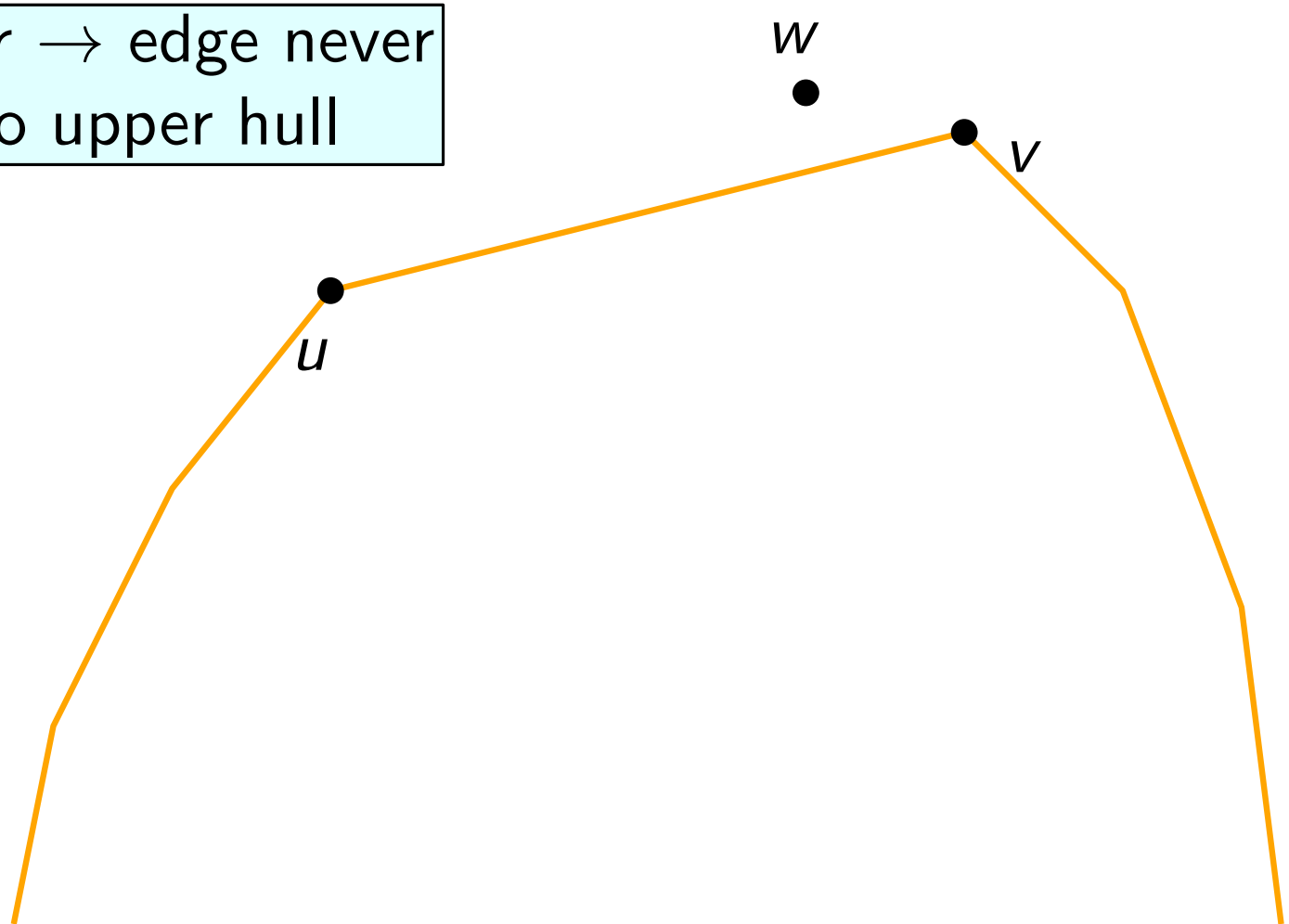

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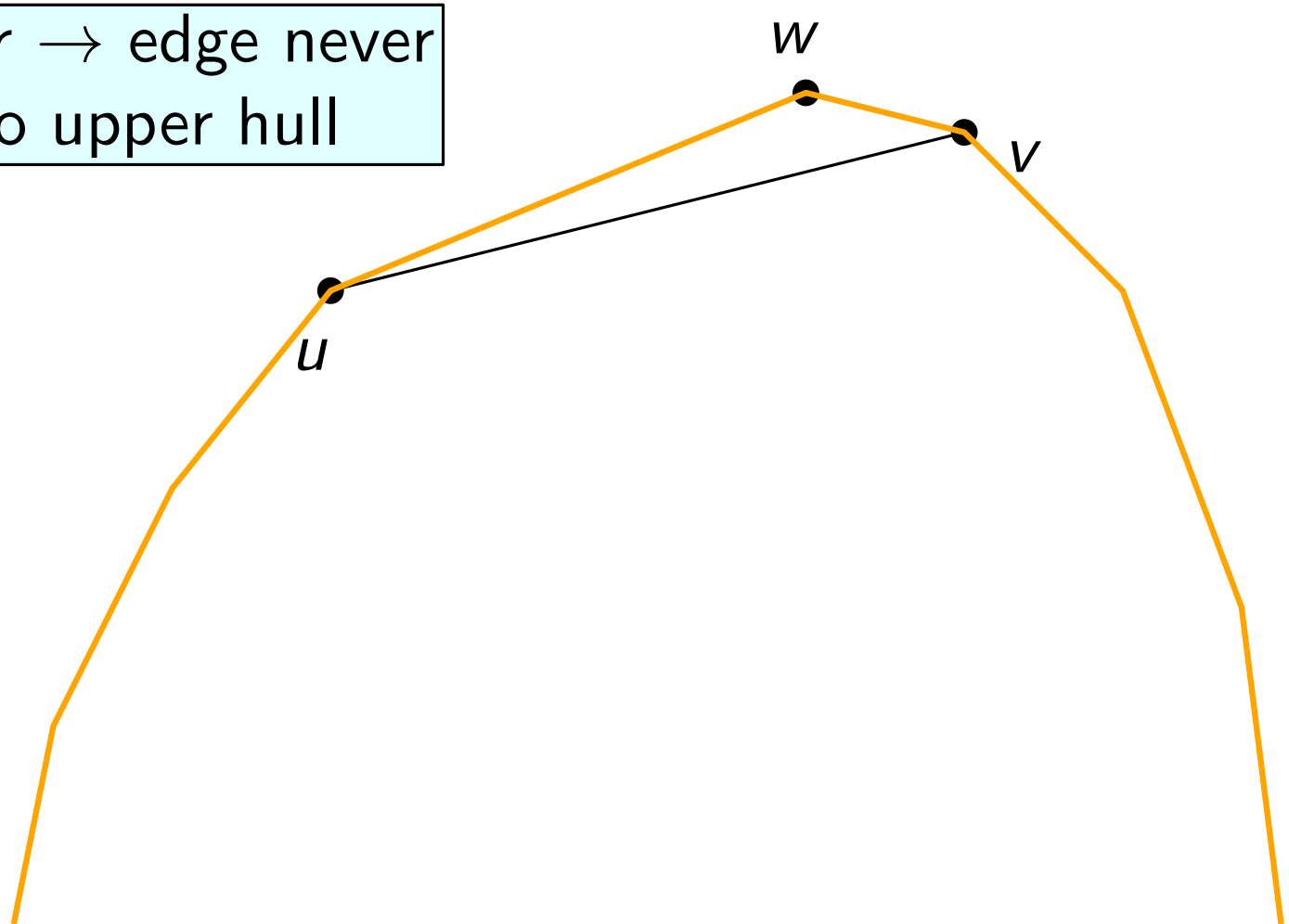

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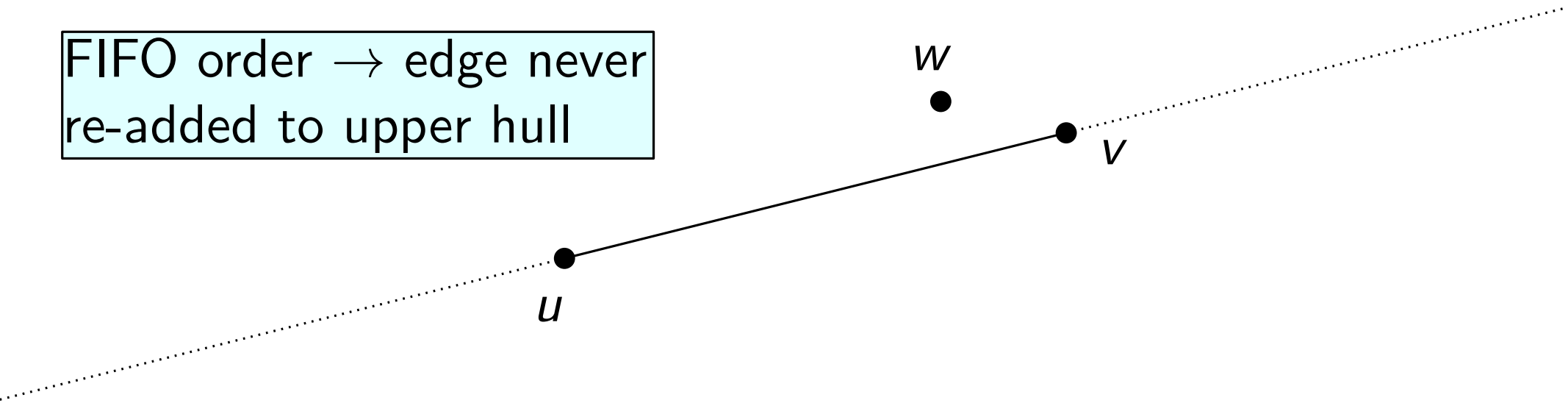

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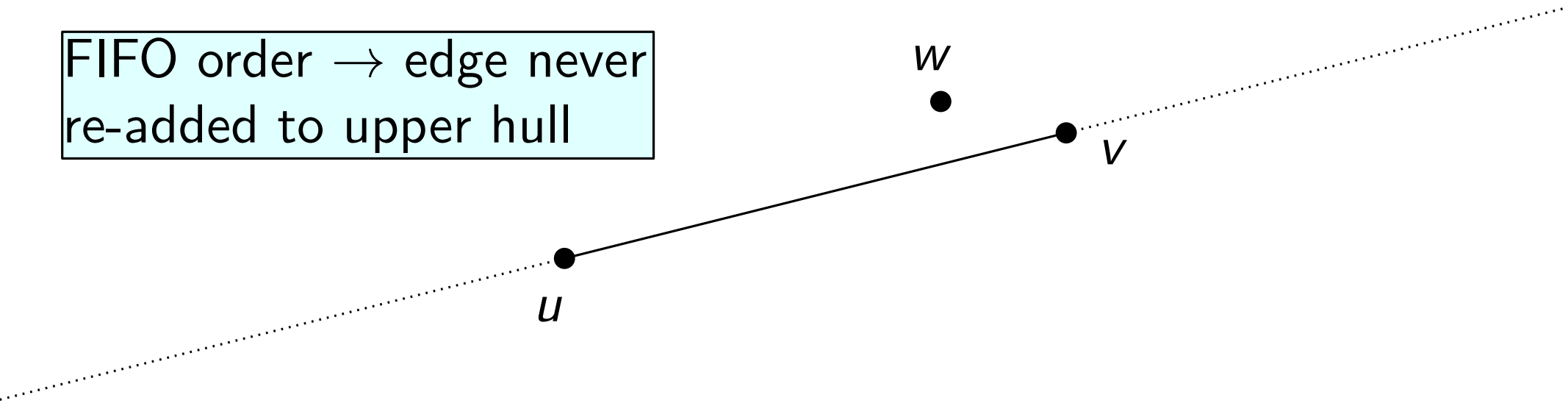

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Result:
The number of structural changes to the upper hull of a set of points in $\mathbb{R}^2$ over n FIFO updates is at most $O(n \log n)$.

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Can this combinatorial bound be improved? Perhaps to $O(n)$?